Counting

Suppose a password consists of 6, 7, or 8 characters and at least 1 character must be a number.

How many passwords are possible?

1) Product rule
2) Sum rule
The product rule

Suppose a procedure can be broken down into a sequence of 2 tasks. If there are \( N_1 \) ways to do the first task and, for each of these ways of doing the first task, there are \( N_2 \) ways of doing the second task, then there are

\[ N_1 \cdot N_2 \]

ways to do the procedure.

Ex: How many strings of length
3 can be formed from the letters A, B, C, D, E if repetition is not allowed?

**Task 1**: choose first letter: 5
**Task 2**: choose the remaining letters: 4 \cdot 3 = 12
  **Task 2.1**: choose the second letter: 4
  **Task 2.2**: choose the third letter: 3

\[ n_1 = 5 \quad n_1 \cdot n_2 = 60 \text{ strings} \]

\[ n_2 = 12 \]

**Ex**: Same problem. Repetition is allowed.
Task 1: Choose first letter: 5
Task 2: Choose remaining 2 letters: \(5 \times 5 = 25\)
Task 2.1: Choose second letter: 5
Task 2.2: Choose third letter: 5

\[ n_1 = 5 \quad n_1 \cdot n_2 = 125 \text{ strings} \]
\[ n_2 = 25 \] 

Example: How many bitstrings of length 7 are there?

\[ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \]

\[ = 2^7 = 128 \text{ bitstrings} \]
The Sum Rule

If a task can be done either in one of \( n_1 \) ways or in one of \( n_2 \) ways, where none of the set of \( n_1 \) ways is the same as any of the set of \( n_2 \) ways, then there are \( n_1 + n_2 \) ways to do the task.

Ex: A university committee needs 1 representative from CSE. There are 43 faculty and
527 students in CSE.

In how many ways can one representative from CSE be chosen?

43 ways to choose a faculty member
527 ways to choose a student

570 ways to choose a rep.

Ex: The food court at the mall has 3 restaurants. Their menus have 23, 15, and 19 entrees respectively. No one entree is on more than 1 menu.
How many options do you have to order 1 entree?

23 entrees @ Taco Bell
15 entrees @ McDonald's
19 entrees @ Chick-fil-A

57 entrees to choose from

Use product rule and sum rule together to count passwords.

How many 6, 7, or 8 character passwords have at least 1 number?

Let $\mathcal{Z}$ be the set of characters that can be used in a password.
Let $\sigma = |\Sigma|$ be the number of characters. Then $(\sigma - 10)$ is the number of non-digit characters.

$P_6 = \text{number of passwords with at least 1 number}$

$= (\# 6\text{-char passwords with no restriction}) - (\# 6\text{-char passwords with no numbers})$

$= \sigma^6 - (\sigma - 10)^6$

$P_7 = \sigma^7 - (\sigma - 10)^7$

$+ \cdots + 8$
\[ r_8 = \sigma^8 - (\sigma^2 - 10) \]

Total number of passwords with at least 1 number is

\[ P = P_6 + P_7 + P_8 \]

167,410,949,583,040 passwords

\( Z = \) \{letters and numbers\}

3) Subtraction rule

4) Division rule

The Subtraction Rule

(principle of inclusion-exclusion)
If a task can be done either in \( N_1 \) ways or \( N_2 \) ways, then the number of ways to do the task is

\[ N_1 + N_2 - (\text{# ways to do the task that are common to both sets of ways}) \]

\[ |A \cup B| = |A| + |B| - |A \cap B| \]

Ex: How many bitstrings of length 8 begin with 111 or end with 0000?
\[
\frac{1}{1} \frac{1}{1} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} = 2^5 = N_1
\]
\[
\frac{0}{2} \frac{0}{2} \frac{0}{2} \frac{0}{1} \frac{1}{1} \frac{1}{1} = 2^4 = N_2
\]
\[
\frac{0}{6} \frac{0}{6} \frac{0}{6} \frac{0}{1} \frac{1}{1} \frac{1}{1} = 2 = N_3
\]

\[
N = N_1 + N_2 - N_3 = 32 + 16 - 2 - 46
\]

The Division Rule

If a task can be done in \( n \) ways
and, for each way \( w \), exactly \( d \) of the ways correspond to way \( w \).

Then, there are \( N/d \) ways to do the task.

Ex: How many different ways are there to seat 4 people around a circular table, where 2 seatings are the same if everyone has the same left and right neighbors?

\[
\begin{align*}
4! & = \frac{4!}{2!} = \frac{4 \times 3 \times 2}{2} = 3 \times 4 \times 3 \times 2 \times 1
\end{align*}
\]
4 ways to select a person for seat 1
3 ways
2 ways
1 way

4! = 24 settings

Each setting is the same as 3 others (4 including itself)

Therefore, \( \frac{24}{4} = 6 \) settings are different