CSCE 669-601 Computational Optimization

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Assignment # 1

(Due February 19)

1. Here we consider only unweighted graphs. A matching M in a graph G is *maximal* (note: NOT maximum) if for any edge e in G, $e \notin M$, $M \cup \{e\}$ is not a matching.

(1) Prove: for any graph G, the size of a maximal matching in G is at least one half of the size of a maximum matching in G. (The *size* of a matching is the number of edges in the matching.)

(2) Develop a linear-time algorithm that constructs a maximal matching of a graph.

2. We can consider the MAXIMUM-FLOW problem for flow-networks in which vertices also have capacities. Therefore, a flow network G is a directed graph with a source vertex s and a sink vertex t in which each edge $[u, v]$ has a capacity $\text{cap}_e[u, v] > 0$, and each vertex w, $w \neq s, t$, also has a capacity $\text{cap}_v(w) > 0$. The vertex capacity $\text{cap}_v(w)$ means that the total amount of flow getting into the vertex w (which is equal to the amount of flow getting out of w) cannot be larger than $\text{cap}_v(w)$, i.e., $\sum_{[v,w] \text{ is an edge}} f(v, w) \leq \text{cap}_v(w)$.

Develop an algorithm to solve this extended MAXIMUM-FLOW problem. (Hint: First convert it into the regular MAXIMUM-FLOW problem.)

3. Formulate the following problem into LINEAR-PROGRAMMING (you do not need to solve it):

> A farmer has a field of 70 acres in which he plants potatoes and corn. The seed for potatoes costs \$20/acre, the seed for corn costs \$60/acre and the farmer has set aside \$3000 to spend on seed. The profit per acre of potatoes is \$150 and the profit for corn is \$50 an acre. Find the optimal solution for the farmer (the maximum profit and the amount of corn and potatoes it takes to get the maximum profit).

4. The 2-Makespan problem is defined as follows: given n jobs, where the i-th job has processing time t_i, $1 \leq i \leq n$, schedule the jobs on two identical machines (i.e., partition the jobs into two sets and let each set be executed by a machine) so that the time to complete the n jobs is minimized.

Show that the 2-Makespan problem can be polynomial-time reducible to the Integer-Linear-Programming problem.

(Remark. It is known that the 2-Makespan problem is NP-hard. Thus, the above reduction shows that the Integer-Linear-Programming problem is NP-hard.)