

CSCE 222-200 Discrete Structures for Computing

Fall 2024

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Assignment # 5 (Due November 7)

1. Find a big- O estimate for the function $f(n)$ that satisfies the recurrence relation

$$f(2) = 1; \quad \text{and} \quad \text{for } n > 2, f(n) = 2f(\sqrt{n}) + \log_2 n.$$

(*Hint:* Make the substitution $m = \log_2 n$. You can assume that m is a power of 2.)

2. A coin is flipped n times where each flip comes up either heads or tails. How many possible outcomes (assuming that n is even and that $k \leq n$)

- (a) are there in total?
- (b) contain exactly k heads?
- (c) contain at least k heads?
- (d) contain the same number of heads and tails?

Give an explanation to your answer to each of the questions.

3. Suppose that k and n are integers with $1 \leq k \leq n$. Prove the *hexagon identity*:

$$\binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} = \binom{n-1}{k} \binom{n}{k-1} \binom{n+1}{k+1}.$$

4. Prove that if E and F are independent events, then \overline{E} and \overline{F} are also independent events.

5. Suppose that we roll a fair die until a 6 comes up.

- (a) What is the probability that a 6 comes up in our n -th rolling?
- (b) What is the expected number of times we roll the die? (*Hint:* You need to find the value for the sum $1 + 2(5/6) + 3(5/6)^2 + \dots + k(5/6)^{k-1} + \dots$.)