# Modeling Computation

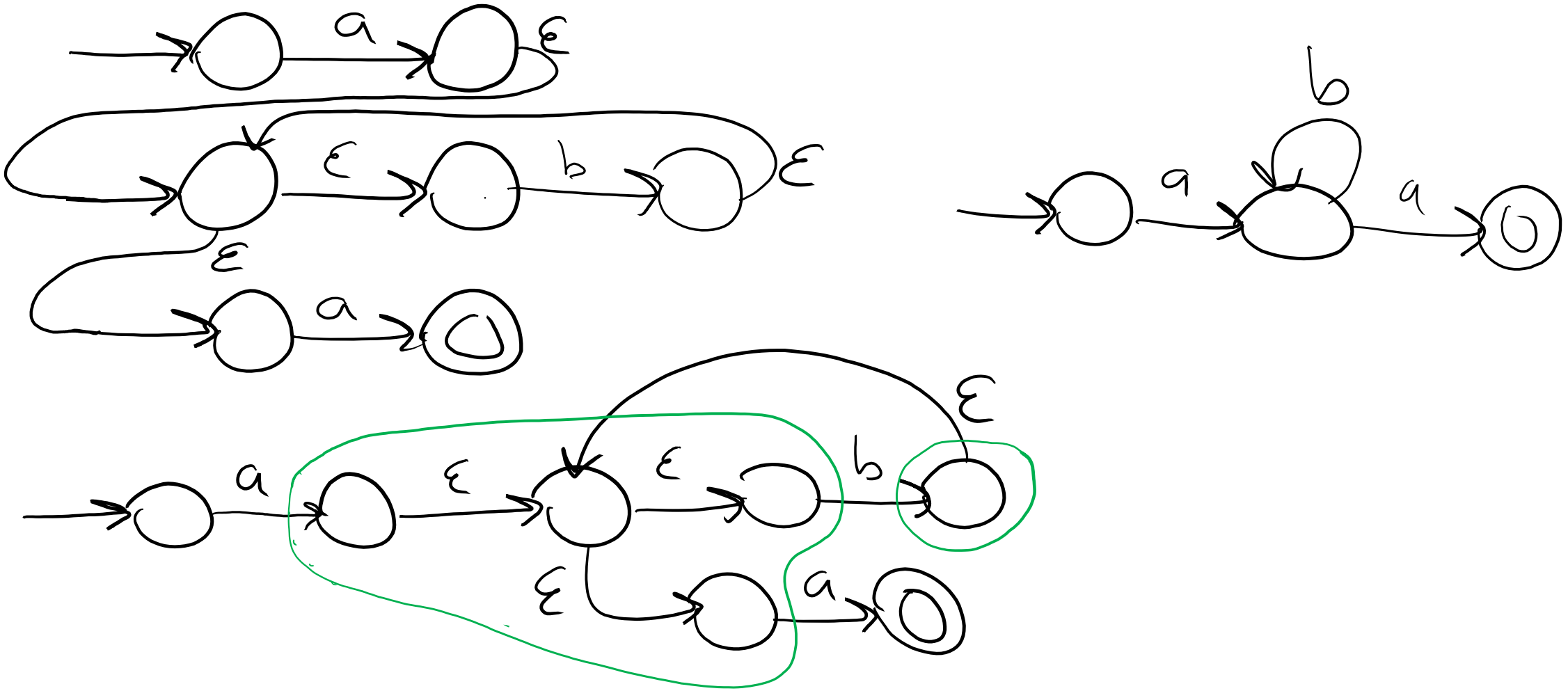
Introduction to Formal Languages and Automata

Regular Expressions and Automata

Any regular expression $\alpha$ can be converted to a finite automaton $A$ such that $L(\alpha) = L(A)$.

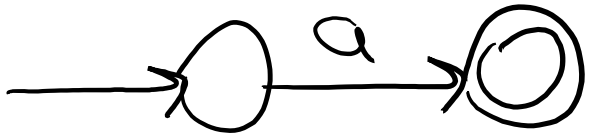
n.b. regular expressions are built from ground elements $a \in \Sigma$, $\varepsilon$, $\emptyset$ to which operations $\cup$, concat, $^*$ are applied.

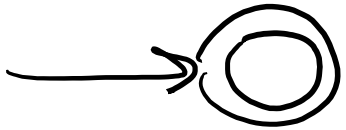
# Example:
# Convert $ab^*a$ to a finite automaton

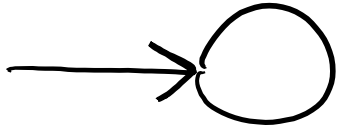


# Algorithm: RegEx to NFA

1. Convert every ground singleton $a \in \Sigma$ or $\epsilon$ or $\emptyset$ (if any) to an automaton accepting just this singleton.

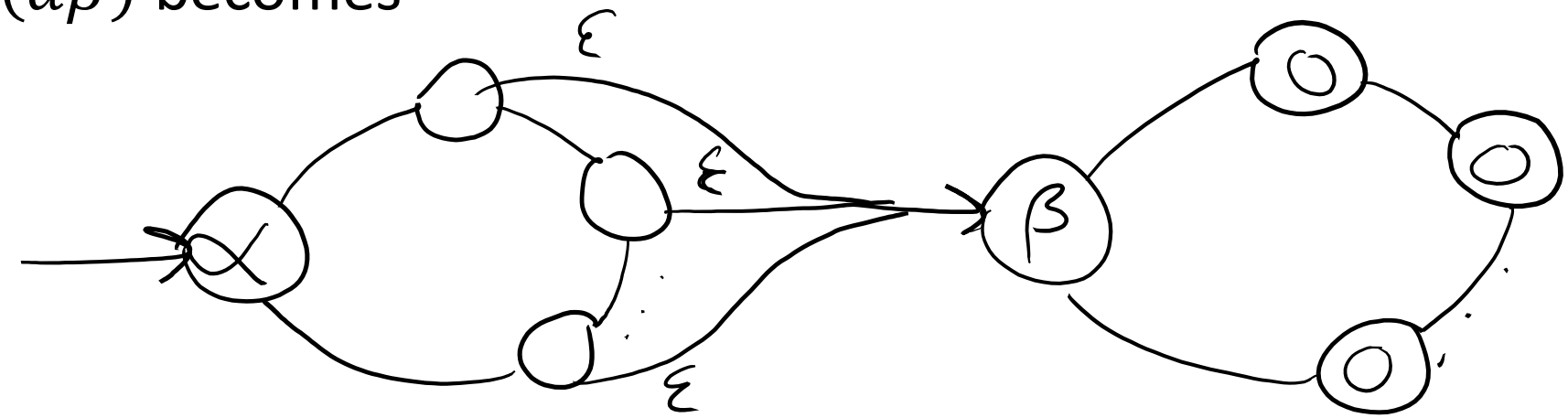
$\{a\}$ becomes 


$\{\epsilon\}$ becomes 

$\emptyset$ becomes 

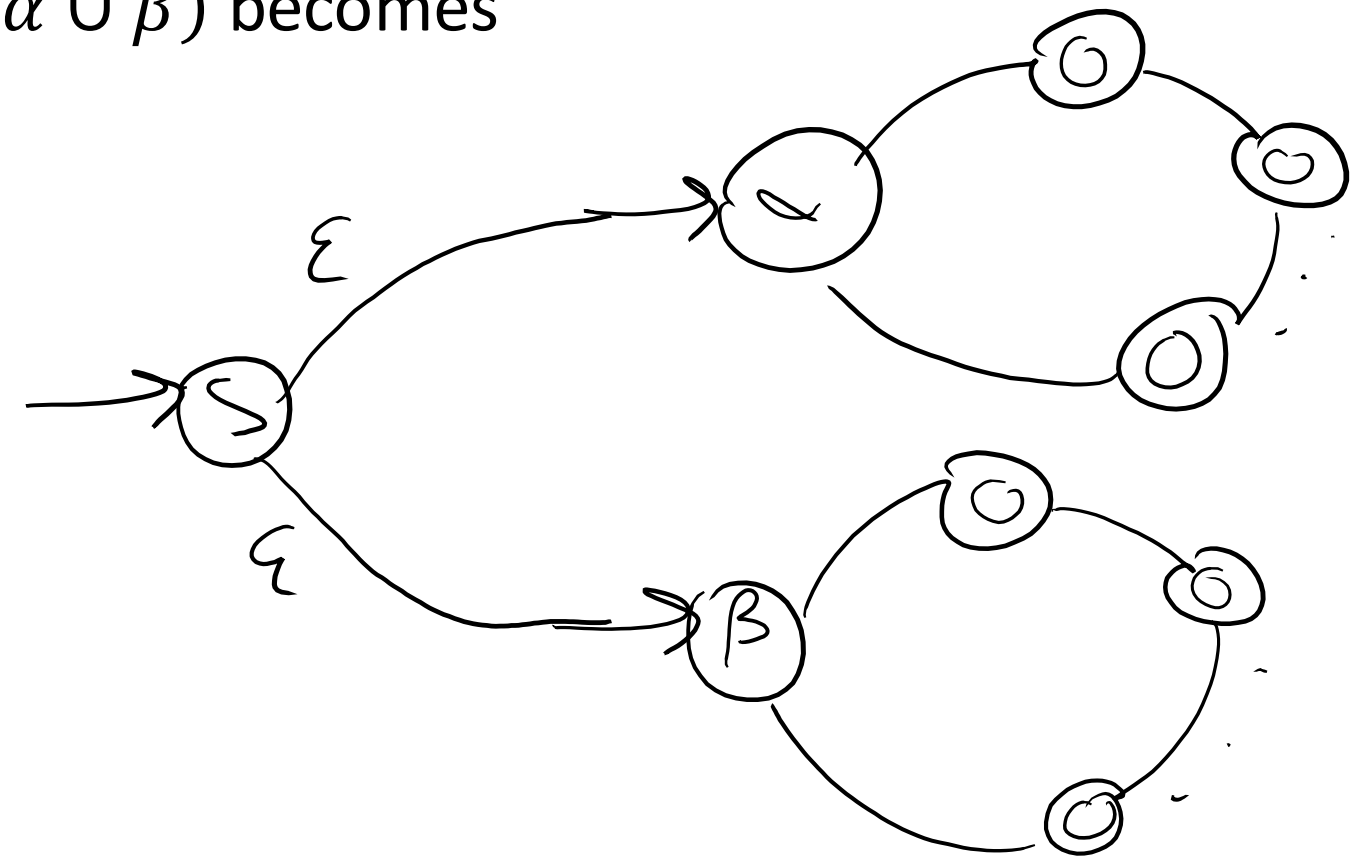
2. Apply **concatenations**, unions, and Kleene stars.

$(\alpha\beta)$ becomes



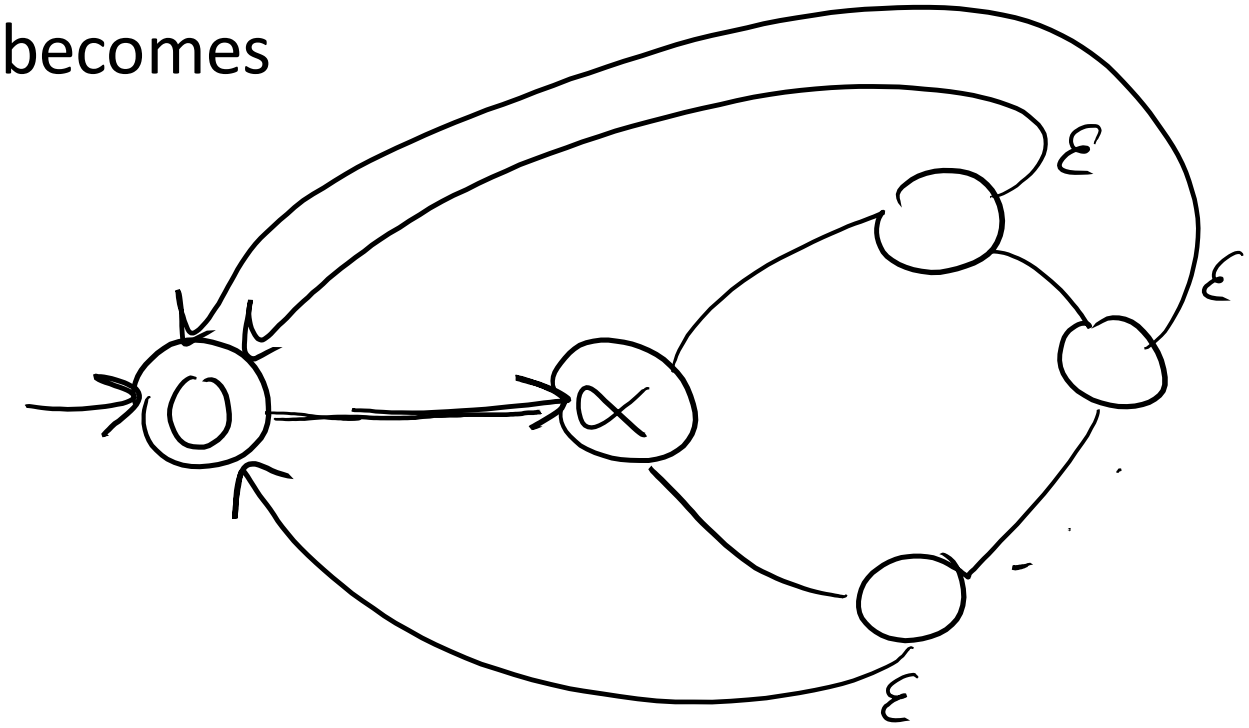
2. Apply concatenations, **unions**, and Kleene stars.

$(\alpha \cup \beta)$ becomes



2. Apply concatenations, unions, and **Kleene stars**.

$\alpha^*$ becomes



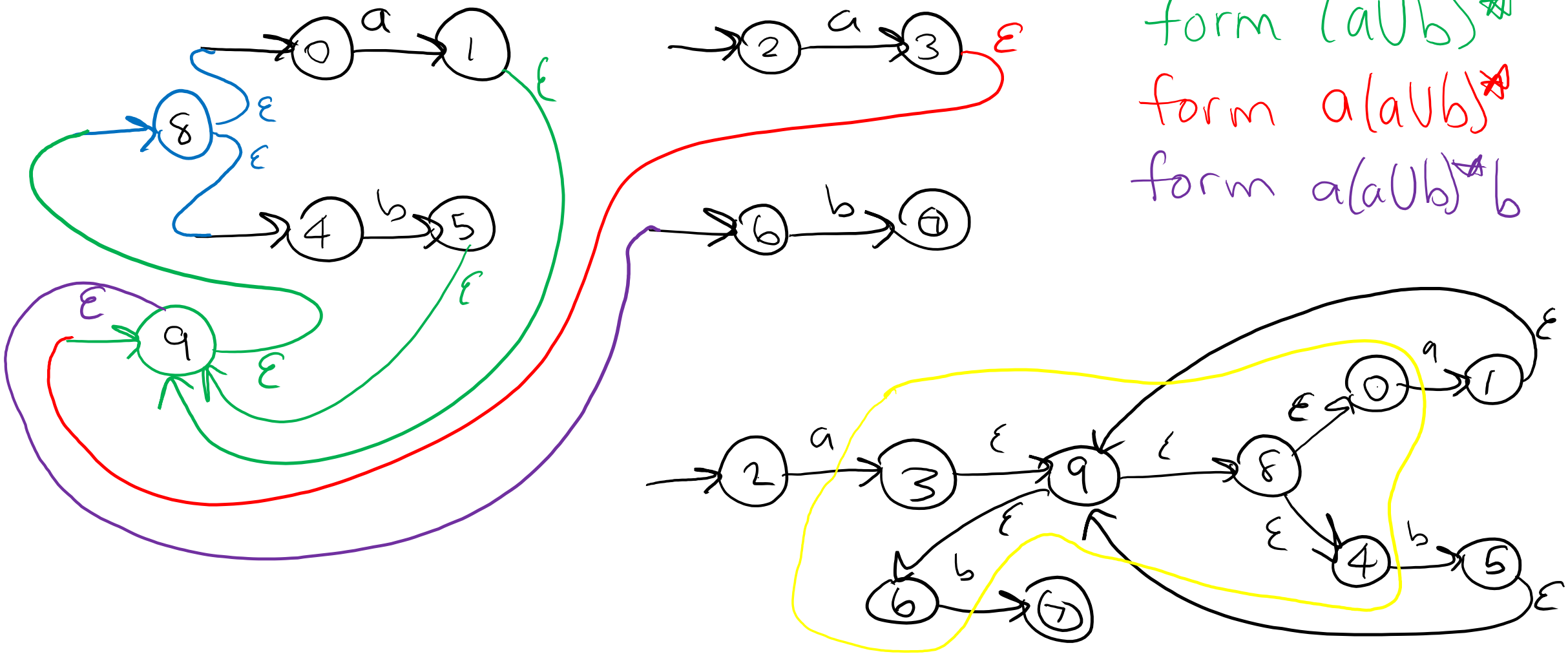
# Exercise: convert $a(a \cup b)^* b$ to an NFA accepting the specified language.

form $(a \cup b)$

form $(a \cup b)^*$

form $a(a \cup b)^*$

form $a(a \cup b)^* b$

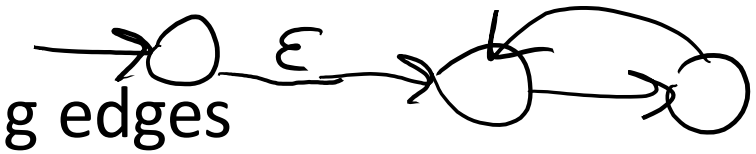


Any finite automaton $A$ can be converted to a regular expression $\alpha$ such that $L(A) = L(\alpha)$.

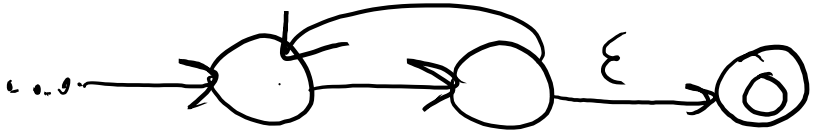
Convert finite state diagram to expression diagram with a single edge from the initial state to the favorable state.

# Assumptions


1. $A$ has a single favorable state
   - If not, make a new favorable state and use ε-jumps to connect old favorable states to new one

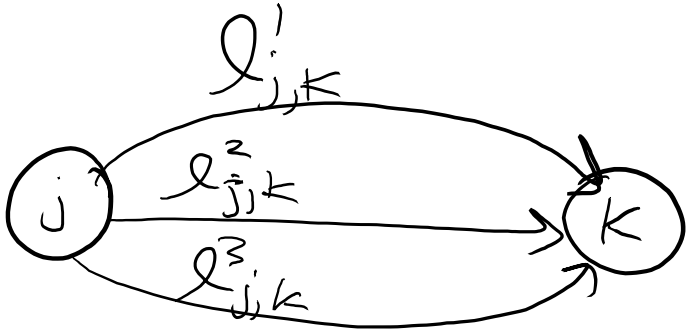



2. $A$'s initial state has no incoming edges
   - If not, make a new initial state and use an ε-jump to connect it to the old initial state




3. $A$'s favorable state has no outgoing edges
   - If not, make a new favorable state and use an ε-jump to connect the old favorable state to it
4. Nodes are labeled 1 (initial state) to $n$ (favorable state)
   - If not, renumber the nodes


# Algorithm: NFA to RegEx




Let $l_{j,k}^{i}$ denote the label of the $i$-th edge from node $j$ to node $k$.

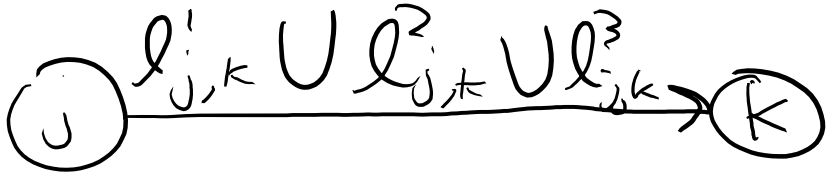
$l_{j,k}$ is used when there is only 1 edge between the nodes.

Whenever there are multiple edges from node $j$ to node $k$ with labels

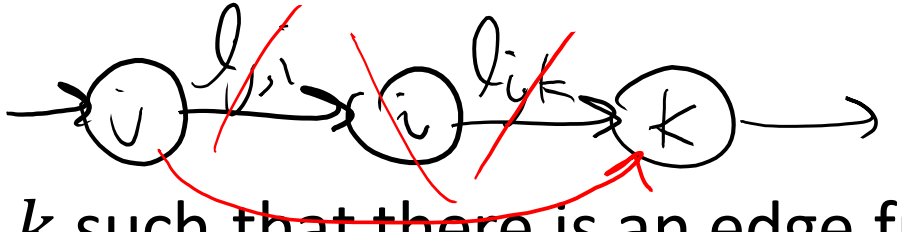
$$l_{j,k}^{1}, l_{j,k}^{2}, \ldots, l_{j,k}^{m}$$

Replace them with a single edge with label

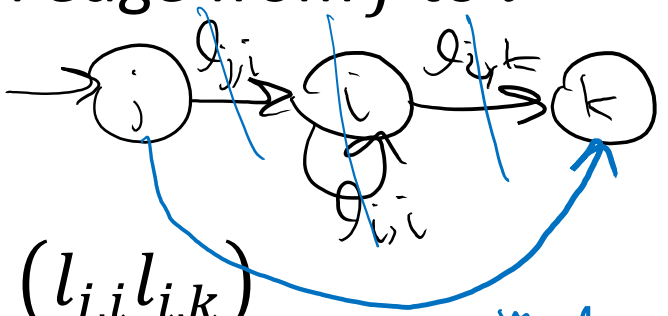
$$l_{j,k}^{1} \cup l_{j,k}^{2} \cup \ \ldots \cup l_{j,k}^{m}$$



**for** $i = 2$ **to** $n - 1$




**for** each pair of nodes $j, k$ such that there is an edge from $j$ to $i$ and an edge from $i$ to $k$,




**if** there is no edge from $i$ to $i$

**then** add an edge from $j$ to $k$ with label $(l_{j,i}l_{i,k})$

**else** add an edge from $j$ to $k$ with label $(l_{j,i}l_{i,i}^{*}l_{i,k})$

**end**

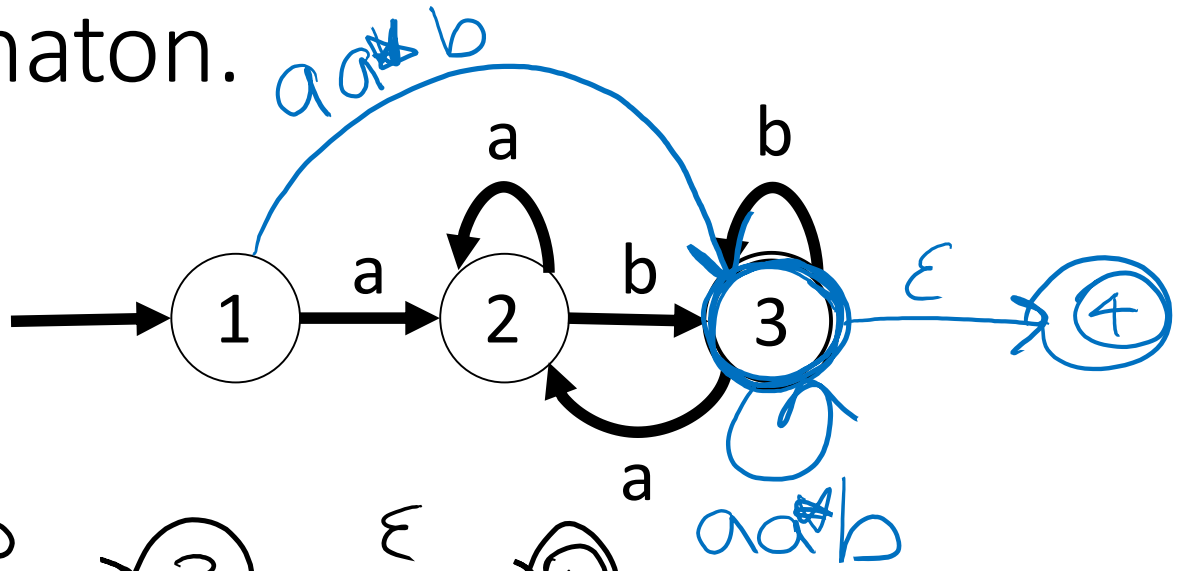
combine multiple edges

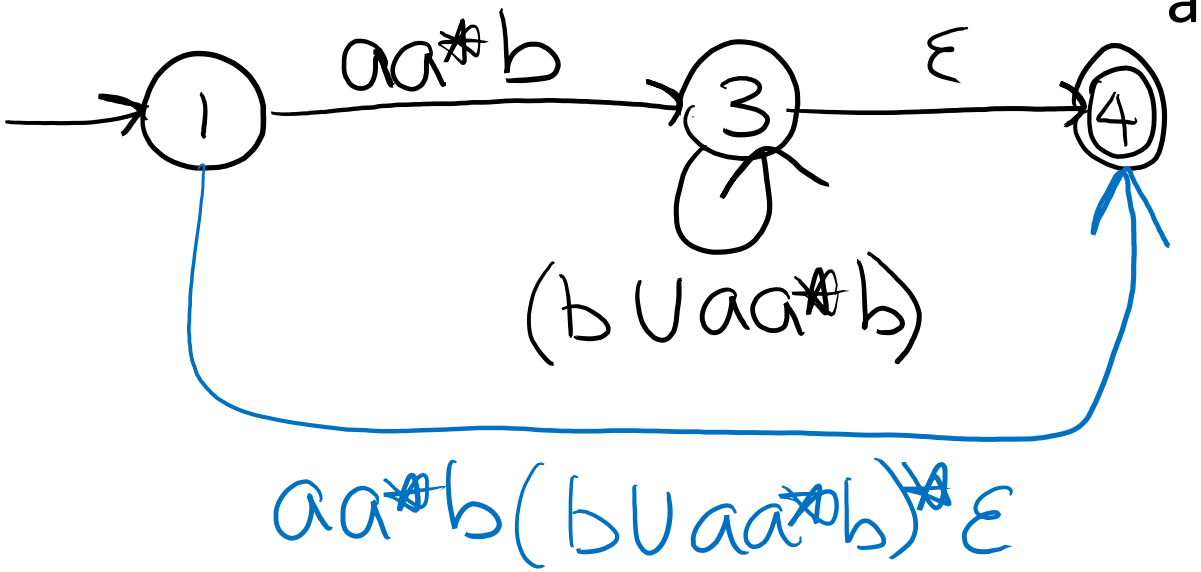
**end**

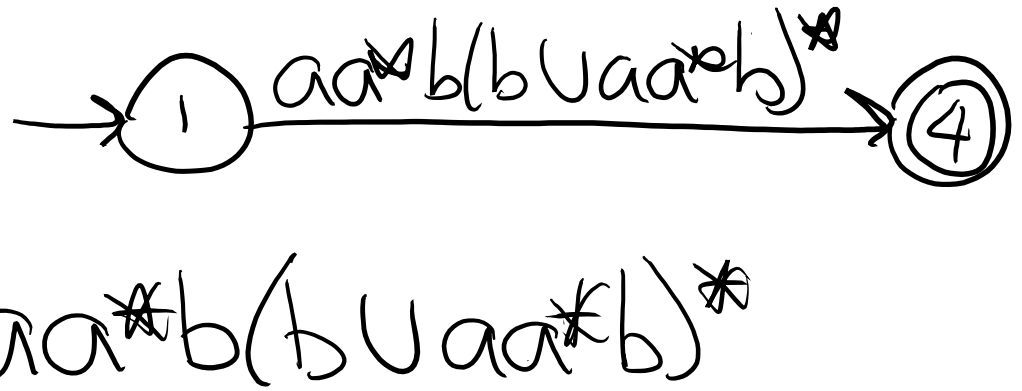
remove from the diagram node $i$ and all edges connected to $i$

**end**

Exercise: convert the automaton to a regular expression representing the language accepted by the automaton.




$aa^*b(b \cup aa^*b)^*$

$aa^*b(b \cup aa^*b)^*$ $= a(a \cup b)^*b$

$aa^*b((\varepsilon \cup aa^*)b)^*$

not easily simplified

verify equivalence by comparing the DFAs