CSCE-637 Complexity Theory

Fall 2020

Instructor: Dr. Jianer Chen
Office: HRBB 338C
Phone: 845-4259
Office Hours: TR 1:30 pm–3:00 pm

Assignment # 3
(Due November 10)

1. Recall that a directed graph G is *strongly connected* if for each pair of vertices (v, w) in G, there exist a directed path from v to w and a directed path from w to v. Define a language

$$\text{S-CONN} = \{G \mid G \text{ is a strongly connected directed graph}\}.$$

Prove that S-CONN is in NL (you only need to present a high-level description of the NL machine that accepts S-CONN).
  Extra Credit. Prove that S-CONN is NL-complete under the L-reduction.

2. A complexity class $\mathcal{C}$ is *closed under union and intersection* if for any two languages L_1 and L_2 in $\mathcal{C}$, the languages $L_1 \cup L_2$ and $L_1 \cap L_2$ are also in $\mathcal{C}$. Prove that the probabilistic classes BPP and R are closed on union and intersection.

3. Prove: if SAT is in BPP, then it is in R. You may look at Example 10.3 in the textbook for a hint for solving this problem. By this result, show that if NP $\subseteq$ BPP, then NP = R.