

CSCE 629-601 Analysis of Algorithms

Fall 2022

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Assignment # 2 (Due September 30)

1. Let U be a (large) set with $|U| \geq nm$. Suppose that we are storing a set of n elements of U using a hash function from U to $[0..m-1]$. Show that no matter what hash function h we use, there is a subset S_h of n elements of U that are all hashed by h to the same slot. Note that this shows that in the worst case, searching in a set of n elements by hashing can be very bad and take time $\Theta(n)$.

2. Let S_n be a set of n elements in $[0..N-1]$. Suppose that a prime number $p \geq N$, and an array $H[0..n^2-1]$ are also given, where all elements in $H[0..n^2-1]$ have an initialized value -1 . Develop a randomized algorithm of time $O(n)$ that, with a probability at least 99%, constructs a hash function h from $[0..N-1]$ to $[0..n^2-1]$ such that h is perfect from S_n . (**Hint** Your randomized algorithm is allowed to randomly pick any number, with equal probability, from a domain $[N_1..N_2]$.)

3. Based on Breadth-First-Search, write algorithms that solve the following problems, respectively:

- (1) Given an undirected graph G , decide if G is connected.
- (2) Given an undirected graph G , decide if G is a tree.
- (3) Given an undirected graph G , decide if G is bipartite.
- (4) Given an unweighted and undirected graph G and two vertices v and w in G , either construct a shortest path from v to w in G , or report that there is no path from v to w in G .

4. Based on Depth-First-Search, write algorithms that solve the following problems, respectively:

- (1) Given an undirected graph G , decide if G is connected.
- (2) Given an undirected graph G , decide if G is a tree.
- (3) Given an undirected graph G , decide if G is bipartite.
- (4) Given an undirected graph G , either construct a cycle in G or report that G contains no cycle.