CSCE 222 [503] Discrete Structures for Computing
Spring 2015 – Philip C. Ritchey

Problem Set 10

Due dates: Electronic submission of the PDF file for this homework is due on **4/16/2015 (Thursday) before 11:59 p.m.** on `http://ecampus.tamu.edu`. A signed and stapled paper copy of the PDF is due on **4/17/2015 (Friday)** at the beginning of class.
You must show your work. **No work → no credit.**

Name: YOUR NAME

Resources. Discrete Mathematics and Its Applications by Rosen, ADDITIONAL PEOPLE, BOOKS, ARTICLES, WEB PAGES, ETC. THAT HAVE BEEN CONSULTED WHEN PRODUCING THIS HOMEWORK. FAILURE TO CITE SOURCES WILL RESULTS IN FAILURE TO PASS THIS CLASS.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. (10 points)

a) What is the coefficient of x^8 in $(1+x)^{12}$?

b) What is the coefficient of x^4 in $\left(x+\frac{1}{x}\right)^8$?

Problem 2. (10 points) How many solutions does $x_0 + x_1 + x_2 + x_3 + x_4 = 19$ have when

a) $x_i \geq 0$?

b) $x_i \geq i$?

Problem 3. (10 points) How many different strings can be formed from the letters ENGINEERING using all of the letters?

Problem 4. (10 points) Find the next permutation in lexicographic order after each of these permutations:

a) 1234

b) 25134

c) 241563

d) 5472361

e) 68754321

Problem 5. (10 points) A robot can climb stairs 1, 2, or 3 steps at a time. Give a recurrence relation for the number of ways the robot can climb n stairs and use it to compute the number of ways the robot can climb 9 stairs.

Problem 6. (10 points) A *ternary string* is a string that contains only 0s, 1s, and 2s.

a) Find a recurrence relation for the number of ternary strings of length n that contain two consecutive symbols that are the same.

b) Find a closed formula for your recurrence relation.

c) Prove the correctness of your closed formula using induction.

d) How many ternary strings of length six contain two consecutive symbols that are the same?

Problem 7. (10 points) Which of these are linear homogeneous recurrence relations with constant coefficients? For each that is, find its degree.

a) $a_n = 4a_{n-3}$

b) $a_n = 3a_{n-1}^2$

c) $a_n = \dfrac{a_{n-1}}{2n}$

d) $a_n = a_{n-1} + a_{n-2} + n + 3$

e) $a_n = 2a_{n-1} + 4a_{n-3} + 6a_{n-6}$

Problem 8. (10 points) Solve these linear homogeneous recurrence relations of degree 2 with constant coefficients:

a) $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 2, a_0 = 2, a_1 = 3$

b) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 2, a_1 = 1$

Problem 9. (10 points) Suppose that $f(n) = 2f(n/3) + 2$ when n is an even positive integer, and $f(1) = 3$. Find:

a) $f(3)$

b) $f(27)$

c) $f(729)$

d) $f(6561)$

Problem 10. (10 points) Give big-O estimates for the increasing function f that satisfies each recurrence realtion:

a) $f(n) = 2f(n/3) + 2$

b) $f(n) = 4f(n/2) + 3n^2$

c) $f(n) = 5f(n/3) + 4n$

d) $f(n) = 7f(n/2) + 5n^3$

Checklist:

( ) Did you add your **name**?

( ) Did you disclose all **resources** that you have used?
(This includes all people, books, websites, etc. that you have consulted)

( ) Did you **sign** that you followed the Aggie honor code?

( ) Did you solve **every problem**?

( ) Did you submit the PDF file of your homework on **eCampus**?

( ) Did you submit a **signed and stapled** hardcopy of the PDF file **in class**?