## CSCE 629-601 Analysis of Algorithms

## Fall 2022

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## Assignment # 6(Due December 5, 2021)

**1.** A vertex cover in an undirected graph G is a set C of vertices in G such that every edge in G has at least one end in C. Consider the following two versions of the VERTEX-COVER problem:

VC-D: Given a graph G and an integer k, decide whether G contains a vertex cover of at most k vertices.

VC-O: Given a graph G, construct a minimum vertex cover for G.

Prove: VC-D is in  $\mathcal{P}$  if and only if VC-O is in  $\mathcal{P}$ .

In Q2-Q3, you need to prove the  $\mathcal{NP}$ -completeness of some problems. To prove a problem is  $\mathcal{NP}$ -complete, you need to prove (1) the problem is in  $\mathcal{NP}$ , and (2) the problem is  $\mathcal{NP}$ -hard.

2. Using the fact that the INDEPENDENT SET problem is  $\mathcal{NP}$ -complete, prove that the following problem is  $\mathcal{NP}$ -complete:

CLIQUE: Given a graph G and an integer k, is there a set C of k vertices in G such that for every pair v and w of vertices in C, v and w are adjacent in G?

**3.** Using the fact that the PARTITION problem is  $\mathcal{NP}$ -complete, prove that the following problem is  $\mathcal{NP}$ -complete:

KNAPSACK: given n items of sizes  $s_1, s_2, \ldots, s_n$  and values  $v_1, v_2, \ldots, v_n$ , resectively, a knapsack of size S, and a value objective V, can we select some of these items to fit into the knapsack so that the total value of the selected items is at least V?