

Binomial coefficients

$\binom{n}{r}$ aka binomial coefficient

$(x+y)^n$

$$\text{Ex: } (x+y)^3 = (x+y)(x+y)(x+y)$$

$$= (x^2 + 2xy + y^2)(x+y)$$

$$= x^3 + x^2y + 2x^2y + 2xy^2 + xy^2 + y^3$$

$$= x^3 + 3x^2y + 3xy^2 + y^3$$

The Binomial Theorem

$$(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$$

$$\begin{aligned} (x+y)^3 &= \binom{3}{0} x^3 y^0 + \binom{3}{1} x^2 y + \binom{3}{2} x y^2 + \binom{3}{3} x^0 y^3 \\ &= x^3 + 3x^2 y + 3x y^2 + y^3 \end{aligned}$$

$$\text{Ex: } \sum_{i=0}^n \binom{n}{i} = (1+1)^n = 2^n$$

Generalized Permutations and Combinations

Permutations:

with repetition: n^r

product rule

without repetition: $\frac{n!}{(n-r)!}$

product rule,
division rule

Combinations:

without repetition: $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

with repetition: New! $\binom{n+r-1}{r}$

Ex: A fruit bowl has 3 kinds of fruit:
Apples, Bananas, Oranges.

How many ways can you choose 4
fruit? (order doesn't matter, ≥ 1 of each in bowl).

represent solution $\ast / \ast\ast / \ast$, $\ast\ast / \ast\ast / \ast$
A B O, A B O

How many ways to order 4 stars and 2 bars?

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$$\frac{6!}{4!2!} = \binom{6}{2} = \binom{6}{4}$$

$$\binom{a+b}{a} \quad \begin{array}{l} a \text{ stars} \\ b \text{ bars} \end{array}$$

3 kinds = $n \rightarrow$ bars + 1
 Choose 4 = $r \rightarrow$ stars

$$\binom{4+3-1}{4} \quad \begin{pmatrix} n+r-1 \\ r \\ n+r-1 \\ n-1 \end{pmatrix}$$

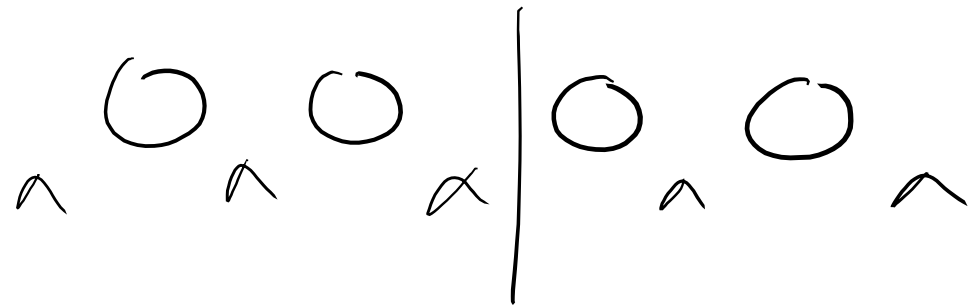
How many solutions?

$$x_1 + x_2 + x_3 = 4 \quad x_i \geq 0$$

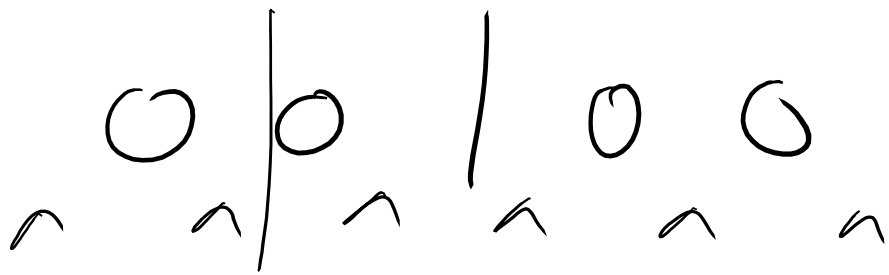
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$$\binom{4+3-1}{4} = \binom{4+3-1}{2} = 15$$



4 fruits
5 places to
partition



6 places to
partition

$$\frac{5 \cdot 6}{2} = 15$$