Permutations with Indistinguishable Objects

Ex: How many strings can be formed from the letters SUCCESS? & use every letter

7 objects, indisting. S×3 indisting. C×2

$7!/3!2!$

$S_1U C_1 C_2 E S_2 S_3$

In general: $\frac{n!}{n_1! n_2! \cdots n_k!}$

$n_i = \#$ indistinguishable objects of type i.

$n =$ total # of objects

Distributing Objects into Boxes

Objects: distinguishable or indistinguishable

Boxes: distinguishable or indistinguishable

D-objects, D-boxes

ex: How many ways can 3 hands of 5 cards each be dealt?

$$\binom{52}{5}\binom{47}{5}\binom{42}{5} \checkmark$$

$$\left(\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}\right)\left(\frac{47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{5!}\right)\left(\frac{42 \cdot 41 \cdot 40 \cdot 39 \cdot 38}{5!}\right)$$

$$= \frac{52!}{37! \, 5! \, 5! \, 5!} = \frac{n!}{n_1! \, n_2! \, n_3! \, n_4!}$$

Indisting. objects, Distinguishable boxes

Ex: How many ways can 5 indisting. balls be placed into 7 distinguishable boxes?

$x_1 + x_2 + \cdots + x_7 = 5 \quad x_i \geq 0$

5 stars, 6 bars ★★★★★|||||

$$\frac{11!}{5!\,6!}$$

Distinguishable Objects & Indistinguishable Boxes

Indistinguishable Objects and Boxes

No simple closed form

Enumerate possibilities by hand and count them.

Advanced Counting

Exercise: A robot can take 1-meter or 2-meter steps.

How many ways can the robot walk 10 meters?

ssssssssss, sssssssl, ..., llllll

1 $9 = \binom{9}{8} \binom{7}{4} \binom{6}{2}$ 1

recurrence relation:

$A(10) = A(9) + A(8)$

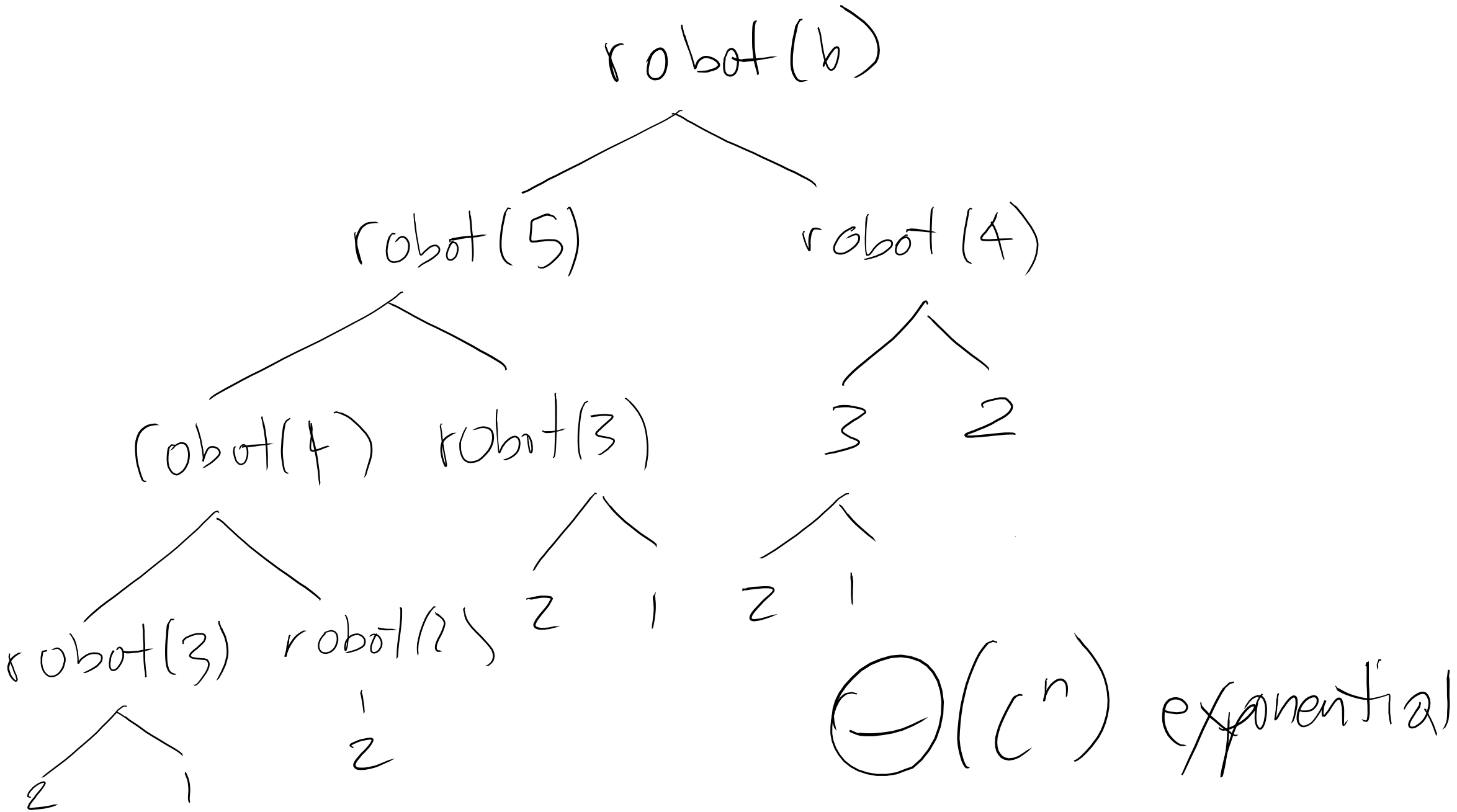
$A(n) = A(n-1) + A(n-2)$

procedure robot(n):

if n=1 or n=2, return n

else, return robot(n-1) + robot(n-2)

complexity?


robot(6)
robot(5)
robot(4)
robot(4)
robot(3)
3
2
robot(3)
robot(2)
2
1
2
1
2
1
$\Theta(c^n)$ exponential

```
procedure robotz(n)
    if n=1, return (1,1)   //(n-1, n)
    (a,b) ← robot(n-1)
    return (b, a+b)
```

robot(6)
|
robot(5)
|
robot(4)
⋮
robot(1)

$\Theta(n)$: linear

Can we go faster?

Yes! Sublinear time algorithm next time