Modeling Computation

Introduction to Formal Languages and Automata

Turing Machines

Finite Automata (DFAs, NFAs)

- Storage: None
- Recognize regular languages

Pushdown Automata (PDAs)

- Storage: Single stack
- Recognize context-free languages

What languages can be recognized by **any computational device whatsoever**?

A more powerful model of a computer

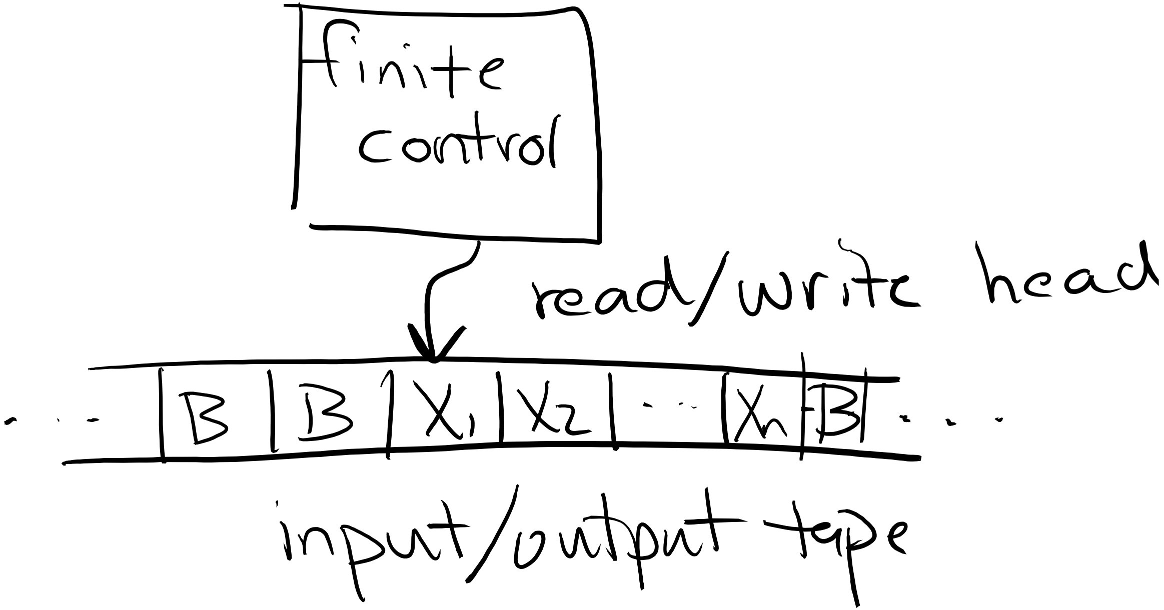
1928: Hilbert’s ***Entscheidungsproblem***.

1936: Alan Turing proposes the ***a-machine***,
a model of any possible computation.

Shows that a general solution to the *entscheidungsproblem* is **impossible**.

Turing Machine

Finite control



Read/Write head

- moves left and right

Input/Output tape - infinite length

Turing Machine (TM)

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Q = finite set of states

Σ = finite input alphabet

$\Gamma \supseteq \Sigma$ finite set of tape symbols

$\delta : ((Q-F) \times \Gamma) \longrightarrow (Q \times \Gamma \times \{\leftarrow, \rightarrow\})$

q_0 : initial state $q_0 \in Q$

$B \in (\Gamma - \Sigma)$ is the blank symbol

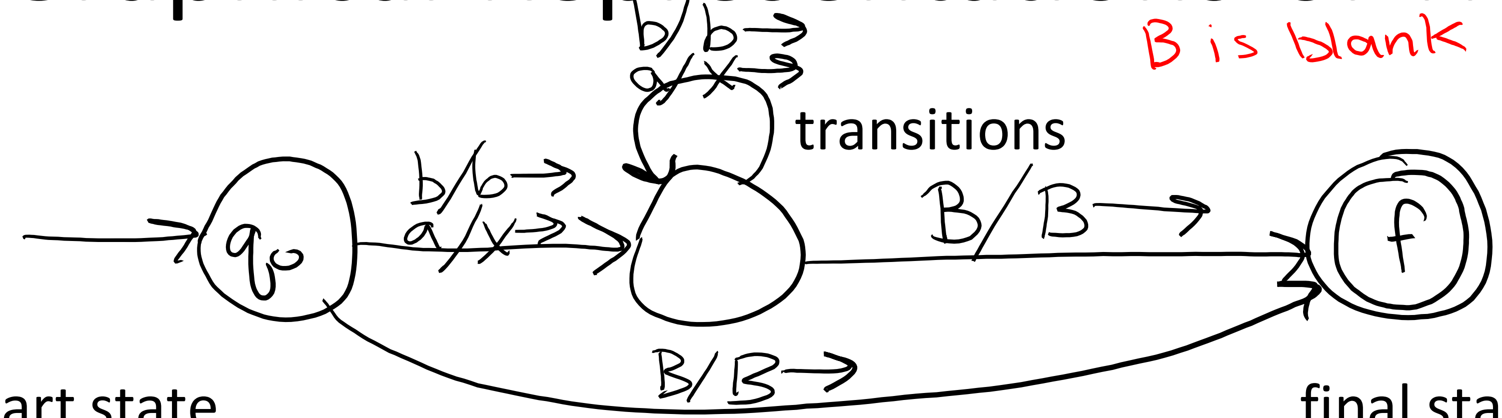
$F \subseteq Q$ is the set of final or accepting states.
(halting)

The language $L(M)$ of TM M is the set of strings $w \in \Sigma^*$ such that M halts on w.

The class of languages recognized by Turing machines is called recursively enumerable.

Graphical Representations of TMs

B is blank



start state

final state

replace "a"s with "X"s, leave "b"s alone.

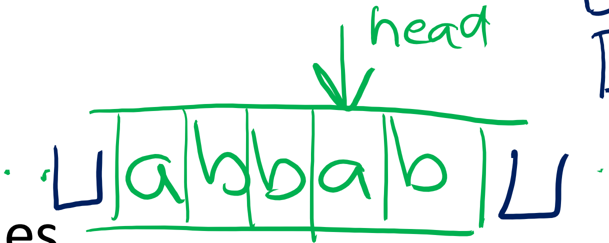
"$a/X \rightarrow$" means

- read "a" from tape
- write "X" to tape
- move head right

Computations by TMs

Let $M(w)\uparrow$ denote that TM M never halts on input w.

blanks
$B=\sqcup$



A **configuration** $C=(s,\alpha\underline{a}\beta)$ specifies

- current state
- string to left of tape head
- symbol under tape head
- string to right of tape head

$C=(s,abb\underline{a}b)$

\vdash := $\vdash$

Let $C_1 \vdash C_2$ denote that configuration C_1 yields C_2 in one step by applying a transition $\delta(s, a), s \in Q, a \in \Sigma$.

$C_1 \vdash^* C_n$ denotes $C_1 \vdash C_2 \vdash \cdots \vdash C_n$

sequence of transition in $C_1 \vdash^* C_n$ is a computation.

Suppose M starts in $\underline{(q_0, w)}$ with head over the leftmost symbol in w and halts in $\underline{(h, u)}$ for $h \in F$ and $u \in \Sigma^*$.

$$(q_0, w) \vdash^* (h, u)$$

Denote $\underline{u}$ by $M(w)$ and call it the <u>output</u> of M on w.

Let $f: \Sigma^* \to \Sigma^*$ partial or ~~total~~ function

Let $Dom(f) = \{w \mid f(w) \text{ is defined}\}$

M **computes** f if M halts on every $w \in Dom(f)$ with $M(w) = f(w)$

and $M(w)\uparrow$ on every $w \notin Dom(f)$

Partial Turing Computable function f

$\exists$ TM M that computes f,
and $Dom(f) \subseteq \Sigma^*$

Turing Computable function f

Partial Turing Computable, and
$Dom(f) = \Sigma^*$

Turing Computable language L : (decidable language)

characteristic function is Turing Computable.

$$\eta_L(w) = \begin{cases} 1 & w \in L \\ 0 & \text{otherwise} \end{cases}$$

Undecidable problems are those whose corresponding language is

Not Turing Computable ____________

no algorithm (TM that always halts) to solve the problem.

The Church-Turing Thesis:

Every effectively calculable function is computable.

Building Turing Machines

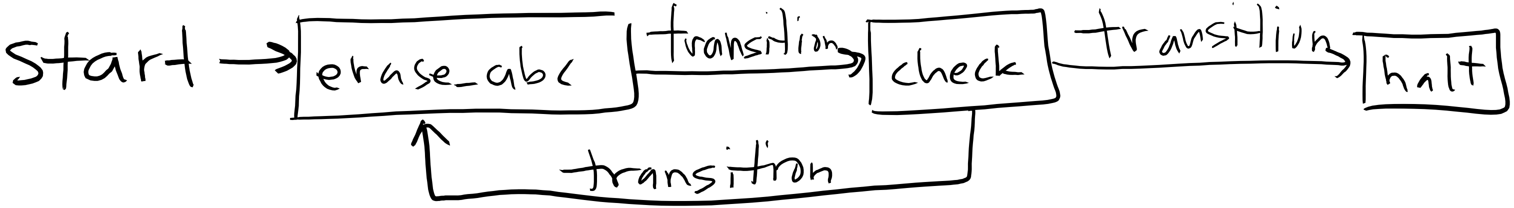
Build up large TMs using smaller ones like subroutines and define correct interfaces.

Example: Accept $w \in \{a^n b^n c^n \mid n > 0\}$

Strategy: (1) overwrite sets of a, b, c with X, X, X
(first a, first b, first c found)

(2) check that only Xs remain
if so, halt, else goto 1

Accept $w \in \{a^n b^n c^n \mid n > 0\}$

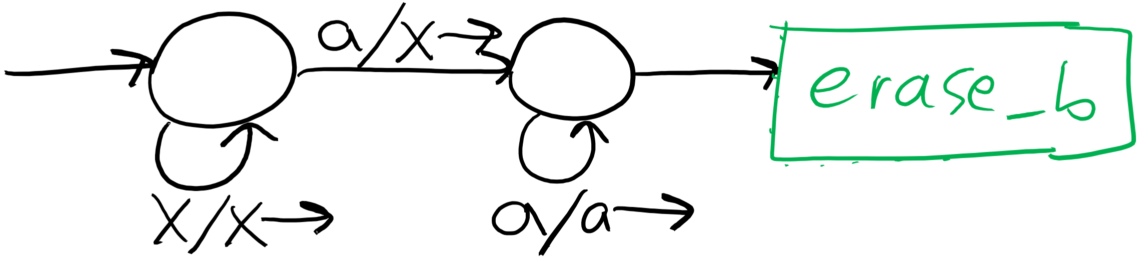


erase_abc: find 1st "a", replace w/ "X", then
find 1st "b", replace w/ "X", then
find 1st "c", replace w/ "X", then
return head to the front.

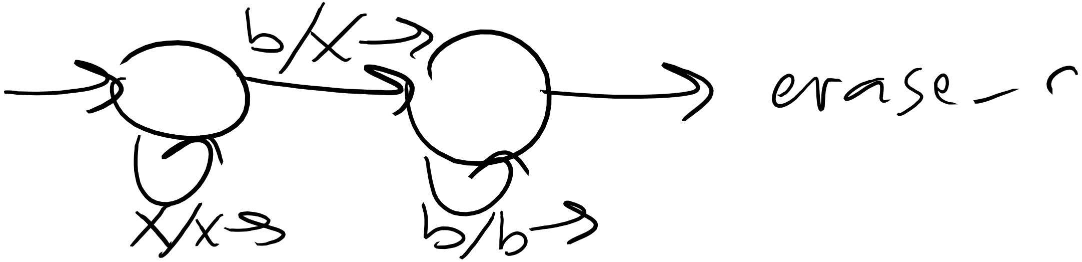
check: stream right through "X"s
if end at "Λ" return to the front.

halt: accept

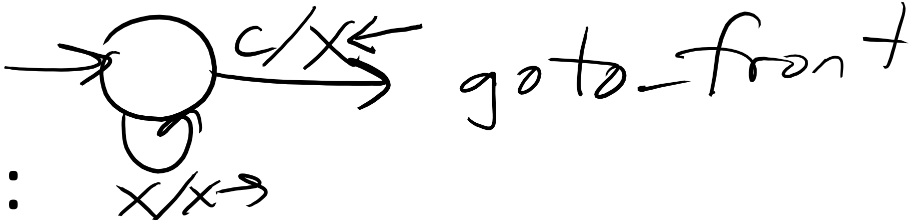
erase_a:



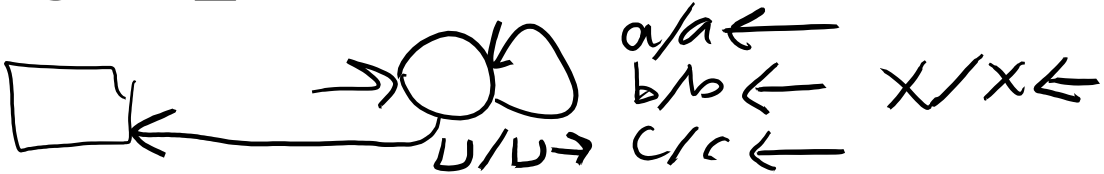
erase_b:



erase_c:



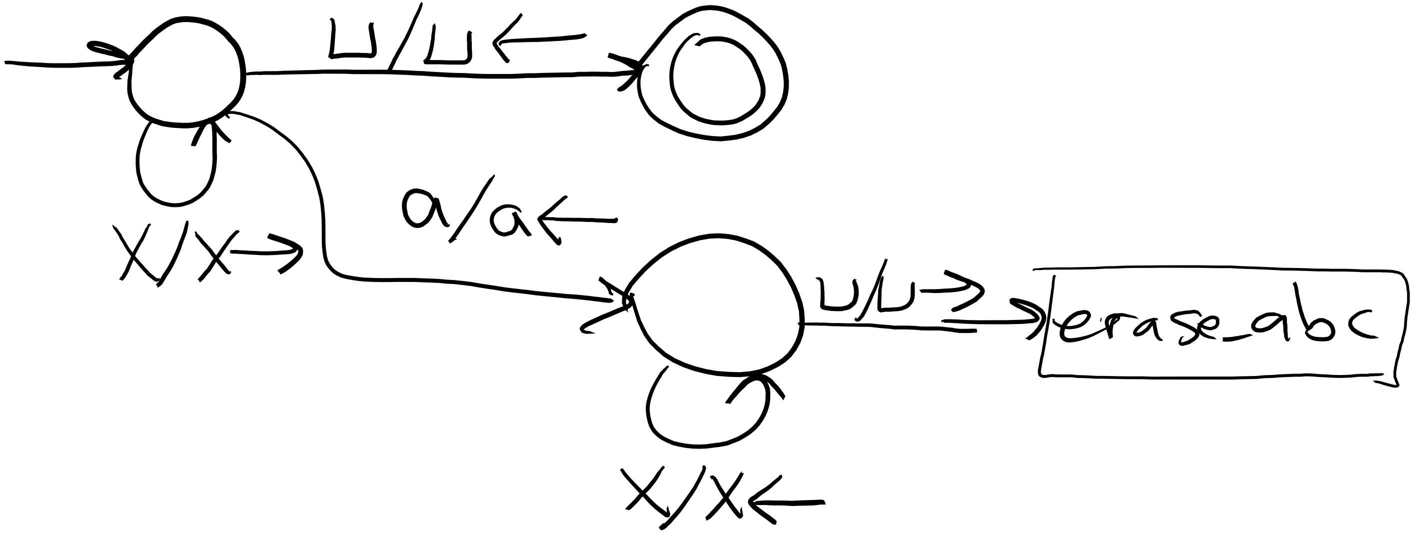
goto_front:



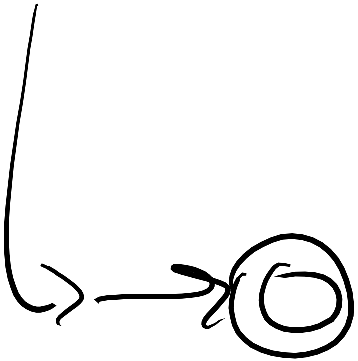
check: stream right through Xs

finds ⊔ blank → goto halt

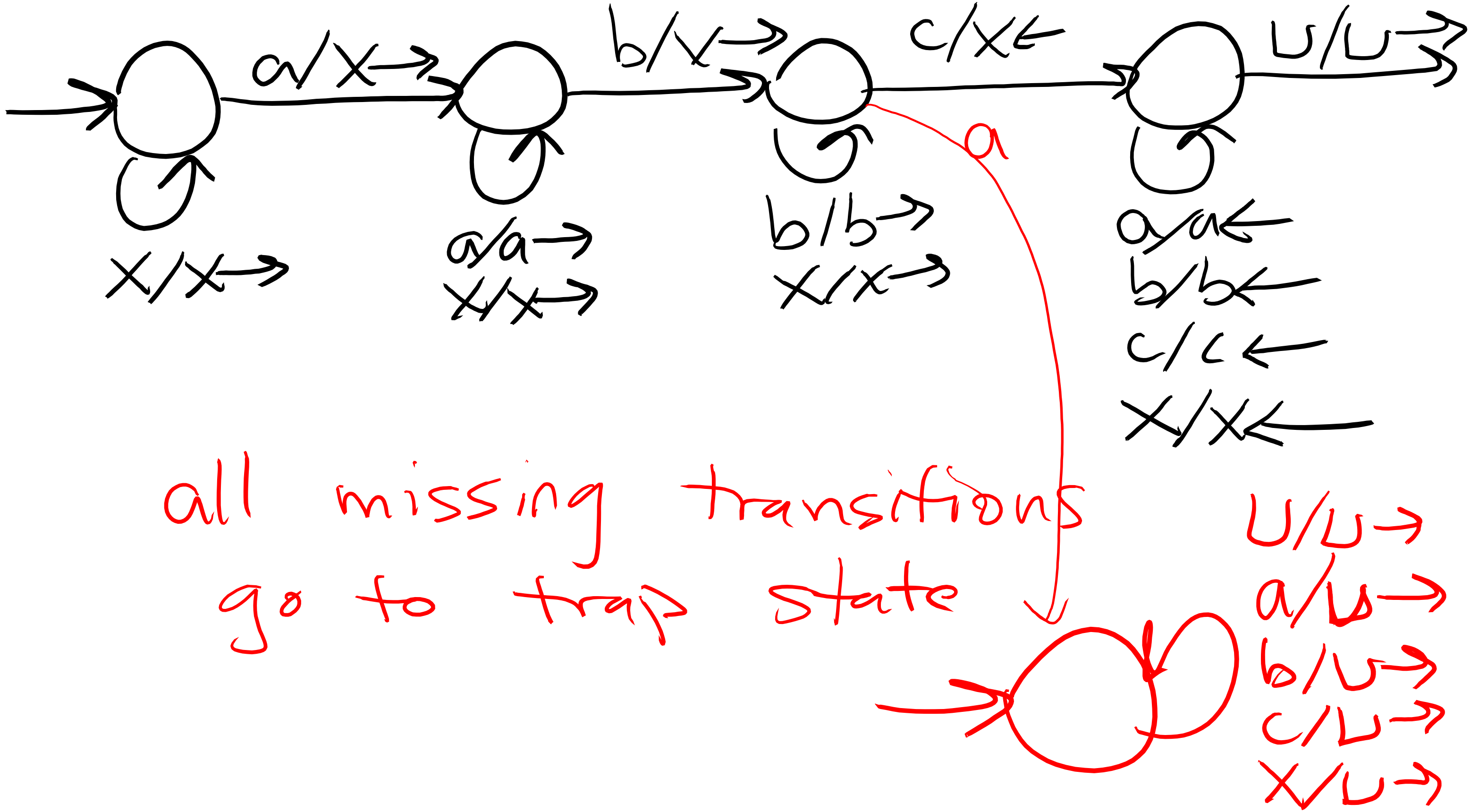
finds a → goto front → goto erase_abc



halt:

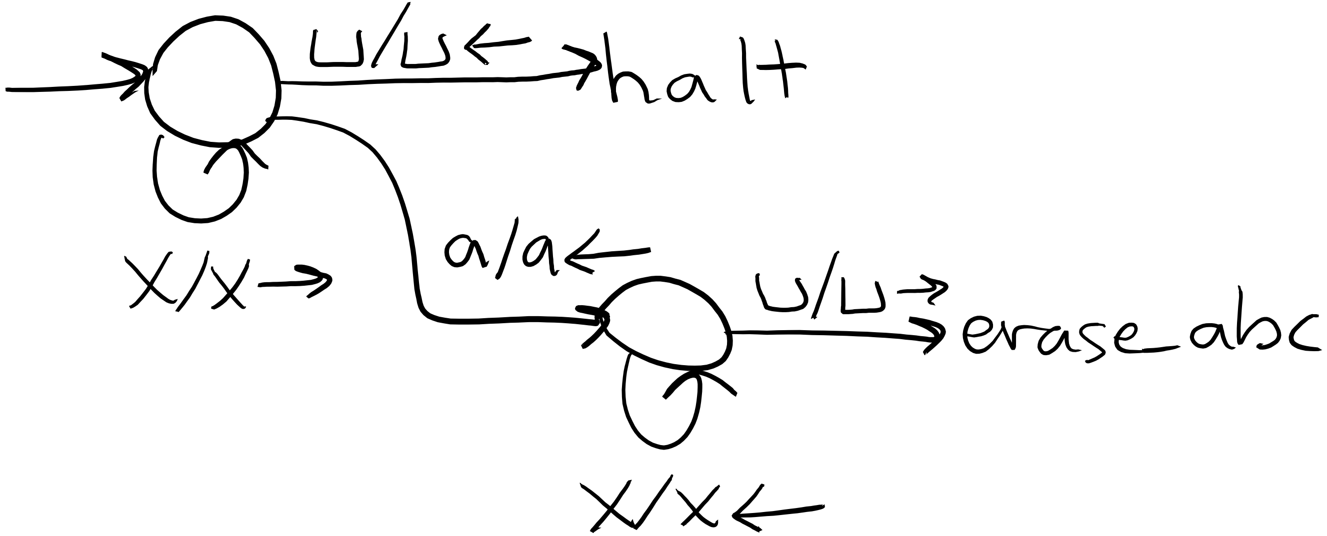


erase_abc :



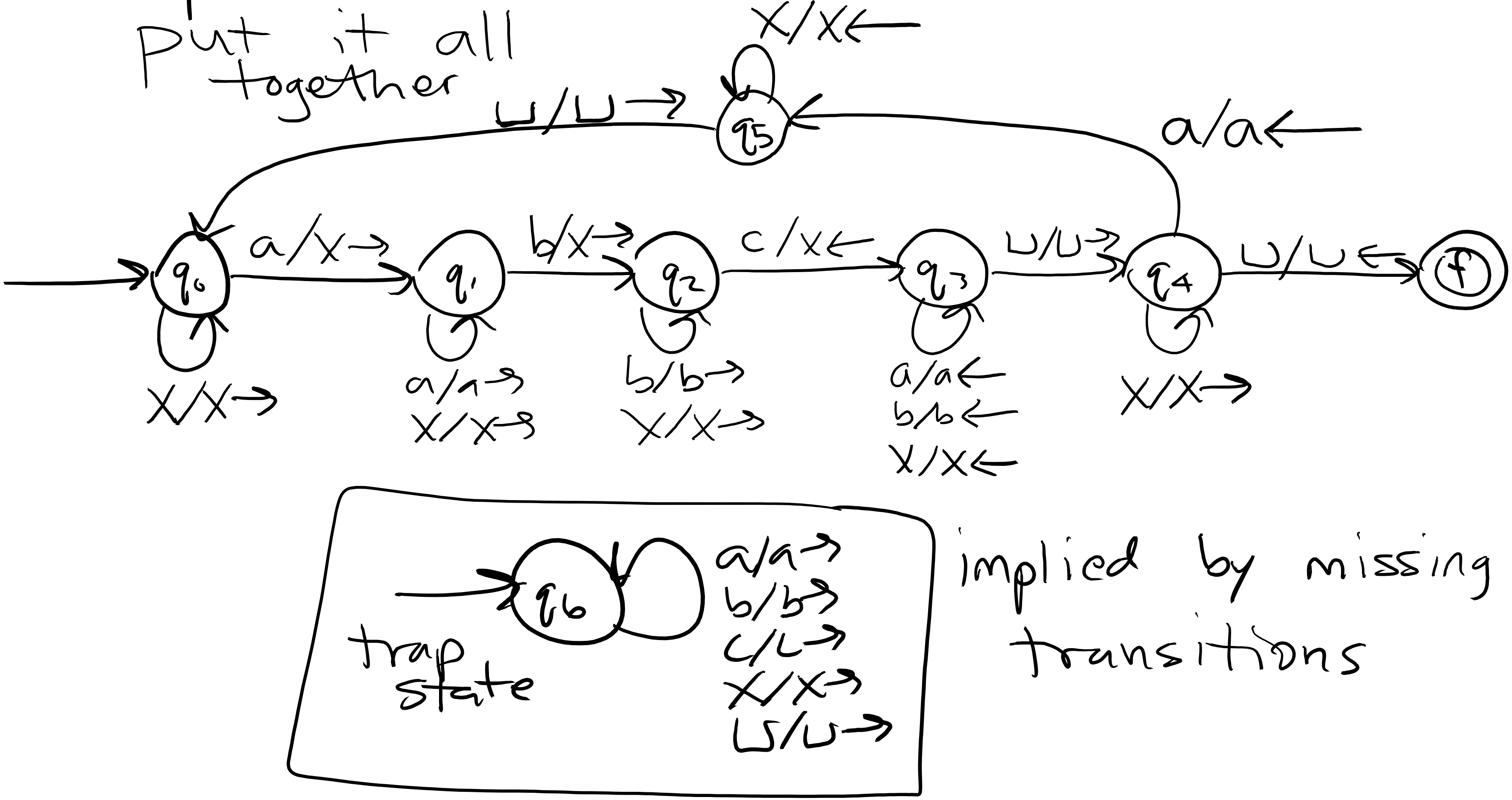
all missing transitions go to trap state

Check:



Complete machine:

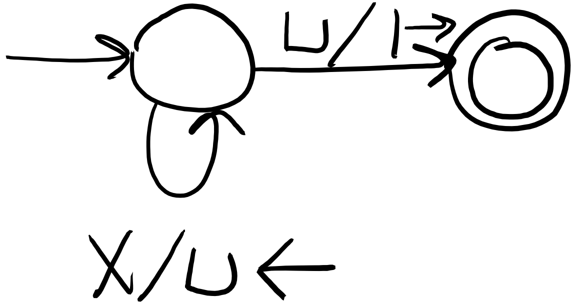
put it all together



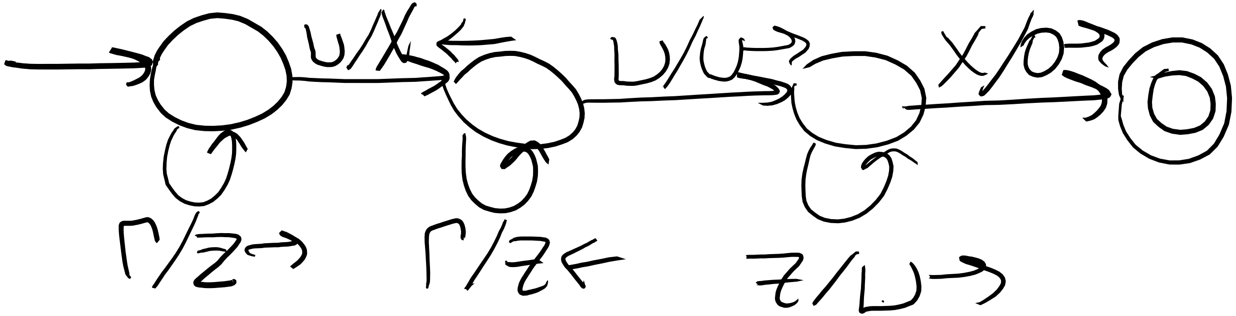
implied by missing transitions

accept by characteristic function

halt:



trap:



Verify $aabbcc \in L$

Verify $ababcc \notin L$

$(q_0, \underline{a}abbcc) \vdash (q_1, X\underline{a}bbcc) \vdash \cdots$

Does the machine accept abcabc?