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Counting

Suppose a password consists of 6, 7, or 8 characters and at least 1 character must be a number.

How many passwords are possible?

1) Product rule

2) Sum rule

The product rule

Suppose a procedure can be broken down into a sequence of 2 tasks. If there are n_1 ways to do the first task and, for each of these ways of doing the first task, there are n_2 ways of doing the second task, then there are

$$n_1 \cdot n_2$$

ways to do the procedure.

Ex: How many strings of length

3 can be formed from the letters A, B, C, D, E if repetition <u>is not</u> allowed?

task 1: choose first letter : 5

task 2: choose the remaining letters : $4 \cdot 3 = 12$

task 2.1: choose the second letter: 4

task 2.2: choose the third letter: 3

$n_1 = 5$

$n_2 = 12$

$\boxed{n_1 \cdot n_2 = 60 \text{ strings}}$

Ex: Same problem.

Repetition <u>is</u> allowed.

task 1: choose first letter: 5

task 2: choose remaining 2 letters: $5 \cdot 5 = 25$

task 2.1: choose second letter: 5

task 2.2: choose third letter: 5

$n_1 = 5$

$n_2 = 25$

$\boxed{n_1 \cdot n_2 = 125 \text{ strings}}$

Ex: How many bitstrings of length 7 are there?

$\underline{}\ \underline{}\ \underline{}\ \underline{}\ \underline{}\ \underline{}\ \underline{}$

2 2 2 2 2 2 2

$= 2^7 = \boxed{128 \text{ bitstrings}}$

The Sum Rule

If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are

$$n_1 + n_2$$

ways to do the task.

Ex: A university committee needs 1 representative from CSE. There are 43 faculty and

527 students in CSE.

In how many ways can the representative from CSE be chosen?

43 ways to choose a faculty member
527 ways to choose a student

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570 ways to choose a rep.

Ex: The food court at the mall has 3 restaurants. Their menus have 23, 15, and 19 entrees respectively. No one entree is on more than 1 menu.

How many options do you have to order 1 entree?

23 entrees @ Taco Bell

15 entrees @ McDonald's

19 entrees @ Chick-fil-A

57 entrees to choose from

Use product rule and sum rule together to count passwords.

How many 6, 7, or 8 character passwords have at least 1 number?

Let Σ be the set of characters that can be used in a password can be used in a password:

Let $\sigma = |\Sigma|$ be the number of characters.

Then $(\sigma - 10)$ is the number of non-digit characters.

P_6 = number of passwords with at least 1 number

= (# 6-char passwords with no restriction) − (# 6-char passwords with no numbers)

$= \sigma^6 - (\sigma - 10)^6$

$P_7 = \sigma^7 - (\sigma - 10)^7$

$P_8 = \sigma^8 - (\sigma - 10)^8$

$P_8 = 62^8 - (62-10)^8$

Total number of passwords with at least 1 number is

$$P = P_6 + P_7 + P_8$$

167,410,949,583,040 passwords

$\Sigma = \{\text{letters and numbers}\}$

3) Subtraction rule

4) Division rule

The Subtraction Rule

(principle of inclusion–exclusion)

If a task can be done either in n_1 ways or n_2 ways, then the number of ways to do the task is

$n_1 + n_2 -$ (# ways to do the task that are common to both sets of ways)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Ex: How many bitstrings of length 8 begin with 111 or end with 0000?

$$\frac{1}{1}\ \frac{1}{1}\ \frac{1}{1}\ \frac{\ }{2}\ \frac{\ }{2}\ \frac{\ }{2}\ \frac{\ }{2}\ \frac{\ }{2} \to 2^5 = N_1$$

$$\frac{\ }{2}\ \frac{\ }{2}\ \frac{\ }{2}\ \frac{\ }{2}\ \frac{0}{1}\ \frac{0}{1}\ \frac{0}{1}\ \frac{0}{1} = 2^4 = N_2$$

$$\frac{1}{1}\ \frac{1}{1}\ \frac{1}{1}\ \frac{\ }{2}\ \frac{0}{1}\ \frac{0}{1}\ \frac{0}{1}\ \frac{0}{1} = 2 = N_3$$

$$\boxed{N = N_1 + N_2 - N_3 = 32 + 16 - 2 = 46}$$

The Division Rule

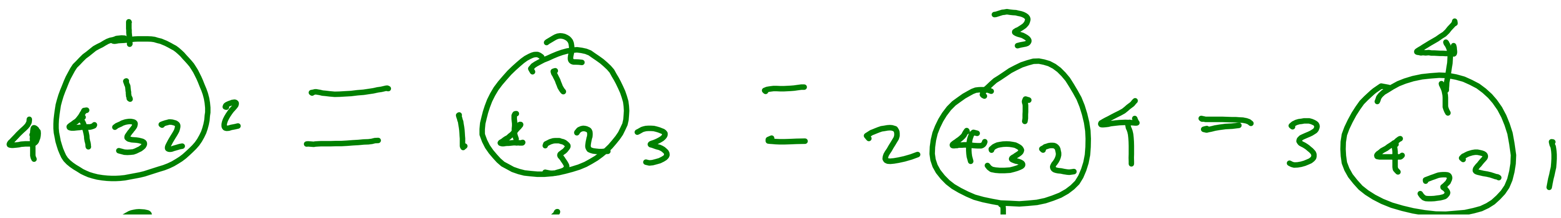
If a task can be done in n ways

and, for each way w, exactly d of the ways correspond to way w, Then, there are

$$n/d$$

ways to do the task.

Ex: How many different ways are there to seat 4 people around a circular table, where 2 seatings are the same if everyone has the same left and right neighbor?



3 4 1 2

4 ways to select a person for seat 1

3 ways 2

2 ways 3

1 way 4

4! = 24 seatings

Each seating is the same as 3 others (4 including itself)

therefore, $\frac{24}{4} = \boxed{6 \text{ seatings are different}}$