L7: Linear prediction of speech

- **Introduction**
- **Linear prediction**
- **Finding the linear prediction coefficients**
- **Alternative representations**

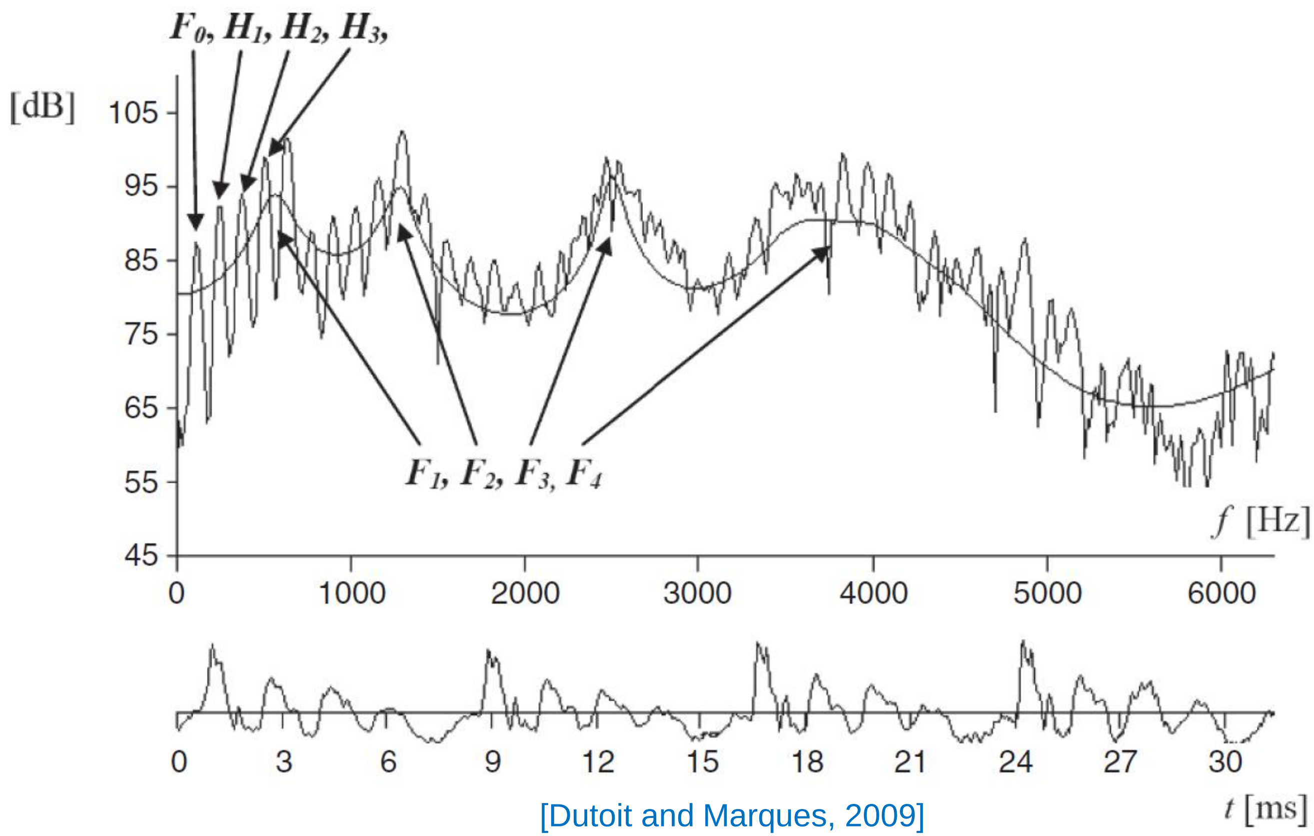
This lecture is based on [Dutoit and Marques, 2009, ch1; Taylor, 2009, ch. 12; Rabiner and Schaefer, 2007, ch. 6]

Introduction

Review of speech production

- Speech is produced by an excitation signal generated in the throat, which is modified by resonances due to the shape of the vocal, nasal and pharyngeal tracts
- The excitation signal can be
  - Glottal pulses created by periodic opening and closing of the vocal folds (voiced speech)
    - These periodic components are characterized by their fundamental frequency (F_0), whose perceptual correlate is the pitch
  - Continuous air flow pushed by the lungs (unvoiced speech)
  - A combination of the two
- Resonances in the vocal, nasal and pharyngeal tracts are called formants

– On a spectral plot for a speech frame
  - Pitch appears as narrow peaks for fundamental and harmonics
  - Formants appear as wide peaks in the spectral envelope



[Dutoit and Marques, 2009]

Linear prediction

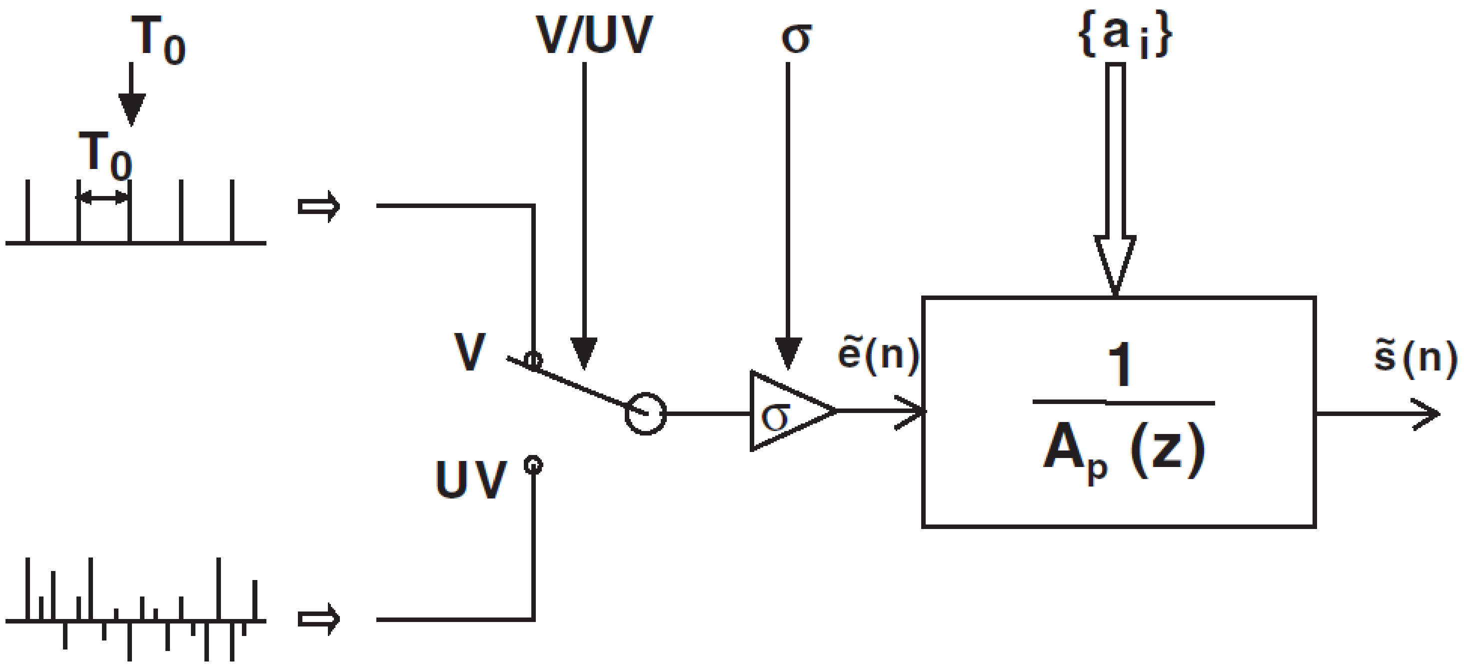
The source-filter model

– Originally proposed by Gunnar Fant in 1960 as a linear model of speech production in which glottis and vocal tract are fully uncoupled
– According to the model, the speech signal is the output $y[n]$ of an all-pole filer $1/A(z)$ excited by $x[n]$

$$Y(z) = X(z) \frac{1}{1 - \sum_{k=1}^{p} a_k z^{-k}} = X(z) \frac{1}{A_p(z)}$$

  - where $Y(z)$ and $X(z)$ are the z transforms of the speech and excitation signals, respectively, and p is the prediction order

– The filter $1/A_p(z)$ is known as the *synthesis filter*, and $\mathrm{A_p}(z)$ is called the *inverse filter*
– As discussed before, the excitation signal is either
  - A sequence of regularly spaced pulses, whose period T_0 and amplitude σ can be adjusted, or
  - White Gaussian noise, whose variance σ^2 can be adjusted



[Dutoit and Marques, 2009]

- The above equation implicitly introduces the concept of linear predictability, which gives name to the model
- Taking the inverse z-transform, the speech signal can be expressed as

$$y[n] = x[n] + \sum_{k=1}^{p} a_k \, y[n-k]$$

- which states that the speech sample can be modeled as a weighted sum of the p previous samples plus some excitation contribution

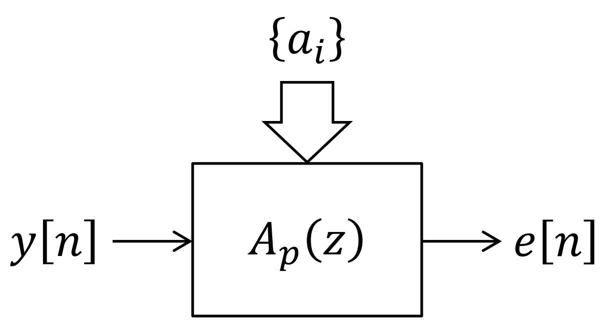
- In linear prediction, the term $x[n]$ is usually referred to as the error (or residual) and is often written as $e[n]$ to reflect this

Inverse filter

- For a given speech signal $x[n]$, and given the LP parameters $\{a_i\}$, the residual $e[n]$ can be estimated as

$$e[n] = y[n] - \sum_{k=1}^{p} a_k\, y[n-k]$$

  - which is simply the output of the inverse filter excited by the speech signal (see figure below)
- Hence, the LP model also allows us to obtain an estimate of the excitation signal that led to the speech signal
  - One will then expect that $e[n]$ will approximate a sequence of pulses (for voiced speech) or white Gaussian noise (for unvoiced speech)



[Dutoit and Marques, 2009]

Finding the LP coefficients

How do we estimate the LP parameters?

- We seek to estimate model parameters $\{a_i\}$ that minimize the expectation of the residual energy $e^2(n)$

$$\{a_i\}^{opt} = \arg\min[e^2(n)]$$

- Two closely related techniques are commonly used
  - the covariance method
  - the autocorrelation method

The covariance method

- Using the term E to denote the sum squared error, we can state

$$E = \sum_{n=0}^{N-1} e^2(n) = \sum_{n=0}^{N-1} \left(y[n] - \sum_{k=1}^{p} a_k \, y[n-k] \right)^2$$

- We can then find the minimum of E by differentiating with respect to each coefficient a_i and setting to zero

$$\frac{\partial E}{\partial a_j} = 0 \Rightarrow \sum_{n=0}^{N-1} \left(2 \left(y[n] - \sum_{k=1}^{p} a_k \, y[n-k] \right) y[n-j] \right) =$$

$$= -2 \sum_{n=0}^{N-1} y[n] y[n-j] + 2 \sum_{n=0}^{N-1} \sum_{k=1}^{p} a_k y[n-k] y[n-j] = 0$$

$$\forall j = 1,2, \ldots p$$

- which gives

$$\sum_{n=0}^{N-1} y[n] y[n-j] = 2 \sum_{k=1}^{p} a_k \sum_{n=0}^{N-1} y[n-k] y[n-j]$$

– Defining $\phi(j,k)$ as

$$\phi(j,k) = \sum_{n=0}^{N-1} y[n-j]y[n-k]$$

– This expression can be written more succinctly as

$$\phi(j,0) = \sum_{k=1}^{p} \phi(j,k)a_k$$

– Or in matrix notation as

$$\begin{bmatrix} \phi(1,0) \\ \phi(2,0) \\ \\ \phi(p,0) \end{bmatrix} = \begin{bmatrix} \phi(1,1) & \phi(1,2) & & \phi(1,p) \\ \phi(2,1) & \phi(2,2) & & \phi(2,p) \\ & & & \\ \phi(p,1) & \phi(p,2) & & \phi(p,p) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \\ a_p \end{bmatrix}$$

– or even more compactly as $\Phi = \Psi a$

– Since Φ is symmetric, this system of equations can be solved efficiently using Cholesky decomposition in $O(p^3)$

- NOTES
  - This method is known as the covariance method (for unclear reasons)
  - The method calculates the error in the region $0 \leq n < N-1$, but to do so uses speech samples in the region $-p \leq n < N-1$
    - Note that to estimate the error at $y[0]$, one needs samples up to $y[-p]$
  - No special windowing functions are needed for this method
  - If the signal follows an all-pole model, the covariance matrix can produce an exact solution
    - In contrast, the method we will see next is suboptimal, but leads to more efficient and stable solutions

The autocorrelation method

- The autocorrelation function of a signal can be defined as

$$R(n) = \sum_{m=-\infty}^{\infty} y[m]y[n-m]$$

- This expression is similar to that of $\phi(j,k)$ in the covariance method but extends over to $\pm\infty$ rather than to the range $0 \le n < N$

$$\phi(j,k) = \sum_{-\infty}^{\infty} y[n-j]y[n-k]$$

- To perform the calculation over $\pm\infty$, we window the speech signal (i.e., Hann), which sets to zero all values outside $0 \le n < N$
- Thus, all errors $e[n]$ will be zero before the window and p samples after the window, and the calculation of the error over $\pm\infty$ can be rewritten as

$$\phi(j,k) = \sum_{n=0}^{N-1+p} y[n-j]y[n-k]$$

- which in turn can be rewritten as

$$\phi(j,k) = \sum_{n=0}^{N-1-(j-k)} y[n]y[n+j-k]$$

- thus, $\phi(j,k) = R(j-k)$
- which allows us to write $\phi(j,0) = \sum_{k=1}^{p} \phi(j,k) a_k$ as

$$R(j) = \sum_{k=1}^{p} R(j-k) a_k$$

- The resulting matrix

$$\begin{bmatrix} R(1) \\ R(2) \\ \\ R(p) \end{bmatrix} = \begin{bmatrix} R(0) & R(1) & & R(p-1) \\ R(1) & R(0) & & R(p-2) \\ & & & \\ R(p-1) & R(p-2) & & R(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \\ a_p \end{bmatrix}$$

- is now a Toeplitz matrix (symmetric, with all elements on each diagonal being identical), which is significantly easier to invert
  - In particular, the Levinson-Durbin recursion provides a solution in $O(p^2)$

Speech spectral envelope and the LP filter

– The frequency response of the LP filter can be found by evaluating the transfer function on the unit circle at angles $2\pi f/f_s$, that is

$$\left|H\left(e^{j2\pi f/f_s}\right)\right|^2 = \left|\frac{G}{1-\sum_{k=1}^{p} a_k e^{-j2\pi kf/f_s}}\right|^2$$

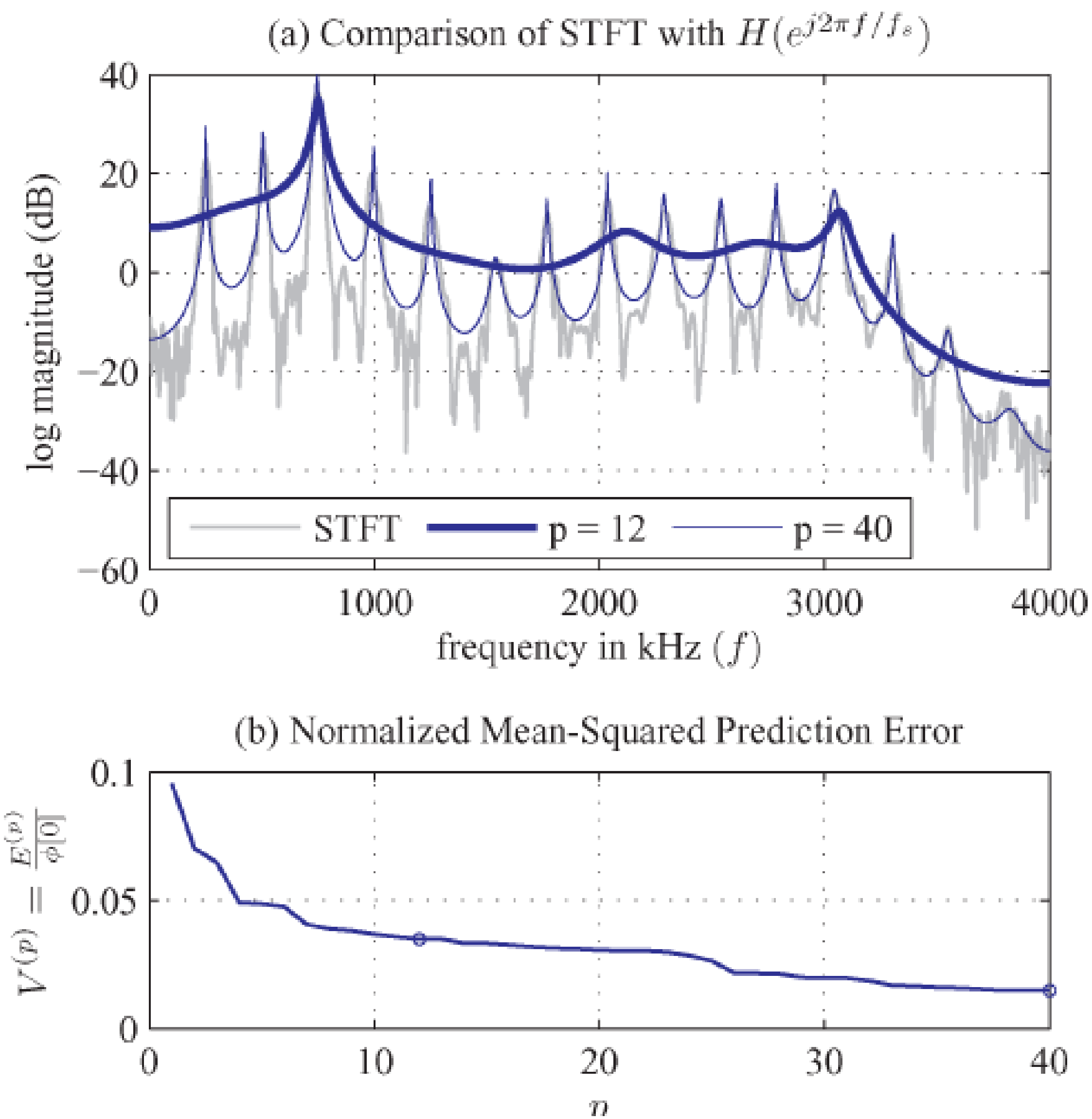
– Remember that this all-pole filter models the resonances of the vocal tract and that the glottal excitation is captured in the residual $e[n]$

– Therefore, the frequency response of $1/A_p(z)$ will be smooth and free of pitch harmonics

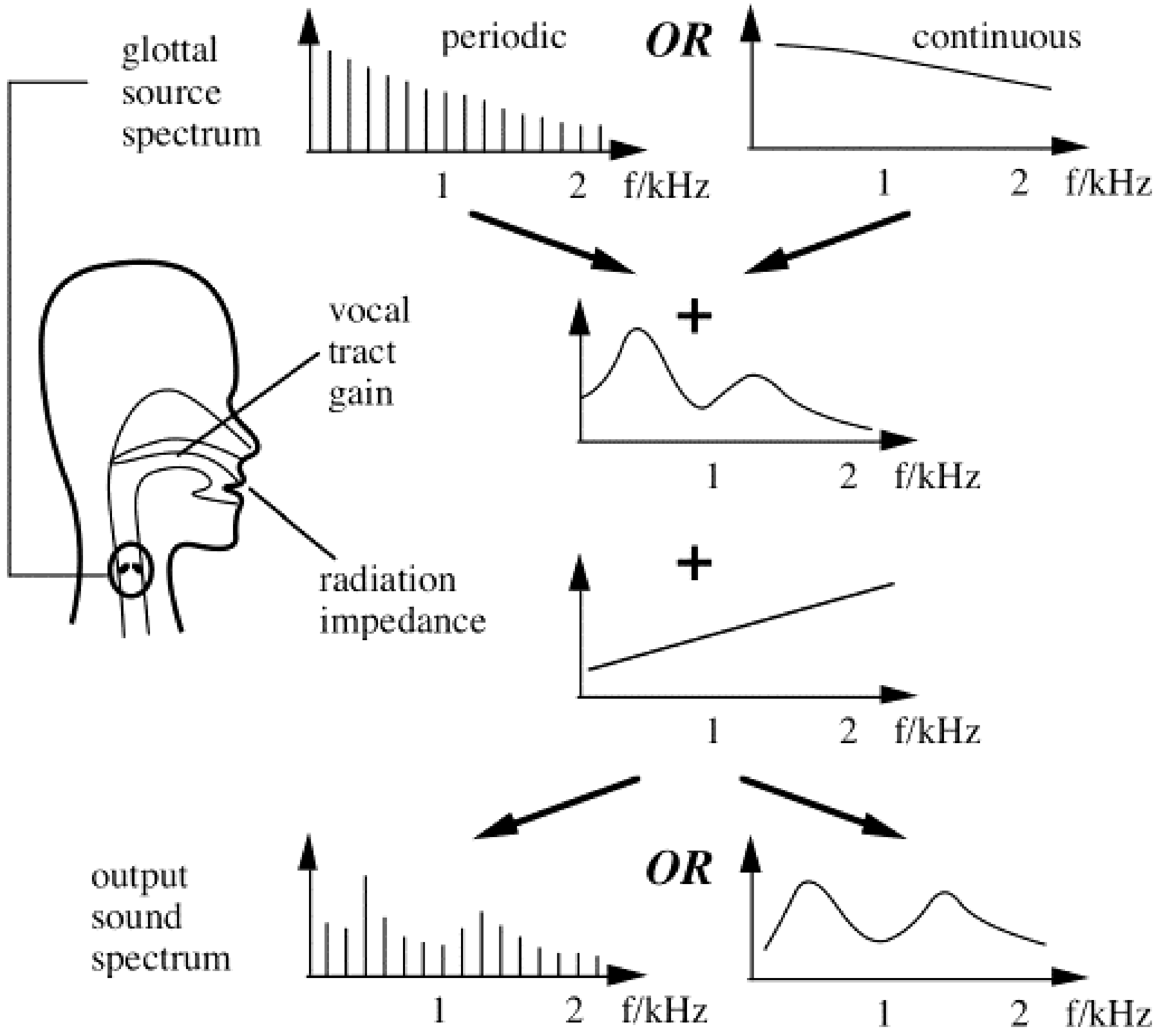
– This response is generally referred to as the spectral envelope

How many LP parameters should be used?

- The next slide shows the spectral envelope for $p = \{12, 40\}$, and the reduction in mean-squared error over a range of values
  - At $p = 12$ the spectral envelope captures the broad spectral peaks (i.e. the harmonics), whereas at $p = 40$ the spectral peaks also capture the harmonic structure
  - Notice also that the MSE curve flattens out above about $p = 12$ and then decreases modestly after
- Also consider the various factors that contribute to the speech spectra
  - Resonance structure comprising about one resonance per 1Khz, each resonance needing one complex pole pair
  - A low-pass glottal pulse spectrum, and a high-pass filter due to radiation at the lips, which can be modeled by 1-2 complex pole pairs
  - This leads to a rule of thumb of $p = 4 + f_s/1000$, or about 10-12 LP coefficients for a sampling rate of $f_s = 8kHz$



[Rabiner and Schafer, 2007]



http://www.phys.unsw.edu.au/jw/graphics/voice3.gif

Examples

ex7p1.m

- Computing linear predictive coefficients
- Estimating spectral envelope as a function of the number of LPC coefficients
- Inverse filtering with LPC filters
- Speech synthesis with simple excitation models (white noise and pulse trains)

ex7p2.m

- Repeat the above at the sentence level

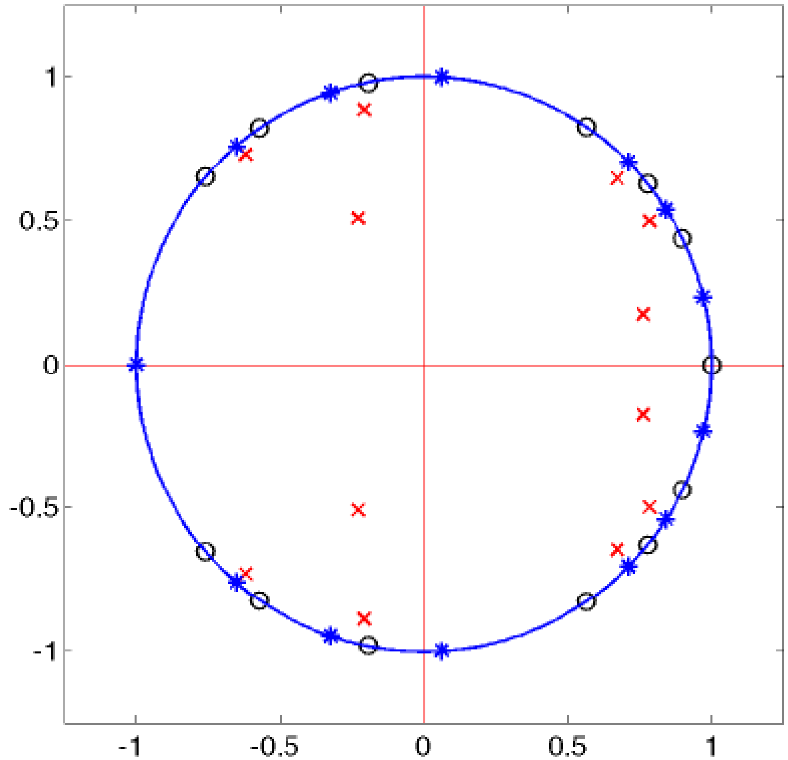
Alternative representations

A variety of different equivalent representations can be obtained from the parameters of the LP model

- This is important because the LP coefficients $\{a_i\}$ are hard to interpret and also too sensitive to numerical precision
- Here we review some of these alternative representations and how they can be derived from the LP model
  - Root pairs
  - Line spectrum frequencies
  - Reflection coefficients
  - Log-area ratio coefficients
- Additional representations (i.e., cepstrum, perceptual linear prediction) will be discussed in a different lecture

Root pairs

- The polynomial can be factored into complex pairs, each of which represents a resonance in the model
  - These roots (poles of the LP transfer function) are relatively stable and are numerically well behaved
- The example in the next slide shows the roots (marked with a $\times$) of a 12-th order model
  - Eight of the roots (4 pairs) are close to the unit circle, which indicates they model formant frequencies
  - The remaining four roots lie well within the unit circle, which means they only provide for the overall spectral shaping due to glottal and radiation influences



[Rabiner and Schafer, 2007]

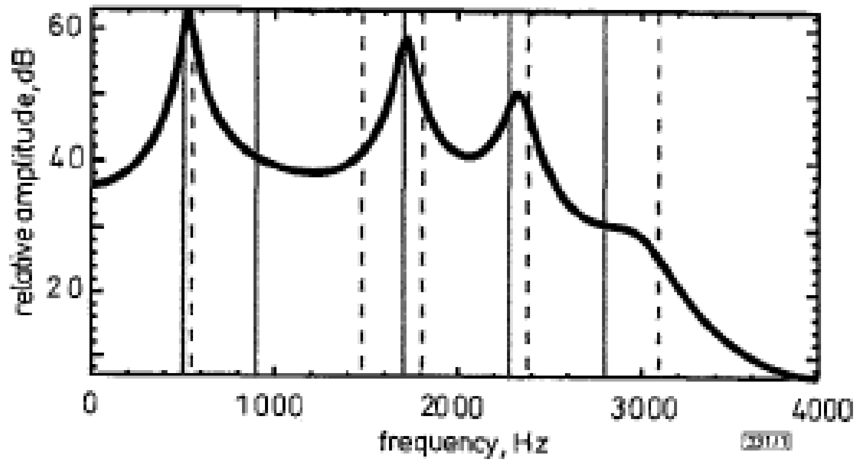


Fig. 1 *LPC spectral speech frame with LSPs overlaid*

[McLoughlin and Chance, 1997]

Line spectral frequencies (LSF)

- A more desirable alternative to quantization of the roots of $A_p(z)$ is based on the so-called line spectrum pair polynomials

$$P(z) = A(z) + z^{-(p+1)}A(z^{-1})$$
$$Q(z) = A(z) - z^{-(p+1)}A(z^{-1})$$

  - which, when added up, yield the original $A_p(z)$

- The roots of $P(z)$, $Q(z)$ and $A_p(z)$ are shown in the previous slide
  - All the roots of $P(z)$ and $Q(z)$ are on the unit circle and their frequencies (angles in the z-plane) are known as the line spectral frequencies
  - The LSFs are close together when the roots of $A_p(z)$ are close to the unit circle; in other words, presence of two close LSFs is indicative of a strong resonance (see previous slide)
  - LSFs are not overly sensitive to quantization noise and are also stable, so they are widely used for quantizing LP filters

Reflection coefficients

- The reflection coefficients represent the fraction of energy reflected at each section of a non-uniform tube model of the vocal tract
- They are a popular choice of LP representation for various reasons
  - They are easily computed as a by-product of the Levinson-Durbin iteration
  - They are robust to quantization error
  - They have a physical interpretation, making then amenable to interpolation
- Reflection coefficients may be obtained from the predictor coefficients through the following backward recursion

$$r_i = a_i^i \quad \forall i = p, \dots, 1$$

$$a_j^{i-1} = \frac{a_j^i + a_i^i a_{i-j}^i}{1 - r_i^2} \quad 1 \le j < i$$

  - where we initialize $a_i^p = a_i$

Log-area ratios

- Log-area ratio coefficients are the natural logarithm of the ratio of the areas of adjacent sections of a lossless tube equivalent to the vocal tract (i.e., both having the same transfer function)
  - While it is possible to estimate the ratio of adjacent sections, it is not possible to find the absolute values of those areas

- Log-area ratios can be found from the reflection coefficients as

$$A_k = \ln\left(\frac{1 - r_k}{1 + r_k}\right)$$

  - where g_k is the LAR and r_k is the corresponding reflection coefficient