CSCE 222-200 Discrete Structures for Computing

Fall 2024

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Assignment # 2
(Due September 19)

1. Determine whether each of the statements below is true or false. Give a one-sentence explanation to your solution to each statement.

(a) $\emptyset \in \{\emptyset\}$;   (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$;   (c) $\{\emptyset\} \in \{\emptyset\}$;   (d) $\{\emptyset\} \in \{\{\emptyset\}\}$;

(e) $\emptyset \subset \{\emptyset\}$;   (f) $\emptyset \subset \{\emptyset, \{\emptyset\}\}$;   (g) $\{\emptyset\} \subset \{\emptyset\}$;   (h) $\{\emptyset\} \subset \{\{\emptyset\}\}$.

2. Prove or disprove that for all sets A, B, and C, we have

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$;   (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

3. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for each positive integer i,

(a) $A_i = [0, i)$, that is, the set of real numbers x with $0 \leq x < i$;

(b) $A_i = [i, \infty)$, that is, the set of real numbers x with $x \geq i$.

4. Give an example of a function from $\mathbf{N}$ to $\mathbf{N}$ (where $\mathbf{N}$ is the set of natural numbers, i.e., $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$) that is

(a) one-to-one but not onto;   (b) onto but not one-to-one;

(c) both onto and one-to-one;   (d) neither one-to-one nor onto.

5. Suppose that g is a function from A to B and f is a function from B to C.

(a) Prove that if $f \circ g$ is onto, then f must be onto;

(b) Prove that if $f \circ g$ is one-to-one, then g must be one-to-one;

(c) Prove that if both f and g are one-to-one, then $f \circ g$ must be one-to-one.

6. Prove the following statements.

(a) If n is an integer, then $n = \lceil n/2 \rceil + \lfloor n/2 \rfloor$;

(b) For all integers n, $\lceil n/2 \rceil \cdot \lfloor n/2 \rfloor = \lfloor n^2/4 \rfloor$.