CSCE-637 Complexity Theory

Fall 2020

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Assignment # 2

(Due October 20)

1. A language L_1 is *Turing reducible* to another language L_2, written as $L_1 \leq_T^p L_2$, if there is a deterministic polynomial-time oracle Turing machine that uses L_2 as its oracle and accepts the language L_1. A language L is *NP-hard under Turing reducibility* if every language in NP is Turing reducible to L. Prove: (1) if an NP-hard language under Turing reducibility is in P, then P = NP; and (2) If a language L_1 is Karp (i.e., polynomial-time many-one) reducible to another language L_2, then L_1 is Turing reducible to L_2.

2. Define a language $\textsc{UnSat} = \{F \mid F \text{ is an unsatisfiable CNF formula}\}$. Prove: $\textsc{UnSat}$ is NP-hard under Turing reducibility, but is unlikely to be NP-hard under Karp reducibility.

3. Prove: the polynomial-time hierarchy PH has no complete languages under the polynomial-time reduction unless PH collapses.

4. In the class, we showed that a problem A is in Σ_k^p if and only if A can be written as

$$A = \{x \mid \exists_{|y_1| \leq p_A(|x|)} y_1 \forall_{|y_2| \leq p_A(|x|)} y_2 \cdots Q_{|y_k| \leq p_A(|x|)} F_A(x, y_1, y_2, \ldots, y_k) = 1\},$$

where F_A is a polynomial-time computable Boolean function. Similarly, a problem B is in Π_k^p if and only if B can be written as

$$B = \{x \mid \forall_{|y_1| \leq p_B(|x|)} y_1 \exists_{|y_2| \leq p_B(|x|)} y_2 \cdots Q_{|y_k| \leq p_B(|x|)} F_B(x, y_1, y_2, \ldots, y_k) = 1,$$

where F_B is a polynomial-time computable Boolean function.

Use these characterizations to prove that if for some $k \geq 1$, $\Sigma_k^p = \Pi_k^p$, then PH $= \Sigma_k^p$.