CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 8

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **11 November 2015 (Wednesday) before 11:30 a.m.** on gradescope (`http://gradescope.com`).

Names of Group Members
YOUR NAME
COLLABORATOR 1
COLLABORATOR 2

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing this homework)

Problem 1. (10 points)
You have an 8-gallon bucket of water an empty 5-gallon bucket, and an empty 3-gallon bucket, Prove or disprove that you can measure 4 gallons of water by successively pouring some or all of the water from one bucket into another bucket.

Solution.

Problem 2. (15 points)
Three numbers are in arithmetic progression.
Three other numbers are in geometric progression.
The corresponding terms of the two progressions sum to 85, 76, 84 respectively.
The sum of all three terms of the arithmetic progression is 126.
Find the terms of both progressions.

Solution.

Problem 3. (15 points)
Show that

$$\sum_{i=1}^{n} i^4 \log^2 i = \Theta(n^5 \log^2 n)$$

Solution.

Problem 4. (15 points)
A guest at a party is a celebrity if this person is known by every other guest, but knows none of them. There is at most one celebrity at a party, for if there were two, they would know each other. A particular party may have no celebrity. Your assignment is to find the celebrity, if one exists, at a party, by asking only one type of question – asking a guest whether they know a second guest. Everyone must answer your questions truthfully. That is, if Alice and Bob are two people at the party, you can ask Alice whether she knows Bob; she must answer correctly. Use mathematical induction to show that if there are n people at the party, then you can find the celebrity, if there is one, with $3(n-1)$ questions.
Hint: First ask a question to eliminate one person as a celebrity. Then use the inductive hypothesis to identify a potential celebrity. Finally, ask two more questions to determine whether that person is actually a celebrity.

Solution.

Problem 5. (15 points)
How many ways are there to travel in $xyzw$ space from the origin (0,0,0,0) to the point (8,6,7,5) by taking steps one unit in the positive x, positive y, positive z, or positive w direction?

Solution.

Problem 6. (15 points)

(a) Set up a divide-and-conquer recurrence relation for the number of modular multiplications required to compute $a^n \pmod{m}$, where a, m, and n are positive integers, using the recursive algorithm from Example 4 in Section 5.4.

(b) Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of modular multiplications used to compute $a^n \pmod{m}$ using the recursive algorithm.

Solution.

Problem 7. (15 points)
Let $R = \{(a, b) \in \mathbb{Z}^2 \mid |a| = |b|\}$.
Show that R is an equivalence relation and specify the equivalence classes of R.

Solution.

Aggie Honor Statement: On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Checklist:

1. Did you type your full name and that of all collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit your PDF file?