# CSCE-637 Complexity Theory

## Fall 2020

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## Assignment # 1

### (Due September 17, 2020)

1. Write a detailed description of a 1-tape Turing machine that accepts the following language:

$$\text{UnEqual} = \{x\#y \mid x \text{ and } y \text{ are binary numbers such that } x \neq y\}.$$

2. A language $L$ is *decidable* if there is a Turing machine that always halts and accepts $L$. We say that *a language $L_1$ is reducible to another language $L_2$* if there is a Turing machine (i.e., an algorithm) that always halts, and on any (yes or no) instance $x_1$ of $L_1$, produces an instance $x_2$ of $L_2$ such that $x_1$ is a yes-instance of $L_1$ if and only if $x_2$ is a yes-instance of $L_2$.

Consider the following language:

$$\textsc{Test} = \{(M; x, y) \mid \text{on input } x, \text{ the Turing machine } M \text{ outputs } y\}.$$

Show that the problem TEST is undecidable. (*Hint:* write an algorithm that reduce HALTING problem to TEST, and use the fact that HALTING is undecidable.)

3. Prove: if $\mathbf{P} = \mathbf{NP}$, then every non-trivial problem in $\mathbf{P}$ is $\mathbf{NP}$-complete. A problem is *non-trivial* if it has both YES-instances and NO-instances.

4. Give a detailed proof for the following statement: if $L_1 \leq_L L_2$ and $L_2 \leq_L L_3$, then $L_1 \leq_L L_3$.