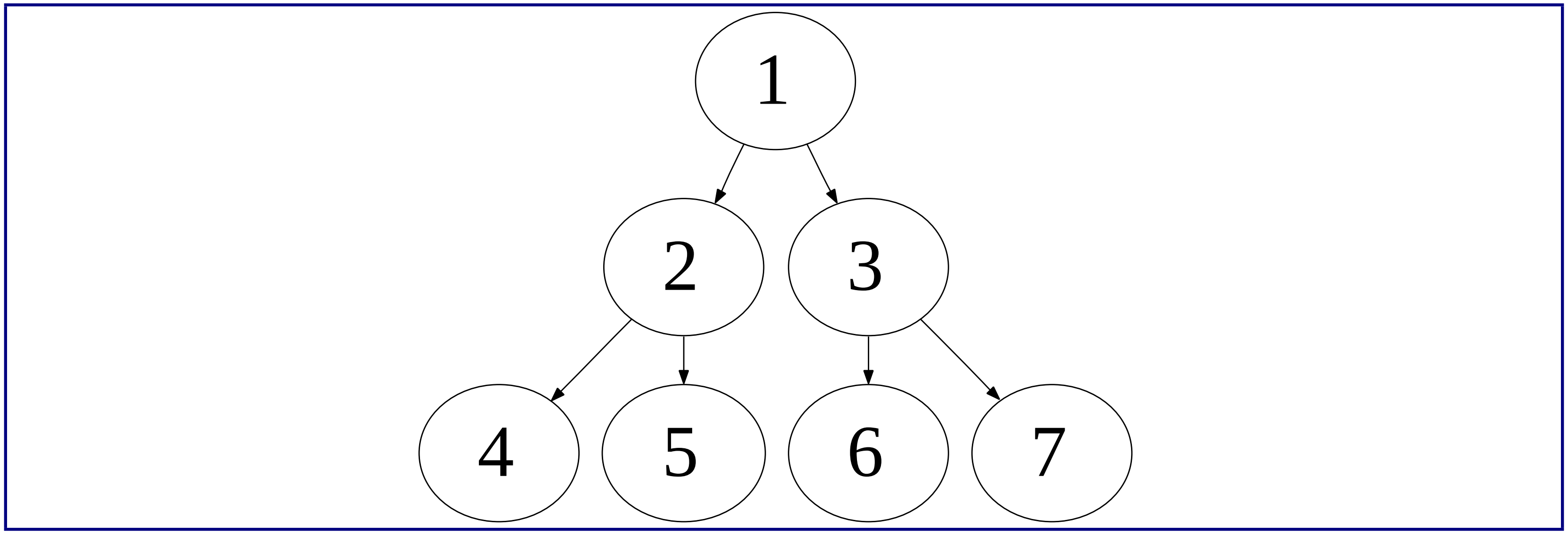
Search and Game Playing

- CSCE 315 Programming Studio
- Material drawn from Gordon Novak's AI course, Yoonsuck Choe's AI course, and Russell and Norvig's *Artificial Intelligence, A Modern Approach*, 2nd edition.

Overview

- Search problems: definition
- Example: 8-puzzle
- General search
- Evaluation of search strategies
- Strategies: breadth-first, uniform-cost, depth-first
- More uninformed search: depth-limited, iterative deepening, bidirectional search

Search Problems: Definition



Search = $<$ initial state, operators, goal states $>$

- Initial State: description of the current situation as given in a problem
- Operators: functions from any state to a set of successor (or neighbor) states
- Goal: subset of states, or test rule

Variants of Search Problems

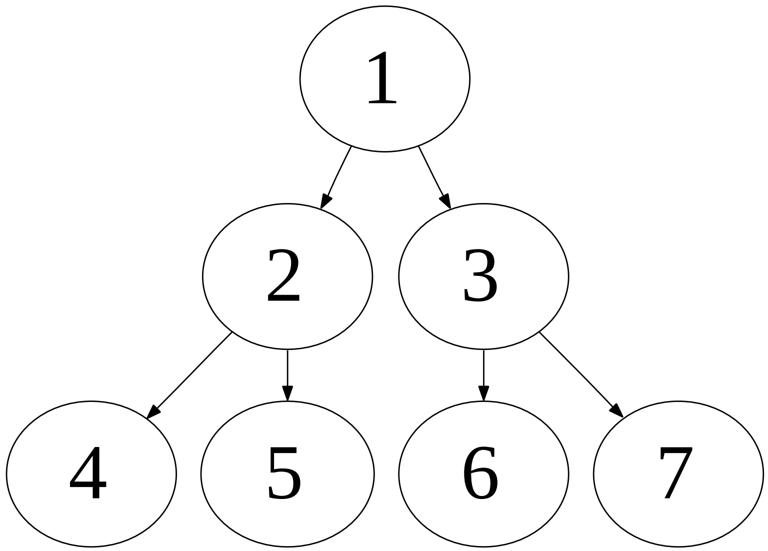
Search = $<$ state space, initial state, operators, goal states $>$

- State space: set of all possible states reachable from the current initial state through repeated application of the operators (i.e. path).

Search = $<$ initial state, operators, goal states, path cost $>$

- Path cost: find **the best** solution, not just **a** solution. Cost can be many different things.

Types of Search



- Uninformed: systematic strategies
- Informed: Use domain knowledge to narrow search
- Game playing as search: minimax, state pruning, probabilistic games

Search State

State as Data Structure

- examples: variable assignment, properties, order in list, bitmap, graph (vertex and edges)
- captures all possible ways world could be
- typically static, discrete (symbolic), but doe snot have to be

Choosing a Good Representation

- concise (keep only the relevant features)
- explicit (easy to compute when needed)
- embeds constraints

Operators

Function from state to subset of states

- drive to neighboring city
- place piece on chess board
- add person to meeting schedule
- slide tile in 8-puzzle

Characteristics

- often requires instantiation (fill in variables)
- encode constraints (only certain operations are allowed)
- generally discrete: continuous parameters $\longrightarrow$ infinite branching

Goals: Subset of states or test rules

Specification:

- set of states: enumerate the eligible states
- partial description: e.g. a certain variable has value over x.
- constraints: or set of constraints. Hard to enumerate all states matching the constraints, or very hard to come up with a solution at all (i.e. you can only verify it; P vs. NP).

Other considerations:

- space, time, quality (exact vs. approximate trade-offs)

An Example: 8-Puzzle

5	4	
6	1	8
7	3	2

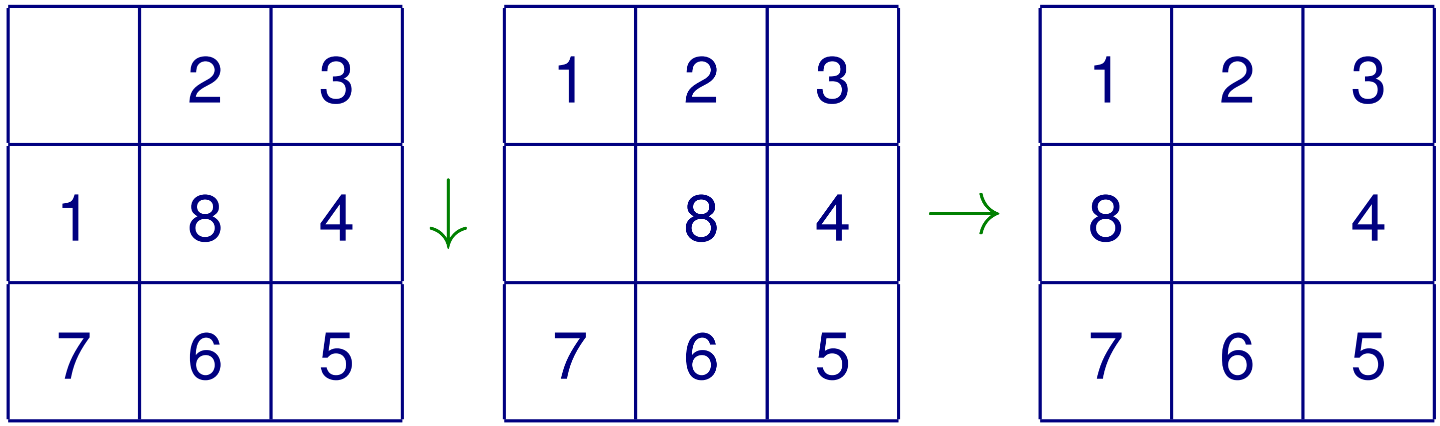
$\rightarrow \ldots \uparrow \ldots \leftarrow \ldots \downarrow$

1	2	3
8		4
7	6	5

- **State**: location of 8 number tiles and one blank tile
- **Operators**: blank moves left, right, up, or down
- **Goal test**: state matches the configuration on the right (see above)
- **Path cost**: each step cost 1, i.e. path length, or search tree depth

Generalization: 15-puzzle, ..., $(N^2 - 1)$-puzzle

8-Puzzle: Example

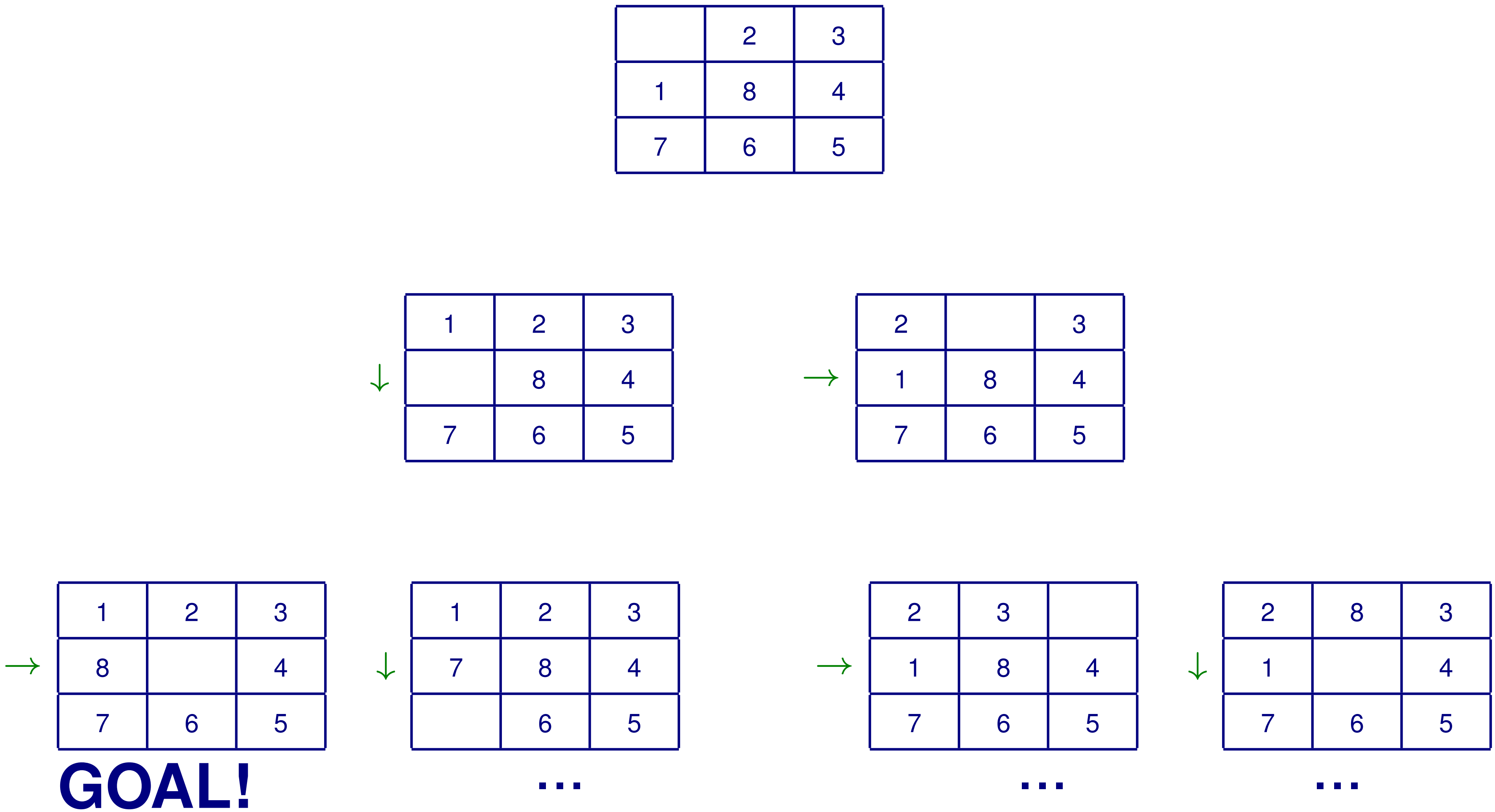


Possible state representations in LISP (0 is the blank):

- `(0 2 3 1 8 4 7 6 5)`
- `((0 2 3) (1 8 4) (7 6 5))`
- `((0 1 7) (2 8 6) (3 4 5))`
- or use the `make-array`, `aref` functions.

How easy to: (1) compare, (2) operate on, and (3) store (i.e. size).

8-Puzzle: Search Tree



General Search Algorithm

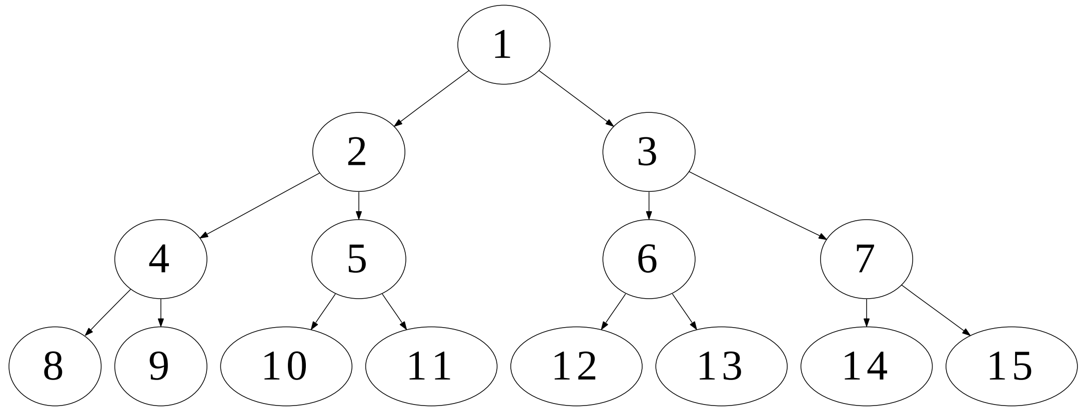
Pseudo-code:

```
function General-Search (problem, Que-Fn)
  node-list := initial-state
  loop begin
      // fail if node-list is empty
      if Empty(node-list) then return FAIL
      // pick a node from node-list
      node := Get-First-Node(node-list)
      // if picked node is a goal node, success!
      if (node == goal) then return as SOLUTION
      // otherwise, expand node and enqueue
      node-list := Que-Fn(node-list, Expand(node))
  loop end
```

Evaluation of Search Strategies

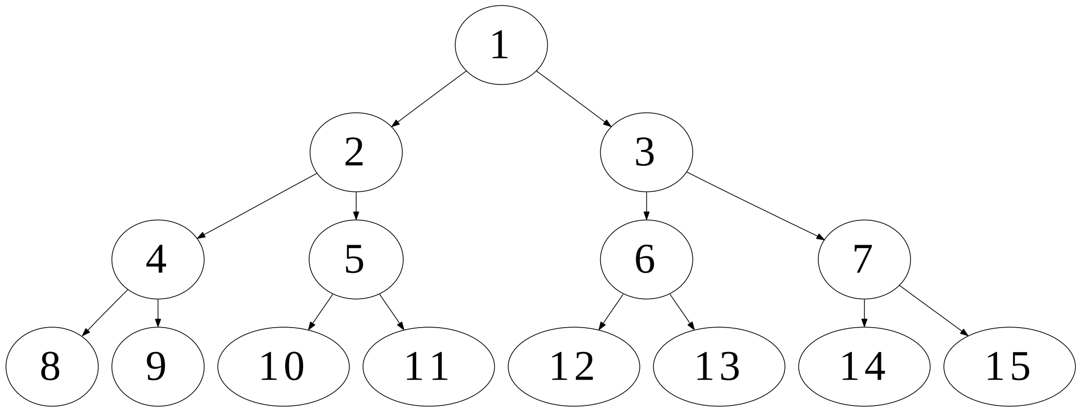
- time-complexity: how many nodes expanded so far?
- space-complexity: how many nodes must be stored in node-list at any given time?
- completeness: if solution exists, guaranteed to be found?
- optimality: guaranteed to find the best solution?

Breadth First Search



- node visit order (goal test): 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
- queuing function: enqueue at end (add expanded node at the end of the list)
- Important: A node taken out of the node list for inspection counts as a single visit!

BFS: Expand Order



Evolution of the queue (**bold**= expanded and added children):

1. **[1]** : initial state
2. **[2][3]** : dequeue 1 and enqueue 2 and 3
3. [3] **[4][5]** : dequeue 2 and enqueue 4 and 5
4. [4] [5] **[6][7]** : **all** depth 3 nodes

...

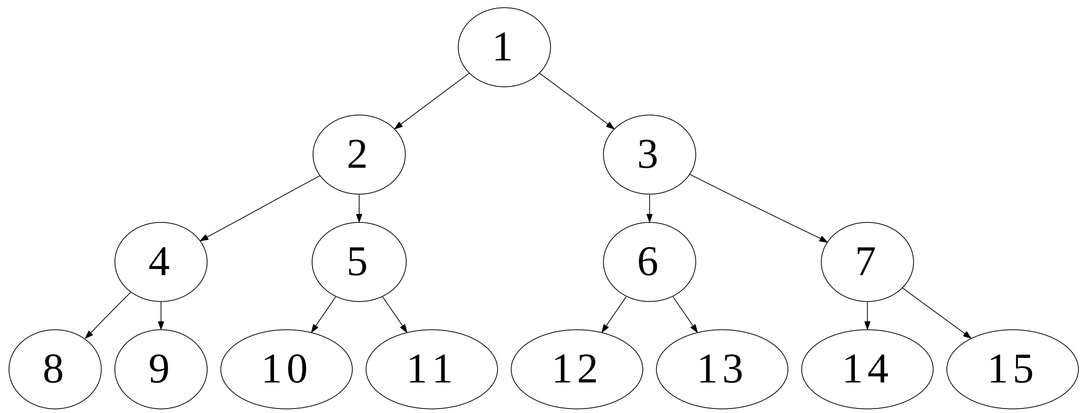
8. [8] [9] [10] [11] [12] [13] **[14][15]** : **all** depth 4 nodes

BFS: Evaluation

branching factor b, depth of solution d:

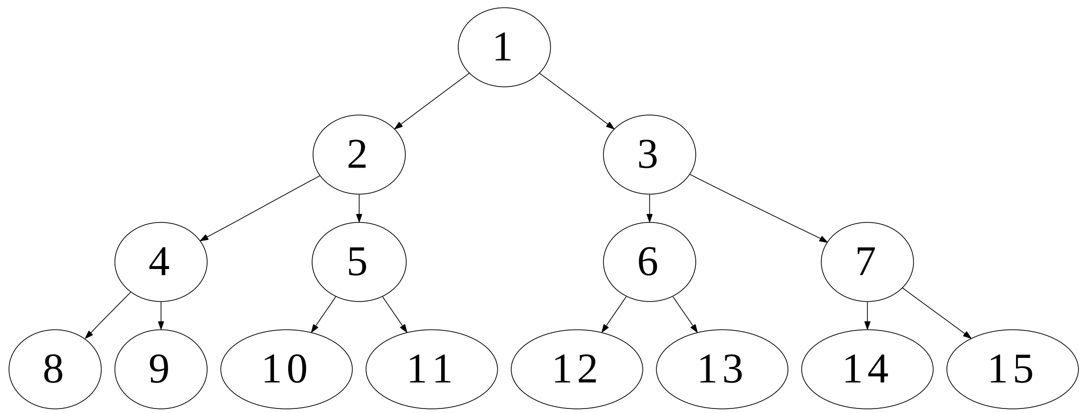
- complete: it will find the solution if it exists
- time: $1 + b + b^2 + ... + b^d$
- space: $O(b^{d+1})$ where d is the depth of the shallowest solution
- space is more problem than time in most cases (p 75, figure 3.12).
- time is also a major problem nonetheless (same as time)

Depth First Search



- node visit order (goal test): 1 2 4 8 9 5 10 11 3 6 12 13 7 14 15
- queuing function: enqueue at left (stack push; add expanded node at the beginning of the list)

DFS: Expand Order



Evolution of the queue (**bold**=expanded and added children):

1. [1] : initial state
2. **[2][3]** : pop 1 and push expanded in the front
3. **[4][5]** [3] : pop 2 and push expanded in the front
4. **[8][9]** [5] [3] : pop 4 and push expanded in the front

DFS: Evaluation

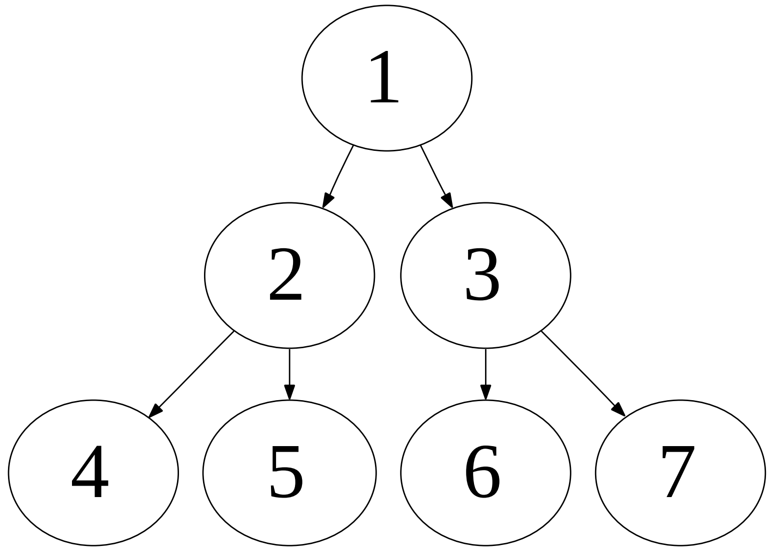
branching factor b, depth of solutions d, max depth m:

- incomplete: may wander down the wrong path
- time: $O(b^m)$ nodes expanded (worst case)
- space: $O(bm)$ (just along the current path)
- good when there are many shallow goals
- bad for deep or infinite depth state space

Key Points

- Description of a search problem: initial state, goals, operators, etc.
- Considerations in designing a representation for a state
- Evaluation criteria
- BFS, DFS: time and space complexity, completeness
- When to use one vs. another
- Node visit orders for each strategy
- Tracking the stack or queue at any moment

Depth Limited Search (DLS): Limited Depth DFS



- node visit order for each depth limit l:

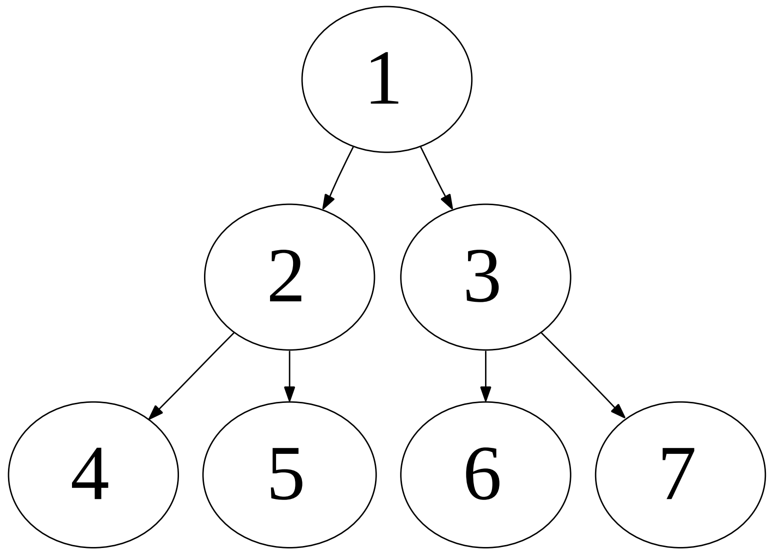
  1 ($l = 1$); 1 2 3 ($l = 2$); 1 2 4 5 3 6 7 ($l = 3$);

- queuing function: enqueue at front (i.e. stack push)

- **push the depth of the node as well**:

  ($<$depth$>$ $<$node$>$)

DLS: Expand Order



Evolution of the queue (**bold**=expanded and then added):

(<depth>,<node>)); Depth limit = 3

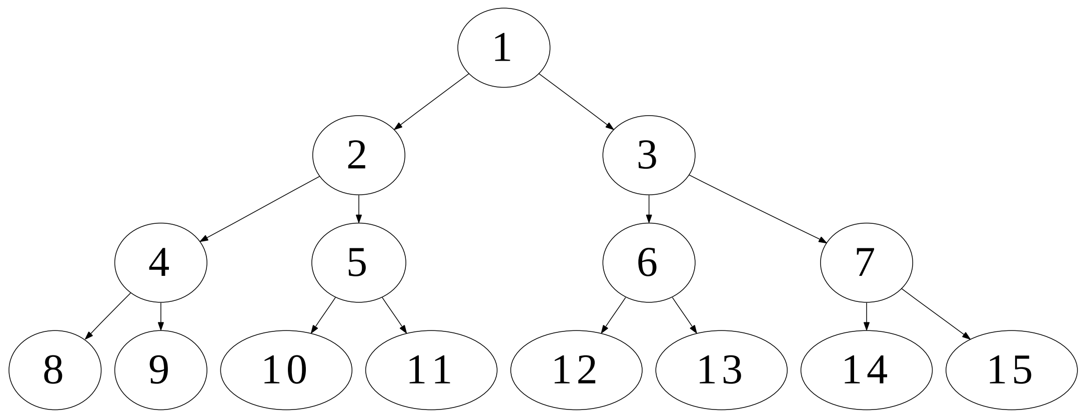
1. [(d1,1)] : initial state
2. **[(d2,2)][(d2,3)]** : pop 1 and push 2 and 3
3. **[(d3,4)][(d3,5)]**[(d2,3)] : pop 2 and push 4 and 5
4. [(d3,5)][(d2,3)]: pop 4, cannot expand it further
5. [(d2,3)]: pop 5, cannot expand it further
6. **[(d3,6)][(d3,7)]**: pop 3, and push 6, 7

DLS: Evaluation

branching factor b, depth limit l, depth of solution d:

- complete: if $l \geq d$
- time: $O(b^l)$ nodes expanded (worst case)
- space: $O(bl)$ (same as DFS, where $l = m$ (m: max depth of tree in DFS)
- good if solution is within the limited depth.
- non-optimal (same problem as in DFS).

Iterative Deepening Search: DLS by Increasing Limit

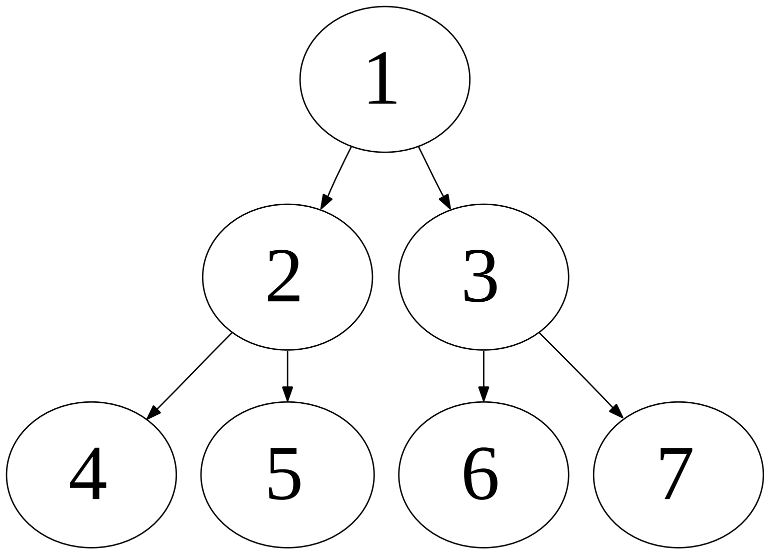


- node visit order:

  1 ; 1 2 3; 1 2 4 5 3 6 7; 1 2 4 8 9 5 10 11 3 6 12 13 7 14 15; ...

- revisits already explored nodes at successive depth limit
- queuing function: enqueue at front (i.e. stack push)
- **push the depth of the node as well**: **(<depth> <node>)**

IDS: Expand Order



Basically the same as DLS: Evolution of the queue (**bold**=expanded and then added): `(<depth>,<node>)`); e.g. Depth limit = 3

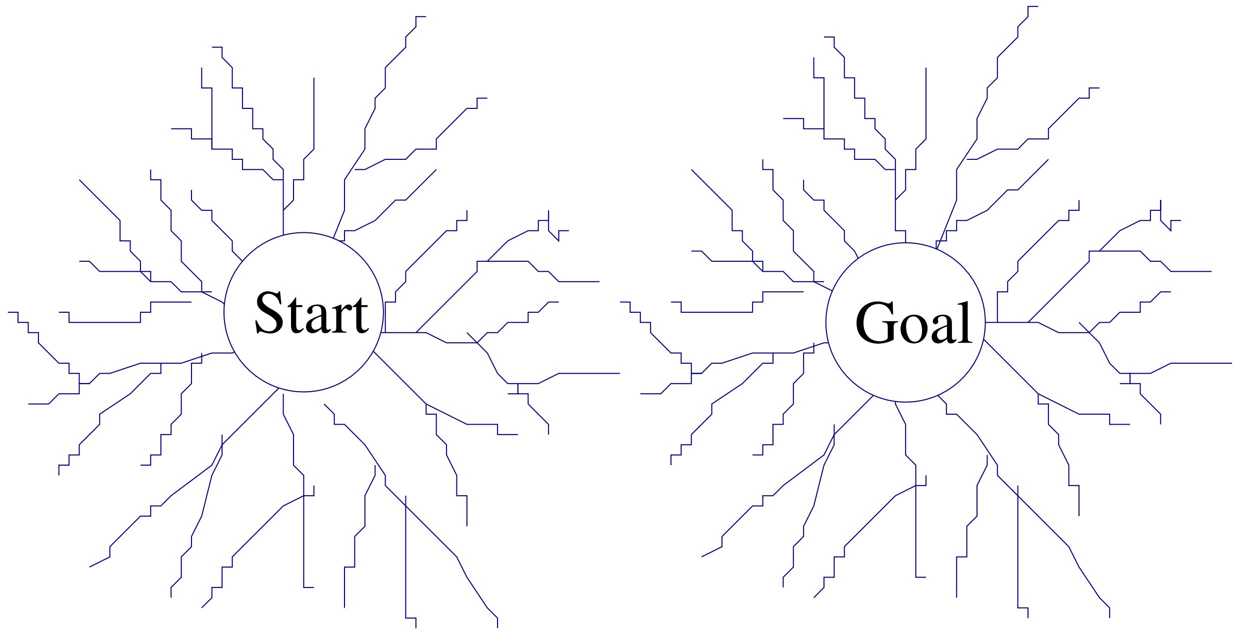
1. `[(d1,1)]` : initial state
2. **[(d2,2)][(d2,3)]** : pop 1 and push 2 and 3
3. **[(d3,4)][(d3,5)]**`[(d2,3)]` : pop 2 and push 4 and 5
4. `[(d3,5)][(d2,3)]`: pop 4, cannot expand it further
5. `[(d2,3)]`: pop 5, cannot expand it further
6. **[(d3,6)][(d3,7)]**: pop 3, and push 6, 7

IDS: Evaluation

branching factor b, depth of solution d:

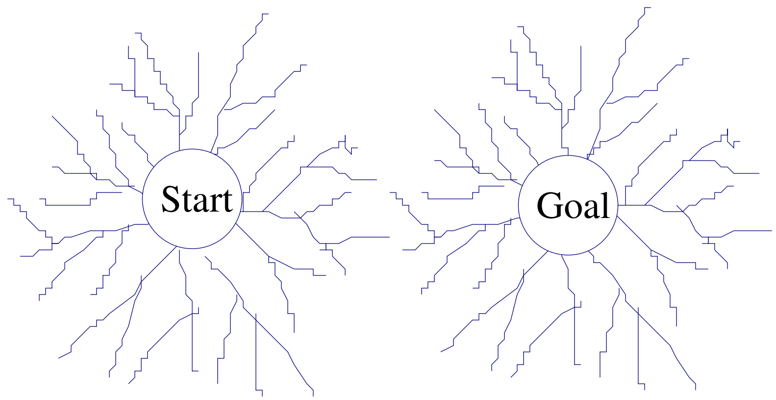
- complete: cf. DLS, which is conditionally complete
- time: $O(b^d)$ nodes expanded (worst case)
- space: $O(bd)$ (cf. DFS and DLS)
- **optimal!**: unlike DFS or DLS
- good when search space is huge and the depth of the solution is not known (*)

Bidirectional Search (BDS)



- Search from both initial state and goal to reduce search depth.
- $O(b^{d/2})$ of BDS vs. $O(b^{d+1})$ of BFS.

BDS: Considerations



1. how to back trace from the goal?
2. successors and predecessors: are operations reversible?
3. are goals explicit?: need to know the goal to begin with
4. check overlap in two branches
5. BFS? DFS? which strategy to use? Same or different?

BDS Example: 8-Puzzle

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

→ ... ←

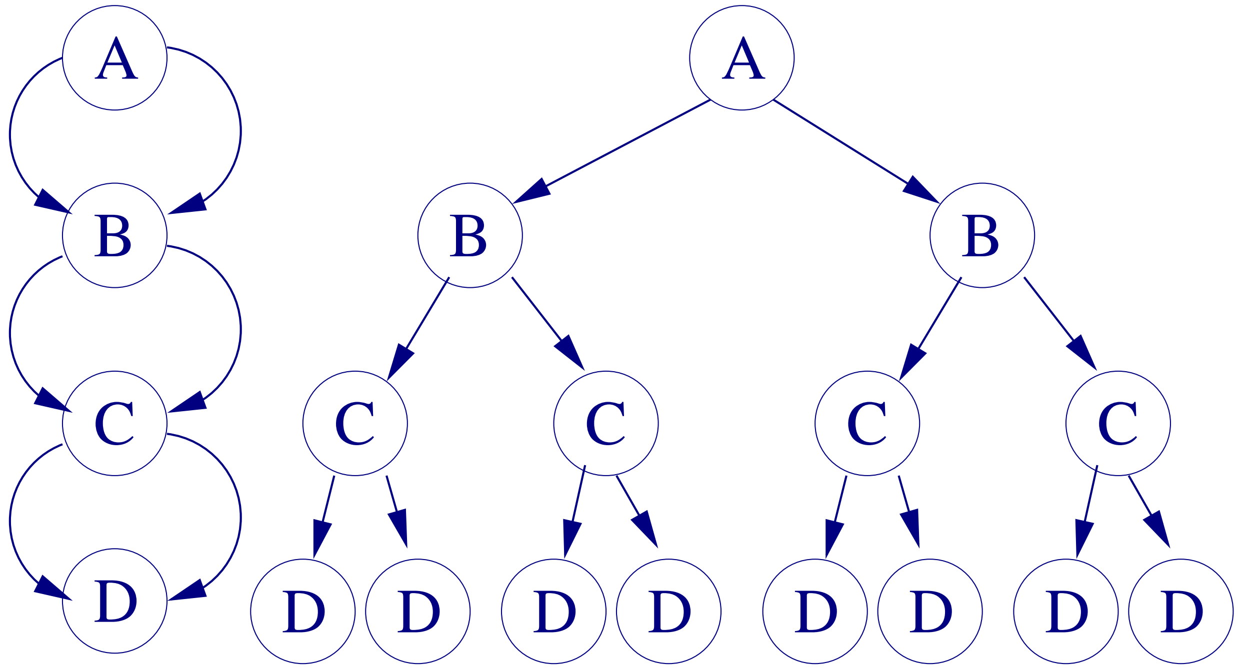
1	2	3
	8	4
7	6	5

←

1	2	3
8		4
7	6	5

- Is it a good strategy?
- What about Chess? Would it be a good strategy?
- What kind of domains may be suitable for BDS?

Avoiding Repeated States



Repeated states can be devastating in search problems.

- Common cases: problems with reversible operators $\longrightarrow$ search space becomes infinite
- One approach: find a spanning tree of the graph

Avoiding Repeated States: Strategies

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

→

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

...

- Do not return to the node's parent
- Avoid cycles in the path (this is a huge theoretical problem in its own right)
- Do not generate states that you generated before: use a hash table to make checks efficient

How to avoid storing every state? Would using a short signature (or a checksum) of the full state description help?

Key Points

- DLS, IDS, BDS search order, expansions, and queuing
- DLS, IDS, BDS evaluation
- DLS, IDS, BDS: suitable domains
- Repeated states: why removing them is important

Overview

- Best-first search
- Heuristic function
- Greedy best-first search
- A^*
- Designing good heuristics
- IDA^*
- Iterative improvement algorithms
  1. Hill-climbing
  2. Simulated annealing

Informed Search

From domain knowledge, obtain an **evaluation function**.

- best-first search: order nodes according to the evaluation function value
- greedy search: minimize estimated cost for reaching the goal – fast, but incomplete and non-optimal.
- A^*: minimize $f(n) = g(n) + h(n)$, where $g(n)$ is the current path cost from start to n, and $h(n)$ is the estimated cost from n to goal.

Best First Search

function Best-First-Search (*problem, Eval-Fn*)

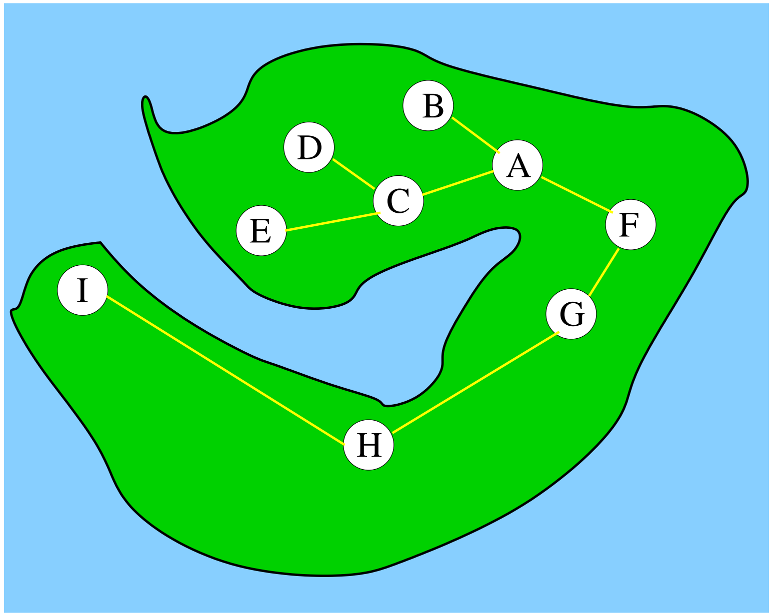
Queuing-Fn $\leftarrow$ sorted list by *Eval-Fn*(node)

return General-Search(*problem, Queuing-Fn*)

- The queuing function queues the expanded nodes, and sorts it every time by the *Eval-Fn* value of each node.

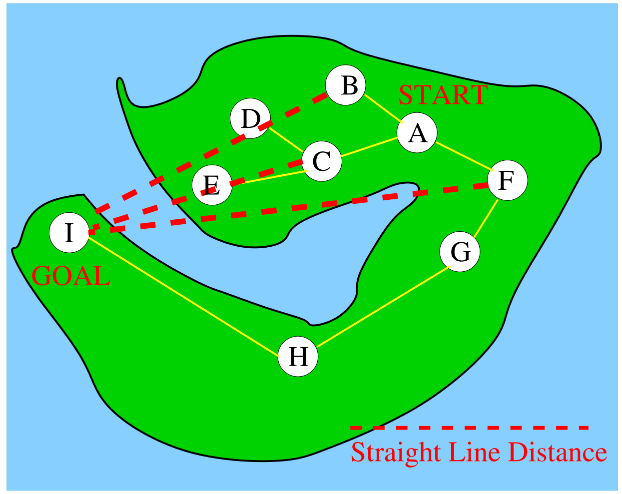
- One of the simplest Eval-Fn: **estimated cost** to reach the goal.

Heuristic Function



- $h(n)$ = estimated cost of the cheapest path from the state at node n to a goal state.
- The only requirement is the $h(n) = 0$ at the goal.
- **Heuristics** means “to find” or “to discover”, or more technically, “how to solve problems” (Polya, 1957).

Heuristics: Example



- $h_{\mathrm{SLD}}(n)$: straight line distance (SLD) is one example.
- Start from **A** and Goal is **I**: **C** is the most promising next step in terms of $h_{\mathrm{SLD}}(n)$, i.e. $h(C) < h(B) < h(F)$
- Requires some knowledge:
  1. coordinates of each city
  2. generally, cities toward the goal tend to have smaller **SLD**.

Greedy Best-First Search

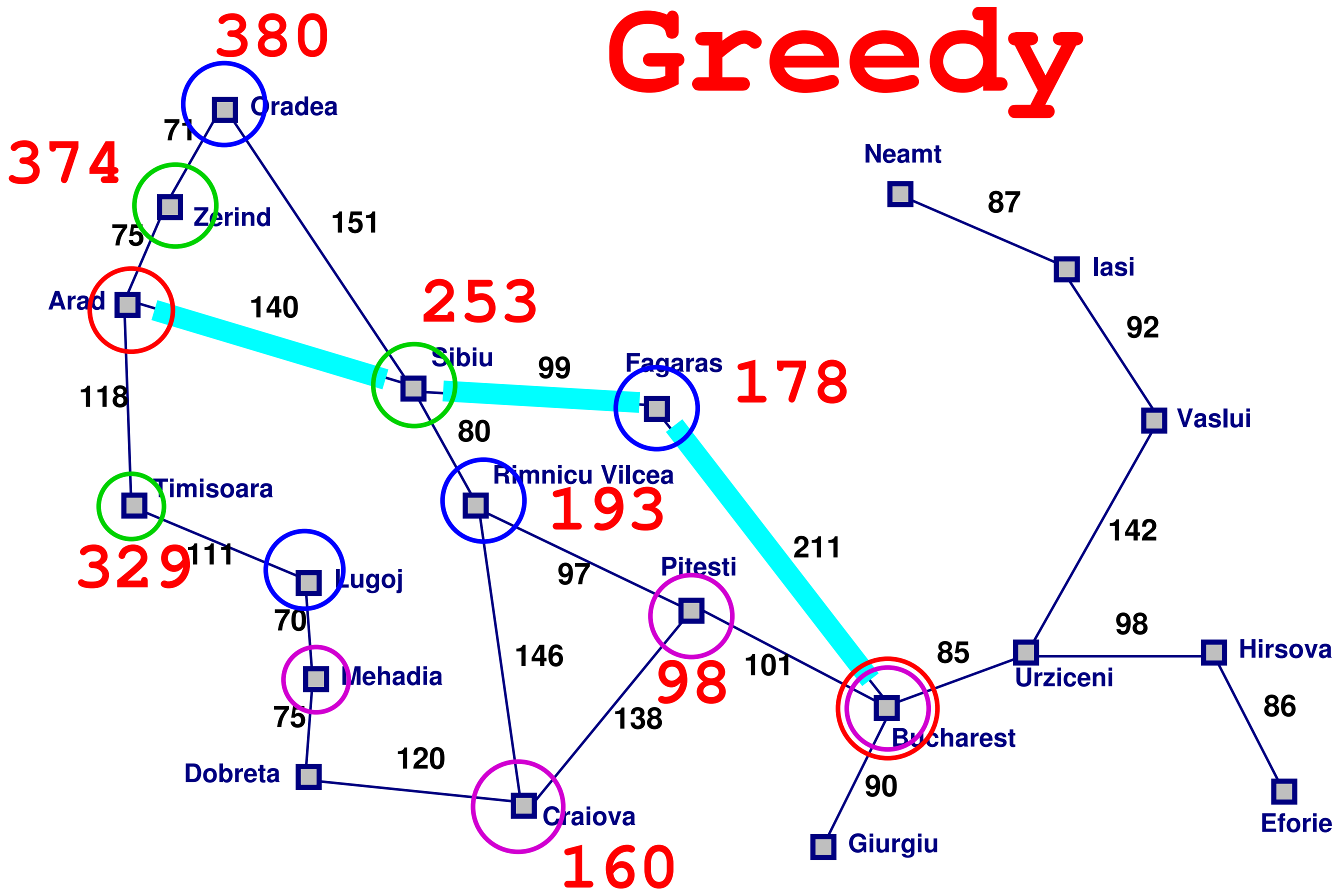
function Greedy-Best-First Search (*problem*)

$h(n)$=estimated cost from n to goal

return Best-First-Search(*problem*,h)

- Best-first with heuristic function $h(n)$

Romania with step costs in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Total Path Cost = 450

Greedy Best-First Search: Evaluation

Branching factor b and max depth m:

- Fast, just like Depth-First-Search: single path toward the goal.
- Time: $O(b^m)$
- Space: same as time – all nodes are stored in sorted list(!), **unlike DFS**
- Incomplete, just like DFS
- Non-optimal, just like DFS

A^*: Uniform Cost + Heuristic Search

Avoid expanding paths that are already found to be expensive:

- $f(n) = g(n) + h(n)$
- $f(n)$: estimated cost to goal through node n
- **provably complete and optimal!**
- **restrictions**: $h(n)$ should be an **admissible heuristic**
- admissible heuristic: one that **never overestimate** the actual cost of the best solution through n
- **NOTE:** $f(n)$ can be different depending on the path taken to $f(n)$ if multiple paths exists from root to n!

A^* **Search**

function A^*-Search (*problem*)

$g(n)$=current cost up till n

$h(n)$=estimated cost from n to goal

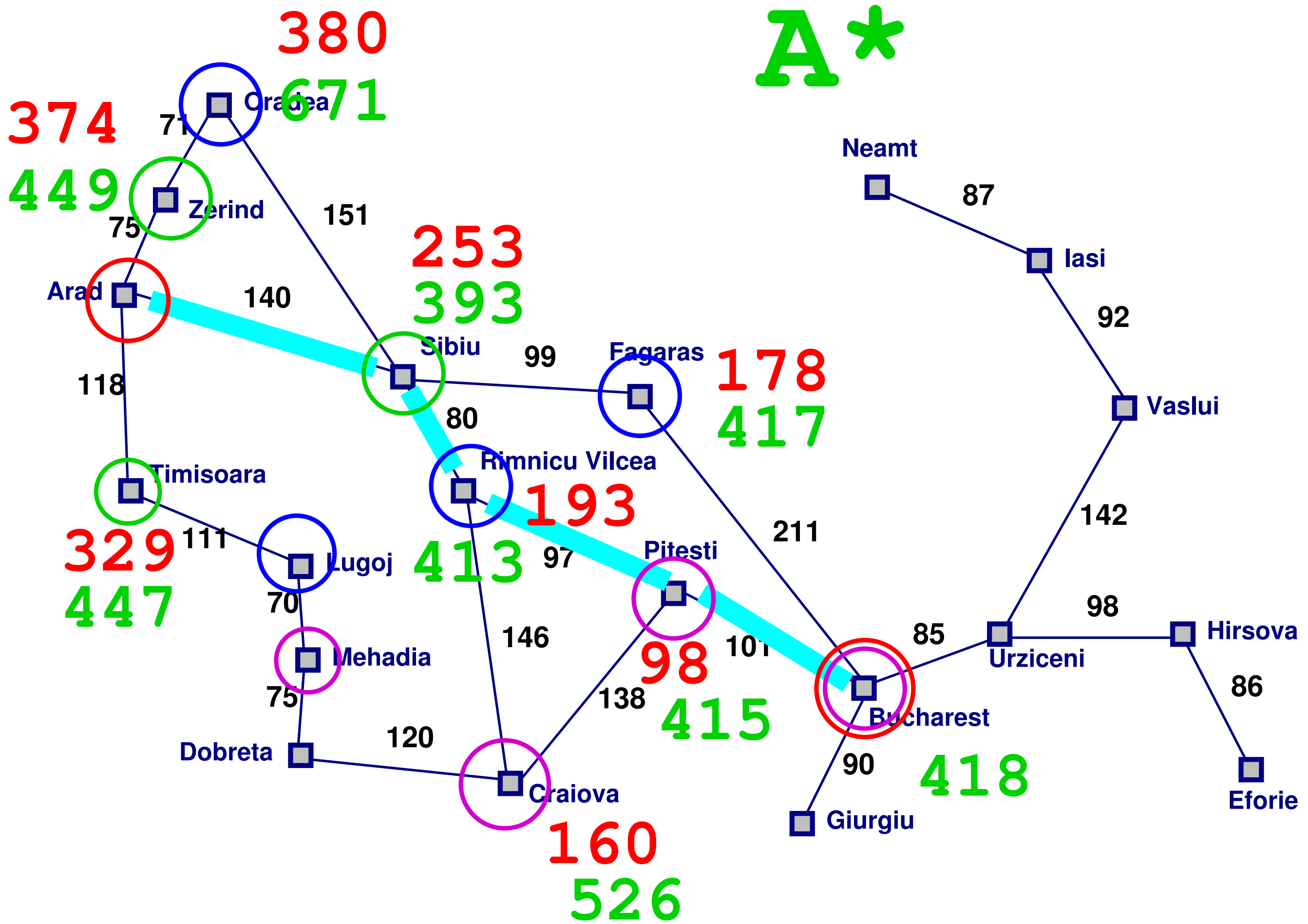
return Best-First-Search(*problem*,$g + h$)

- Condition: $h(n)$ must be an **admissible heuristic function**!
- A^* is **optimal!**

Behavior of A^* Search

- usually, the f value never decreases along a given path: **monotonicity**
- in case it is nonmonotonic, i.e. $f(Child) < f(Parent)$, make this adjustment:
  $f(Child) = max(f(Parent), g(Child) + h(Child))$.
- this is called **pathmax**

Romania with step costs in km



Straight–line distance to Bucharest

Arad	366
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Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Total Path Cost = 418

Optimality of A^*

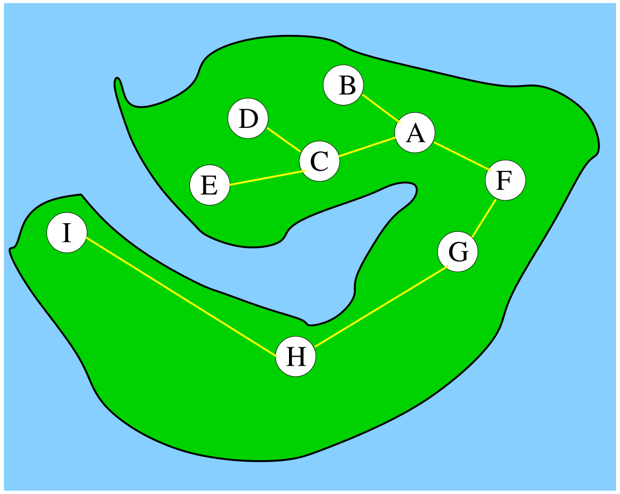
G_2: suboptimal goal in the node-list.

n: unexpanded node on a shortest path to goal G_1

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $> g(G_1)$ since G_2 is suboptimal
- $\geq f(n)$ since h is admissible

Since $f(G_2) > f(n)$, A^* will never select G_2 for expansion.

Optimality of A^*: Example



1. **Expansion of parent allowed**: search fails at nodes **B**, **D**, and **E**.
2. **Expansion of parent disallowed**: paths through nodes **B, D**, and **E** with have an inflated path cost $g(n)$, thus will become nonoptimal.

$$\underbrace{A \rightarrow C \rightarrow E \rightarrow C \rightarrow}_{\text{inflated path cost}} A \rightarrow F \rightarrow ...$$

Lemma to Optimality of A^*

Lemma: A^* expands nodes in order of increasing $f(n)$ value.

- Gradually adds **f-contours** of nodes (cf. BFS adds layers).
- The goal state may have a f value: let's call it f^*
- This means that all nodes with $f < f^*$ will be expanded!

Complexity of A^*

A^* is complete and optimal, but space complexity can become exponential if the heuristic is not good enough.

- condition for **subexponential** growth:

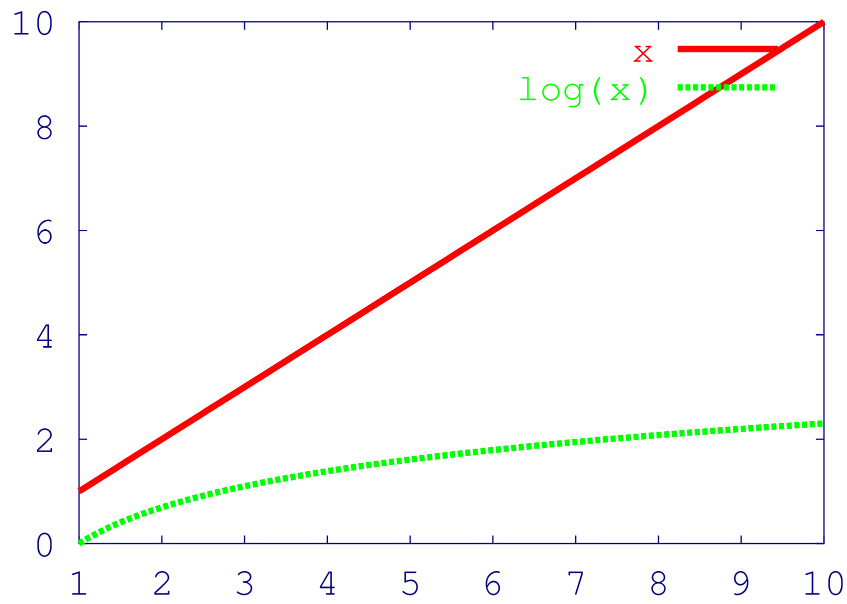
  $$|h(n) - h^*(n)| \leq O(\log h^*(n)),$$

  where $h^*(n)$ is the **true** cost from n to the goal.

- that is, error in the estimated cost to reach the goal should be less than even linear, i.e. $< O(h^*(n))$.

Unfortunately, with most heuristics, error is at least proportional with the true cost, i.e. $\geq O(h^*(n)) > O(\log h^*(n))$.

Linear vs. Logarithmic Growth Error



- Error in heuristic: $|h(n) - h^*(n)|$.
- For most heuristics, the error is at least linear.
- For A^* to have subexponential growth, the error in the heuristic should be on the order of $O(\log h^*(n))$.

Problem with A^*

Space complexity is usually **exponential**!

- we need a memory bounded version
- one solution is: Iterative Deepening A^*, or IDA^*

A^*: Evaluation

- Complete : unless there are infinitely many nodes with $f(n) \leq f(G)$
- Time complexity: exponential in (relative error in $h \times$ length of solution)
- Space complexity: same as time (keep all nodes immediately outside of current f-contour in memory)
- Optimal

Heuristic Functions: Example

Eight puzzle

5	4	
6	1	8
7	3	2

1	2	3
8		4
7	6	5

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

$h_1(n)$ = 7 (not counting the blank tile)

$h_2(n)$ = 2+3+3+2+4+2+0+2 = 18

* Both are admissible heuristic functions.

Dominance

If $h_2(n) \geq h_1(n)$ for all n and both are admissible, then we say that $h_2(n)$ **dominates** $h_1(n)$, and is better for search.

Typical search costs for depth $d = 14$:

- Iterative Deepening : 3,473,941 nodes expanded
- $\mathrm{A}^*(h_1)$: 539 nodes
- $\mathrm{A}^*(h_2)$: 113 nodes

Observe that in A^*, every node with $f < f^*$ is expanded. Since $f = g + h$, nodes with $h(n) < f^* - g(n)$ will be expanded, so larger h will result in less nodes being expanded.

- f^* is the f value for the optimal solution path.

Designing Admissible Heuristics

Relax the problem to obtain an admissible heuristics.

For example, in 8-puzzle:

- allow tiles to move anywhere $\rightarrow h_1(n)$
- allow tiles to move to any adjacent location $\rightarrow h_2(n)$

For traveling:

- allow traveler to travel by air, not just by road: **SLD**

Other Heuristic Design

- Use composite heuristics: $h(n) = max(h_1(n), ..., h_m(n))$
- Use statistical information: random sample h and true cost to reach goal. Find out how often h and true cost is related.

Iterative Deepening A^*: IDA^*

A^* is complete and optimal, but the performance is limited by the available space.

- Basic idea: only search within a certain f bound, and gradually increase the f bound until a solution is found.
- Popular use include path finding in game AI.

IDA^*

function IDA^*(*problem*)

root $\leftarrow$ Make-Node(Initial-State(*problem*))

f-limit $\leftarrow$ f-Cost(*root*)

loop do

solution, f-limit $\leftarrow$ DFS-Contour(*root, f-limit*)

if *solution* != NULL **then return** *solution*

if *f-limit* == ∞ **then return** *failure*

end loop

Basically, iterative deepening depth-first-search with depth defined as the f-cost ($f = g + n$):

DFS-Contour(*root, f-limit*)

Find solution from node **root**, within the f-cost limit of **f-limit**.

DFS-Contour returns **solution sequence** and **new f-cost limit**.

- if f-cost(**root**) $>$ **f-limit**, return fail.
- if **root** is a goal node, return solution and new f-cost limit.
- **recursive call** on all successors and return solution and **minimum f-limit** returned by the calls
- return **null solution** and new **f-limit** by default

Similar to the recursive implementation of DFS.

IDA^*: Evaluation

- complete and optimal (with same restrictions as in A^*)
- space: proportional to longest path that it explores (because it is depth first!)
- time: dependent on the number of different values $h(n)$ can assume.

IDA^*: Time Complexity

Depends on the heuristics:

- small number of possible heuristic function values $\longrightarrow$ small number of f-contours to explore $\longrightarrow$ becomes similar to A^*
- complex problems: each f-contour only contain one new node

  if A^* expands N nodes,
  IDA^* expands
  $1 + 2 + .. + N = \frac{N(N+1)}{2} = O(N^2)$
- a possible solution is to have a **fixed** increment ϵ for the f-limit $\longrightarrow$ solution will be suboptimal for at most ϵ (ϵ-admissible)

Iterative Improvement Algorithms

Start with a complete configuration (all variable values assigned, and **optimal**), and**gradually improve** it.

- Hill-climbing (maximize cost function)
- Gradient descent (minimize cost function)
- Simulated Annealing (probabilistic)

Hill-Climbing

- no queue, keep only the best node
- greedy, no back-tracking
- good for domains where **all nodes are solutions**:
  - goal is to **improve** quality of the solution
  - optimization problems
- note that it is different from greedy search, which keeps a node list

Hill-Climbing Strategies

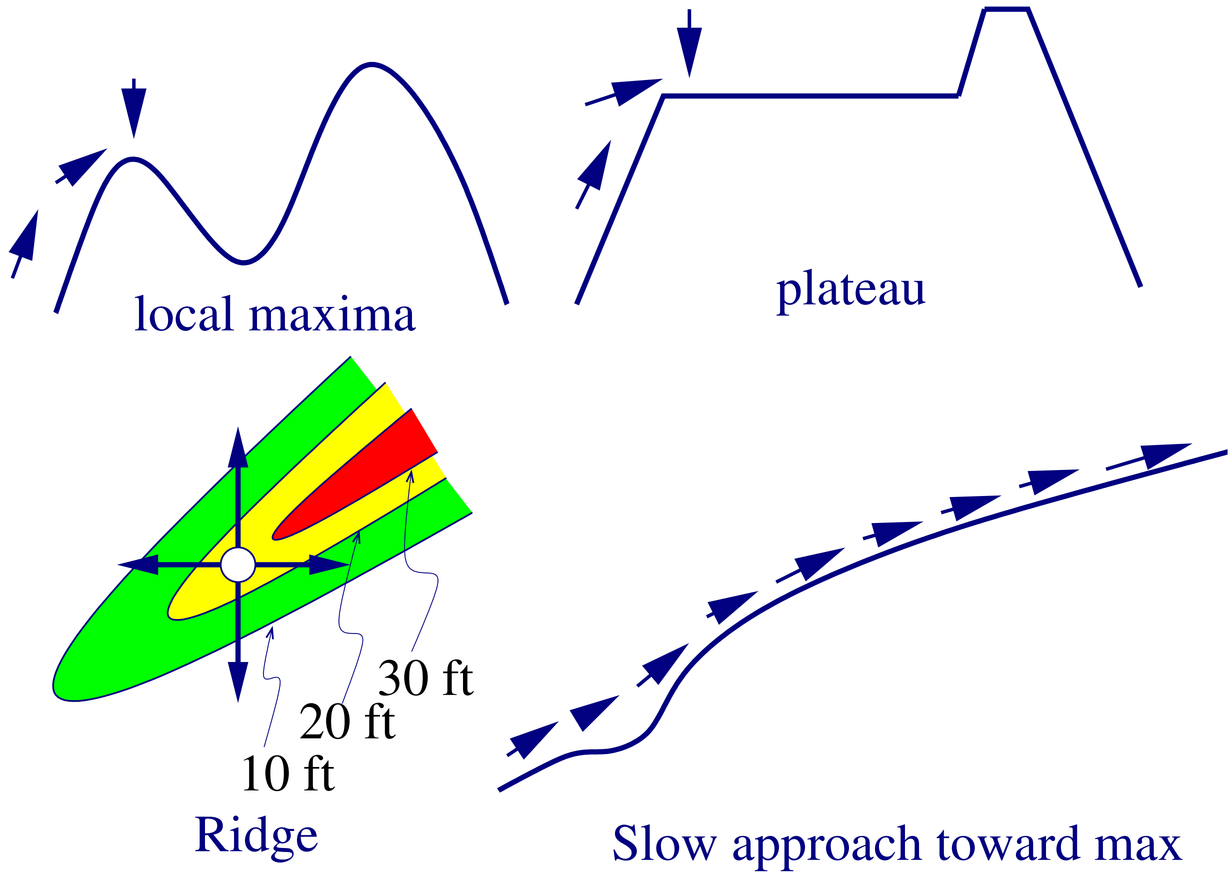
Problems of local maxima, plateau, and ridges:

- try **random-restart**: move to a random location in the landscape and restart search from there
- parallel search
- simulated annealing *

Hardness of problem depends on the **shape of the landscape**.

*: coming up next

Hill-Climbing and Gradient Search: Problems



- Possible solution: **simulated annealing** – gradually decrease randomness of move to attain globally optimal solution (more on this next week).

Simulated Annealing: Overview

Annealing:

- heating metal to a high-temperature (making it a liquid) and then allowing to cool slowly (into a solid); this relieves internal stresses and results in a more stable, lower-energy state in the solid.
- at high temperature, atoms move actively (large distances with greater randomness), but as temperature is lowered, they become more static.

Simulated annealing is similar:

- basically, hill-climbing with randomness that allows going **down** as well as the standard **up**
- randomness (as temperature) is reduced over time

Simulated Annealing (SA)

Goal: **minimize** (not maximize) the energy E, as in statistical thermodynamics.

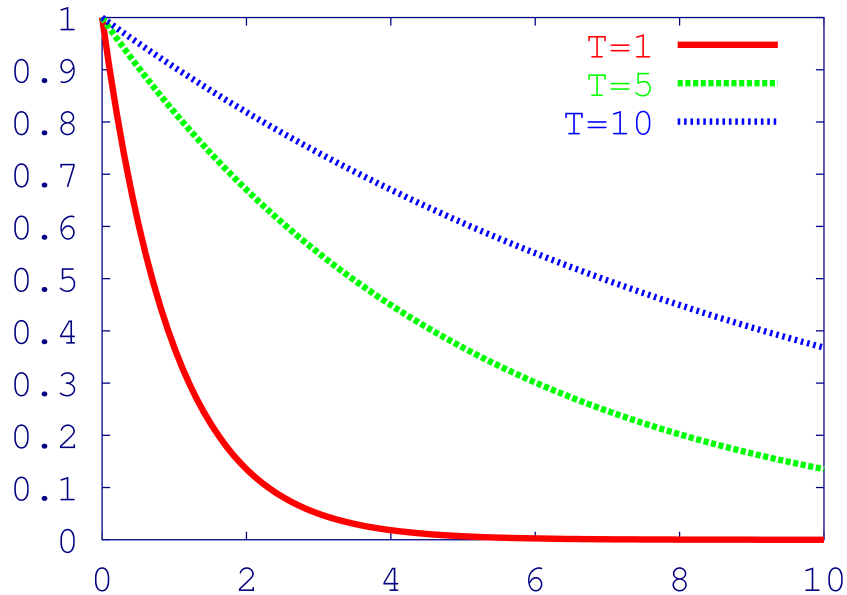
For successors of the current node,

- if $\Delta E \leq 0$, the move is accepted
- if $\Delta E > 0$, the move is accepted with probability $P(\Delta E) = e^{-\frac{\Delta E}{kT}}$, where k is the Boltzmann constant and T is temperature.
- randomness is in the comparison: $P(\Delta E) < \mathrm{rand}(0, 1)$

$\Delta E = E_{\mathrm{new}} - E_{\mathrm{old}}$.

The heuristic $h(n)$ or $f(n)$ represents E.

Temperature and $P(\Delta E) < \text{rand}(0,1)$



Downward moves of any size are allowed at high temperature, but at low temperature, only small downward moves are allowed.

- Higher temperature $T \longrightarrow$ higher probability of **downward** hill-climbing
- Lower $\Delta E \longrightarrow$ higher probability of **downward** hill-climbing

T Reduction Schedule

High to low temperature reduction schedule is important:

- reduction too fast: suboptimal solution
- reduction too slow: wasted time
- question: does the form of the reduction schedule curve matter? linear, quadratic, exponential, etc.?

The proper values are usually found experimentally.

Simulated Annealing Applications

- VLSI wire routing and placement
- Various scheduling optimization tasks
- Traffic control
- Neural network training
- etc.

Key Points

- best-first-search: definition
- heuristic function $h(n)$: what it is
- greedy search: relation to $h(n)$ and evaluation. How it is different from DFS (time complexity, space complexity)
- A^*: definition, evaluation, conditions of optimality
- complexity of A^*: relation to error in heuristics
- designing good heuristics: several rule-of-thumbs
- IDA^*: evaluation, time and space complexity (worst case)
- hill-climbing concept and strategies
- simulated annealing: core algorithm, effect of T and ΔE, source of randomness.

Game Playing

Game Playing

- attractive AI problem because it is **abstract**
- one of the oldest domains in AI
- in most cases, the world state is fully accessible
- computer representation of the situation can be clear and exact
- challenging: uncertainty introduced by the opponent and the complexity of the problem (full search is impossible)
- hard: in chess, branching factor is about 35, and 50 moves by each player $= 35^{100}$ nodes to search
  - compare to 10^{40} possible legal board states
- *game playing is more like real life than mechanical search*

Games vs. Search Problems

"Unpredictable" opponent $\longrightarrow$ solution is a contingency plan

Time limits $\longrightarrow$ unlikely to find goal, must approximate

Plan of attack:

- algorithm for perfect play (Von Neumann, 1944)
- finite horizon, approximate evaluation (Zuse, 1945; Shannon, 1950; Samuel, 1952–57)
- pruning to reduce costs (McCarthy, 1956)

Types of Games

	deterministic	chance
perfect info	chess, checkers, go, othello	backgammon, monopoly
imperfect info	battle ship	bridge, poker, scrabble

Two-Person Perfect Information Game

initial state: initial position and who goes first

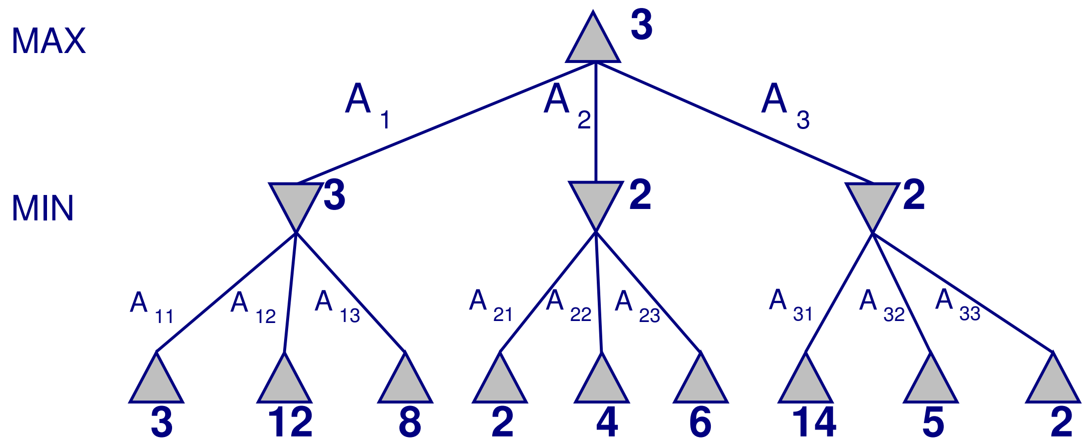
operators: legal moves

terminal test: game over?

utility function: outcome (win:+1, lose:-1, draw:0, etc.)

- two players (**MIN** and **MAX**) taking turns to maximize their chances of winning (each turn generates one **ply**)
- one player's victory is another's defeat
- need a **strategy** to win no matter what the opponent does

Minimax: Strategy for Two-Person Perfect Info Games



- generate the whole tree, and apply util function to the leaves
- go back upward assigning utility value to each node
- at MIN node, assign **min(successors' utility)**
- at MAX node, assign **max(successors' utility)**
- **assumption**: the opponent acts optimally

Minimax Decision

function Minimax-Decision (game) **returns** operator

return operator that leads to a child state with the **max**(Minimax-Value(child state,game))

function Minimax-Value(state,game) **returns** utility value

if Goal(state), **return** Utility(state)

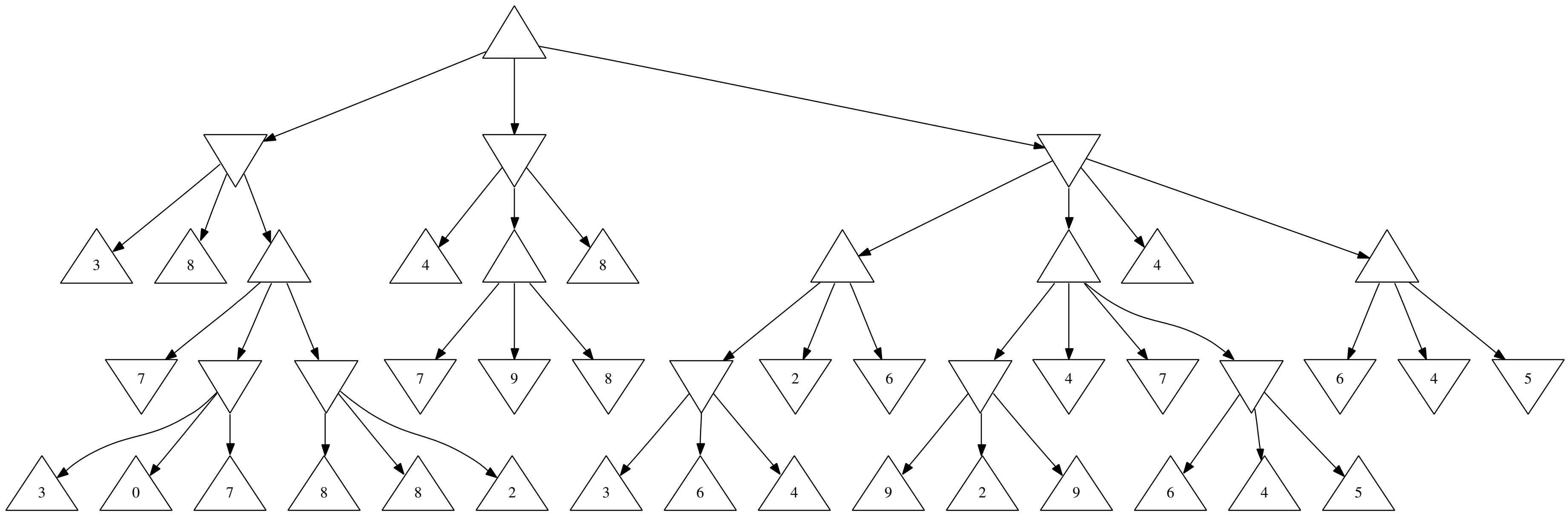
else if Max's move **then**

$\longrightarrow$ **return** max of successors' Minimax-Value

else

$\longrightarrow$ **return** min of successors' Minimax-Value

Minimax Exercise



Minimax: Evaluation

Branching factor b, max depth m:

- **complete**: if the game tree is finite
- **optimal**: if opponent is optimal
- **time**: b^m
- **space**: bm – depth-first (only when utility function values of all nodes are known!)

Resource Limits

- Time limit: as in Chess $\longrightarrow$ can only evaluate a fixed number of paths
- Approaches:

  - **evaluation function** : how desirable is a given state?
  - **cutoff test** : depth limit
  - **pruning**

Depth limit can result in the **horizon effect**: interesting or devastating events can be just over the horizon!

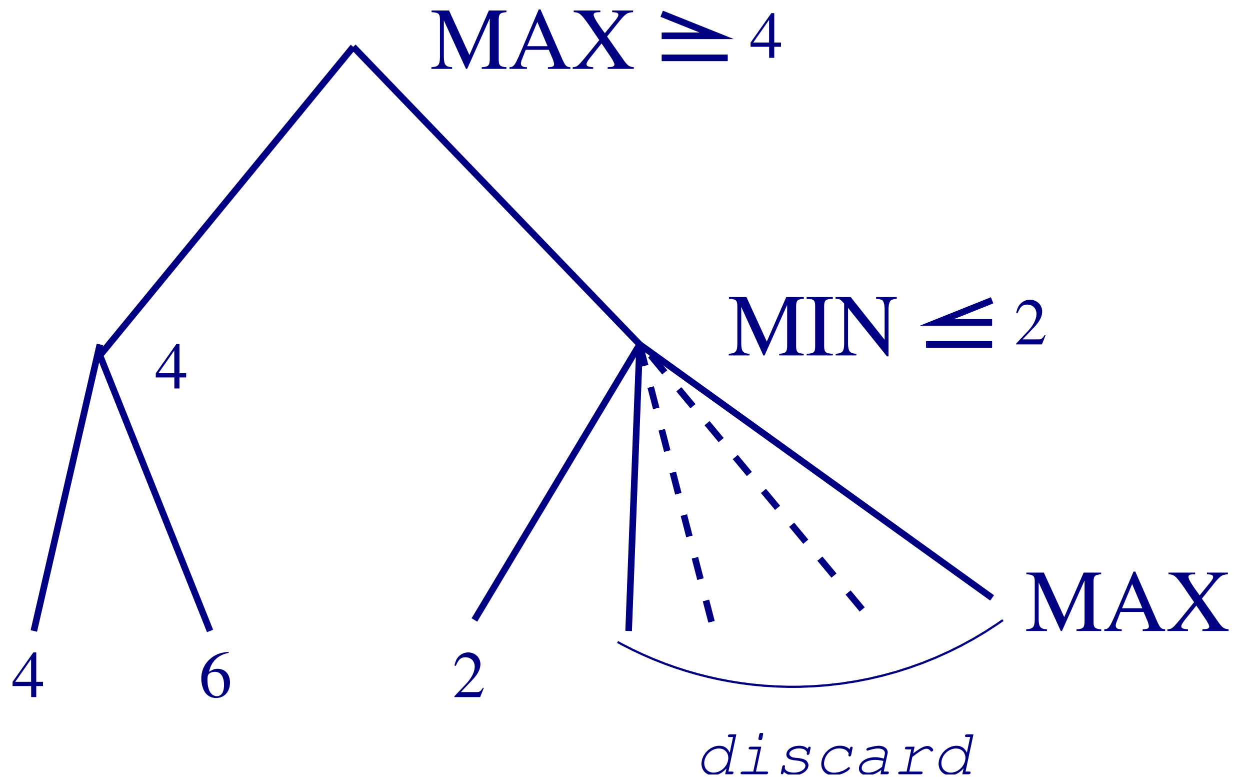
Evaluation Functions

For chess, usually a **linear** weighted sum of feature values:

- $\text{Eval}(s) = \sum_i w_i f_i(s)$
- $f_i(s) =$ (number of white piece X) - (number of black piece X)
- other features: degree of control over the center area
- exact values do not matter: the **order** of Minimax-Value of the successors matter.

α Cuts

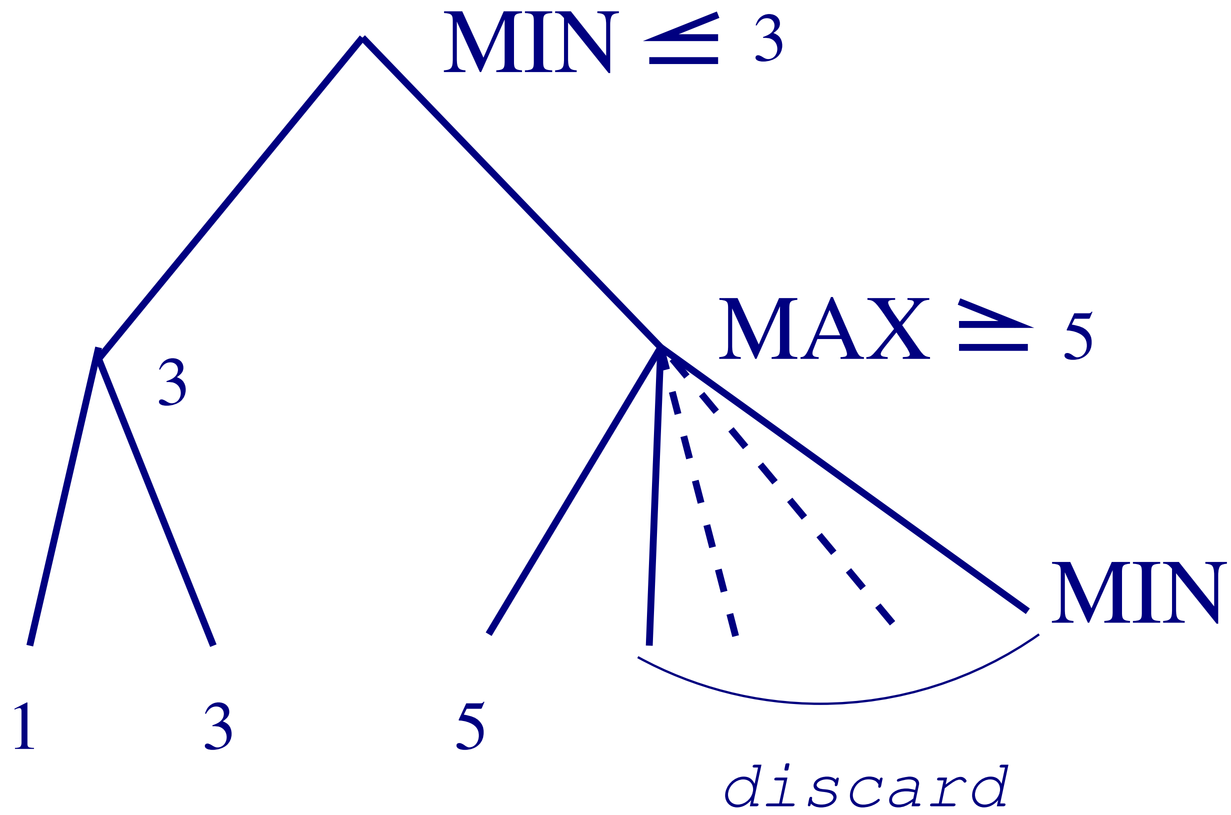
When the current max value is greater than the successor's min value, don't look further on that min subtree:



Right subtree can be **at most** 2, so MAX will always choose the left path regardless of what appears next.

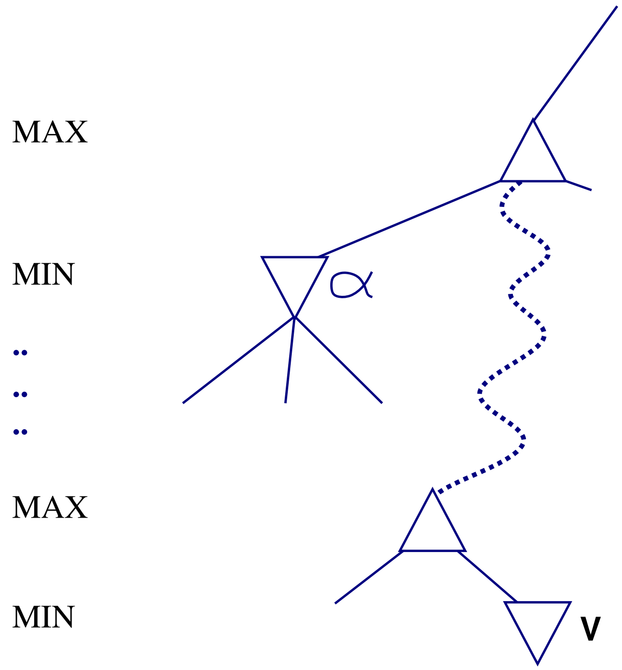
β **Cuts**

When the current min value is less than the successor's max value, don't look further on that max subtree:



Right subtree can be **at least** 5, so MIN will always choose the left path regardless of what appears next.

$\alpha - \beta$ Pruning



- memory of best MAX value α and best MIN value β
- do not go further on any one that does worse than the remembered α and β

$\alpha - \beta$ Pruning Properties

Cut off nodes that are known to be suboptimal.

Properties:

- pruning **does not** affect final result
- good move ordering improves effectiveness of pruning
- with **perfect ordering**, time complexity = $b^{m/2}$
  $\rightarrow$ **doubles** depth of search
  $\rightarrow$ can easily reach 8-ply in chess
- $b^{m/2} = (\sqrt{b})^m$, thus $b = 35$ in chess reduces to $b = \sqrt{35} \approx 6$!!!

Key Points

- Game playing: what are the types of games?
- Minimax: definition, and how to get minmax values
- Minimax: evaluation
- α-β pruning: why it saves time

Overview

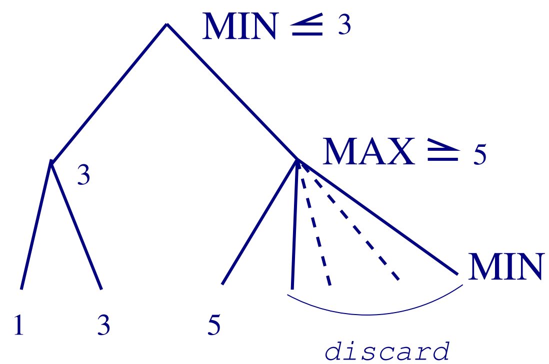
- formal $\alpha - \beta$ pruning algorithm
- $\alpha - \beta$ pruning properties
- games with an element of chance
- state-of-the-art game playing with AI
- more complex games

$\alpha - \beta$ Pruning: Initialization

Along the path from the beginning to the current **state**:

- α: best MAX value
  - initialize to $-\infty$
- β: best MIN value
  - initialize to ∞

$\alpha - \beta$ Pruning Algorithm: Max-Value



function Max-Value (state, game, α, β) **return** utility value

α: best MAX on path to *state* ; β: best MIN on path to *state*

if Cutoff(state) **then return** Utility(state)

$v \leftarrow -\infty$

for each *s* in Successor(state) **do**

· $v \leftarrow$ Max(α, Min-Value(s,game,α,β))

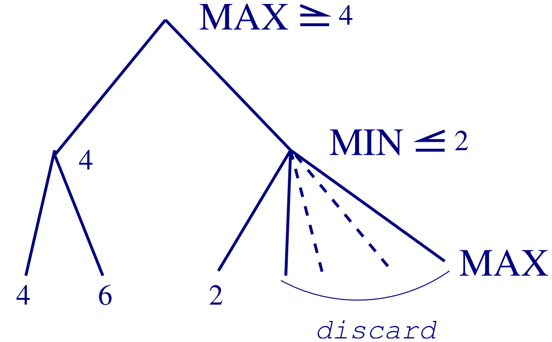
· **if** $v \geq \beta$ **then return** v /* **CUT!!** */

· $\alpha \leftarrow$Max(α, v)

end

return v

$\alpha - \beta$ Pruning Algorithm: Min-Value



function Min-Value (state, game, α, β) **return** utility value

α: best MAX on path to *state* ; β: best MIN on path to *state*

if Cutoff(state) **then return** Utility (state)

$v \leftarrow \infty$

for each *s* in Successor(state) **do**

· $v \leftarrow$ Min(β, Max-Value(s,game,α,β))

· **if** $v \leq \alpha$ **then return** v /* **CUT!!** */

· $\beta \leftarrow$Min(β,v)

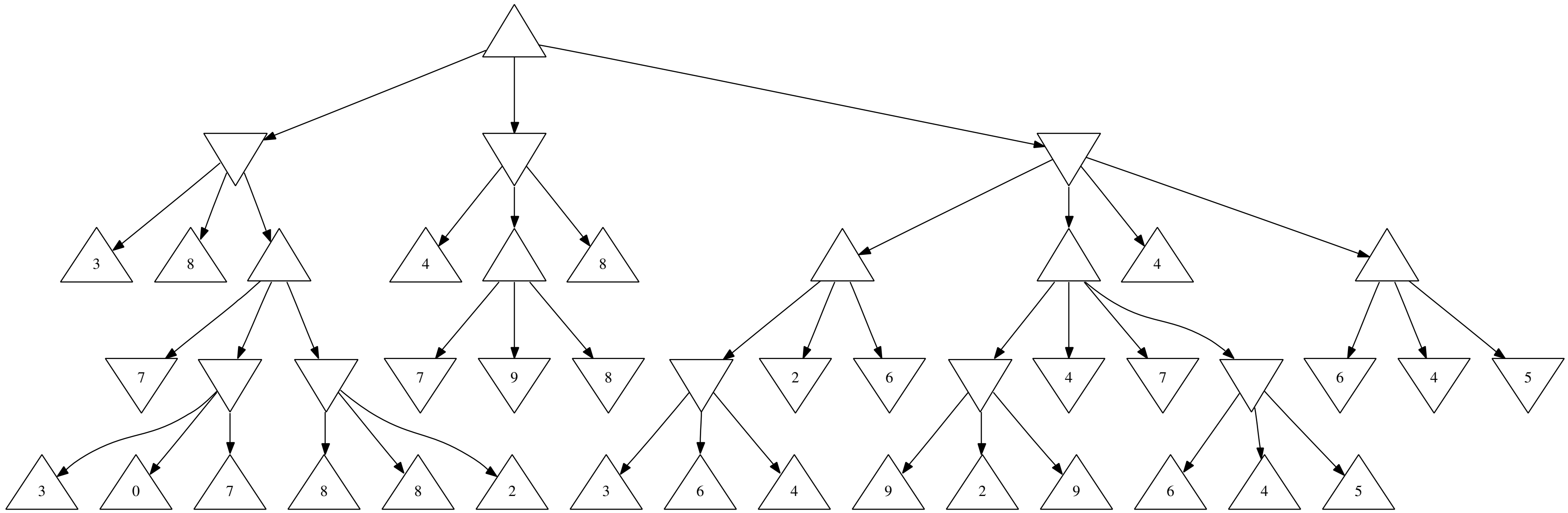
end

return v

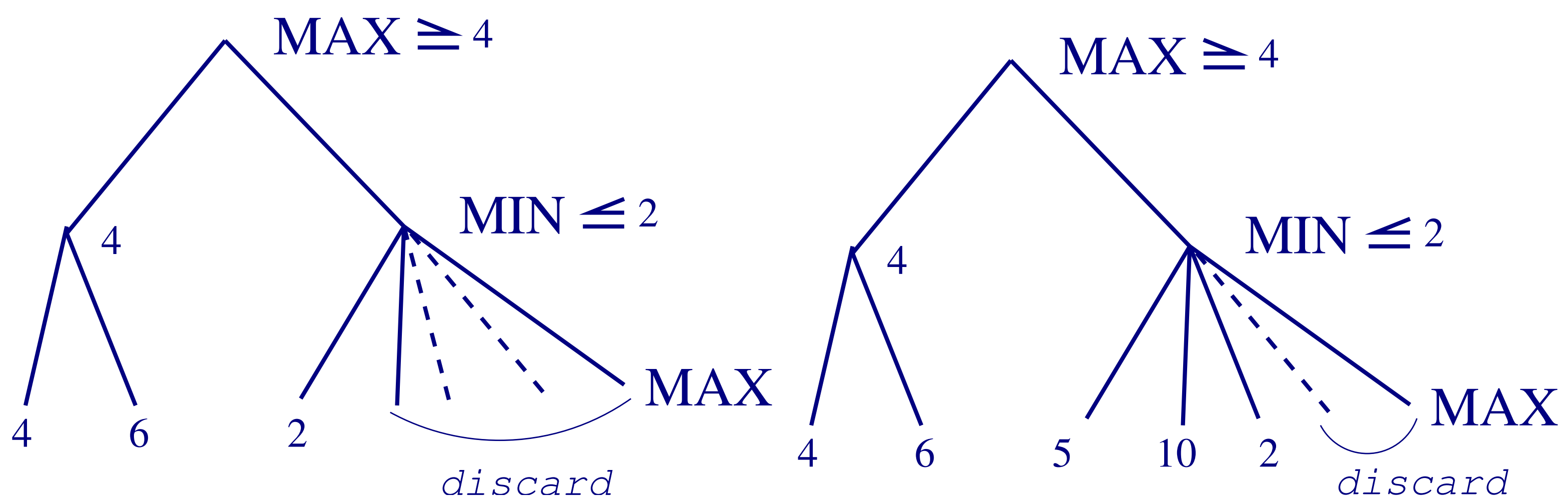
$\alpha - \beta$ Pruning Tips

- At a MAX node:
  - Only α is updated with the MAX of successors.
  - Cut is done by checking if returned $v \geq \beta$.
  - If all fails, MAX(v of succesors) is returned.
- At a MIN node:
  - Only β is updated with the MIN of successors.
  - Cut is done by checking if returned $v \leq \alpha$.
  - If all fails, MIN(v of succesors) is returned.

$\alpha - \beta$ Exercise



Ordering is Important for Good Pruning



- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

Games With an Element of Chance

Rolling the dice, shuffling the deck of card and drawing, etc.

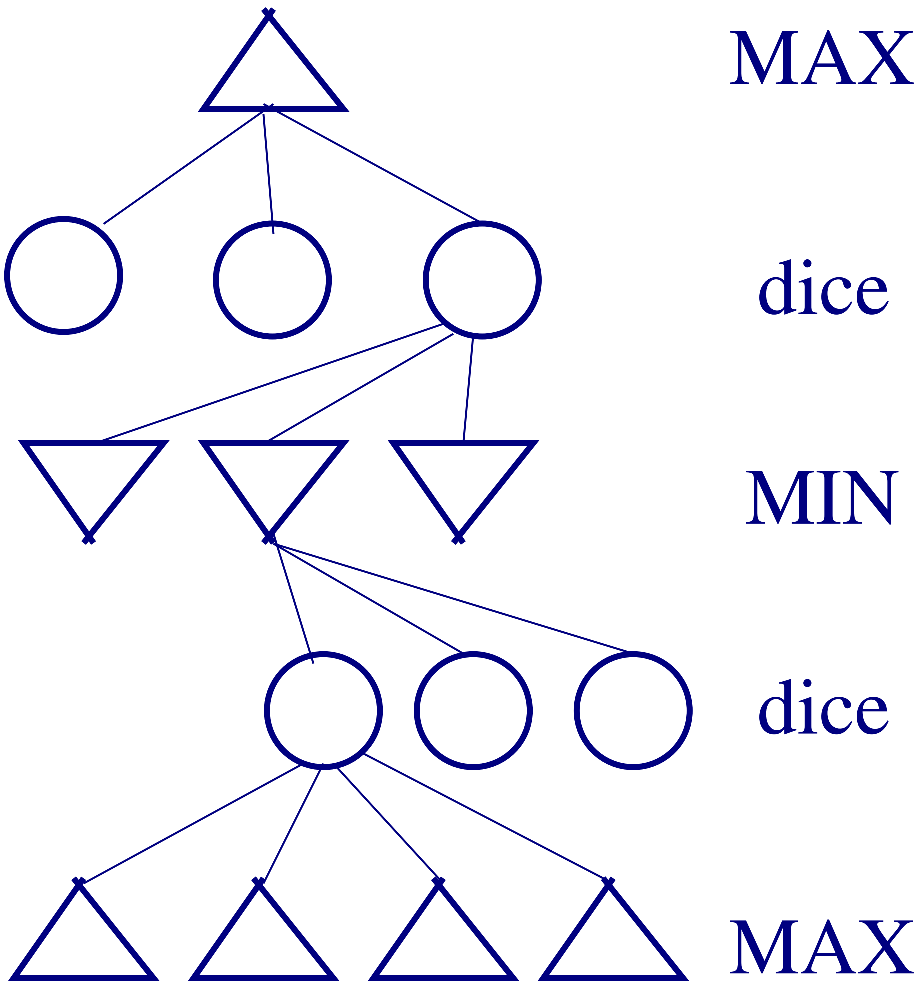
- **chance nodes** need to be included in the minimax tree
- try to make a move that maximizes the **expected value** $\rightarrow$ **expectimax**
- expected value of random variable X:

$$E(X) = \sum_x x P(x)$$

- expectimax

$$\text{expectimax}(C) = \sum_i P(d_i) \text{max}_{s \in S(C, d_i)}(utility(s))$$

Game Tree With Chance Element



- chance element forms a new ply (e.g. dice, shown above)

Design Considerations for Probabilistic Games

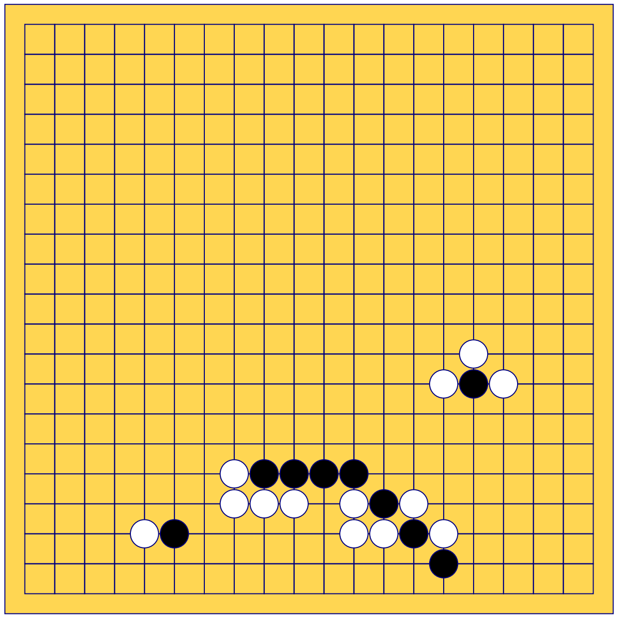
- the **value** of evaluation function, not just the **scale** matters now! (think of what expected value is)
- time complexity: $b^m n^m$, where n is the number of distinct dice rolls
- pruning can be done if we are careful

State of the Art in Gaming With AI

- Chess: IBM's Deep Blue defeated Garry Kasparov (1997)
- Backgammon: Tesauro's Neural Network $\longrightarrow$ top three (1992)
- Othello: smaller search space $\longrightarrow$ superhuman performance
- Checkers: Samuel's Checker Program running on 10Kbyte (1952)

Genetic algorithms can perform very well on select domains.

Hard Games



The game of *Go*, popular in East Asia:

- $19 \times 19 = 361$ grid: branching factor is huge!
- search methods inevitably fail: need more structured rules
- the bet was high: $1,400,000 prize for the first computer program to beat a select, 12-year old player. The late Mr. Ing Chang Ki (photo above) put up the money from his personal funds.

Photo from http://www.samsloan.com/ing.htm.

Key Points

- formal $\alpha - \beta$ pruning algorithm: know how to apply pruning
- $\alpha - \beta$ pruning properties: evaluation
- games with an element of chance: what are the added elements? how does the minmax tree get augmented?