CSCE-433 Formal Languages & Automata CSCE-627 Theory of Computability

Spring 2022

Instructor: Dr. Jianer Chen	Senior Grader: Avdhi Shah
Office: PETR 428	Office: N/A
Phone: 845-4259	Phone: tba
$\mathbf{Email:}$ chen@cse.tamu.edu	$\mathbf{Email:}$ avdhi.shah@tamu.edu
Office Hours: MWF 10:30–11:30am	Office Hours: MW 3:00–4:00pm

Assignment # 6 (Due April 29, 2022)

Instructions.

- Your assignment must be typed using your favorite word processor. You may draw diagrams by hand, but only if you are very neat and the diagram is legible.
- Turn in a PDF file of your homework on Canvas.
- Homework is always due at the **beginning** of the class on the due day.

Questions.

- 1. (20 points) Let A and B be languages and $A \leq_m B$.
 - (a) If B is context-free, does that imply that A is also context-free? Why or why not?
 - (b) If A is context-free, does that imply that B is also context-free? Why or why not?

2. (20 points) Let $L = \{ \langle M \rangle \mid M \text{ is a Turing machine that accepts } w^R \text{ whenever it accepts } w \}$. Show that L is undecidable.

3. (20 points) A useless state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

- 4. (20 points)
 - (a) (CSCE 433 students only) Show that P is closed under union, concatenation, and complement.
 - (b) (CSCE 627 students only) Show that NP is closed under union and concatenation.
- 5. (20 points)
 - (a) (CSCE 433 students only) Let COMPOSITE = $\{N \mid N > 0 \text{ is an integer but not a prime}\}$. Prove that the language COMPOSITE is in NP.
 - (b) (CSCE 627 students only) Two graphs G and H are *isomorphic* if the vertices of G may be renamed so that G becomes identical to H. Prove that the following language is in NP: ISOMORPHISM = { $\langle G, H \rangle | G$ and H are isomorphic}.