

Logical Equivalences

Trivial Equivalences

$$p \text{ with } \mathbf{T} \text{ and } \mathbf{F}: \quad p \vee \mathbf{T} \equiv \mathbf{T}, \quad p \wedge \mathbf{T} \equiv p, \quad p \vee \mathbf{F} \equiv p, \quad p \wedge \mathbf{F} \equiv \mathbf{F}$$

$$p \text{ with } p \text{ and } \neg p: \quad p \vee p \equiv p, \quad p \wedge p \equiv p, \quad p \vee \neg p \equiv \mathbf{T}, \quad p \wedge \neg p \equiv \mathbf{F}$$

$$\text{double negation:} \quad \neg(\neg p) \equiv p$$

$$\text{Commutativity} \quad p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$$

$$\text{Associativity} \quad (p \vee q) \vee r \equiv p \vee (q \vee r), \quad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

$$\text{Distributivity} \quad p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r), \quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$\text{De Morgan} \quad \neg(p \wedge q) \equiv \neg p \vee \neg q, \quad \neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\exists x P(x)) \equiv \forall x(\neg P(x)), \quad \neg(\forall x P(x)) \equiv \exists x(\neg P(x))$$

$$\text{Absorption} \quad p \vee (p \wedge q) \equiv p, \quad p \wedge (p \vee q) \equiv p$$

Equivalences Involving Implications

$$\text{basic:} \quad p \rightarrow q \equiv \neg p \vee q, \quad p \rightarrow q \equiv \neg q \vee \neg \neg p$$

$$\text{one implies two:} \quad (p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r), \quad (p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

$$\text{two implies one:} \quad (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r, \quad (p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

Equivalences Involving Biconditionals

$$\text{basic:} \quad p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p), \quad p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$\text{other intuitives:} \quad p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q), \quad \neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Inference Rules

$$\text{Modus Ponens} \quad p \wedge (p \rightarrow q) \text{ infers } q$$

$$\text{Modus Tollens} \quad \neg q \wedge (p \rightarrow q) \text{ infers } \neg p$$

$$\text{Hypothetical Syllogism} \quad (p \rightarrow q) \wedge (q \rightarrow r) \text{ infers } p \rightarrow r$$

$$\text{Disjunctive Syllogism} \quad (p \vee q) \wedge \neg p \text{ infers } q$$

$$\text{Addition} \quad p \text{ infers } p \vee q$$

$$\text{Simplification} \quad p \wedge q \text{ infers } p$$

$$\text{Resolution} \quad (p \vee q) \wedge (\neg p \vee r) \text{ infers } q \vee r$$

(Remark: by “ p infers q ”, we mean that $p \rightarrow q$ is a tautology.)

Suggested Reading

Please read the entire Chapter 1 of the textbook.