CSCE 222-200 Discrete Structures for Computing

Fall 2024

Instructor: Dr. Jianer Chen
Office: PETR 428
Phone: (979) 845-4259
Email: `chen@cse.tamu.edu`
Office Hours: T+R 2:00pm–3:30pm

Teaching Assistant: Evan Kostov
Office: PETR 445
Phone: (469) 996-5494
Email: `evankostov@tamu.edu`
Office Hours: MW 4:15pm-5:15pm

Notes #2. Proof by Contradiction

To prove a proposition P by contradiction, we start with the contrary of the conclusion of the proposition P, then use strict mathematics to derive a "nonsense" (i.e., a statement that contradicts either the condition of the proposition P or something obtained during the derivations).

Example. (the sentences in bold should be included in every of your proofs by contradiction)

Proposition. $\sqrt{5}$ is irrational.

PROOF. **Assume the contrary that** $\sqrt{5}$ is rational. Thus, $\sqrt{5}$ can be written as

$$\sqrt{5} = \frac{m}{n}, \tag{1}$$

where m and n are integers with no common factor larger than 1. Sqare both sides of (1), then multiply both sides of the resulting equality by n^2. We get

$$5n^2 = m^2. \tag{2}$$

Since n^2 is an integer, m^2 is divisible by 5. As a result, the integer m is also divisible by 5 (remark: please make sure that you know how to prove this statement). Thus, m can be written as $m = 5r$, where r is an integer. This gives $m^2 = 25r^2$. Bringing this equality into (2), we get

$$5n^2 = 25r^2, \qquad \text{or} \qquad n^2 = 5r^2. \tag{3}$$

Using an argument completely similar to that applied to (2), we would derive that n is also divisible by 5. Thus, both m and n are divisible by 5. However, this would contradict our assumption that m and n have no common factor greater than 1.

This contradiction proves that $\sqrt{5}$ must be irrational.

Practice Exercises

Prove the following propositions. Whenever it is possible, prove the propositions by contradiction.

1. Prove that if m and n are integers and mn is even, then m is even or n is even.
2. Prove that if n is an integer and $3n + 2$ is even, then n is even.
3. Prove that if x^3 is irrational, then x is irrational.
4. An integer n is a *perfect square* if $n = m^2$ for some integer m. Prove that if n is a perfect square, then $n + 2$ is not a perfect square.
5. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even. (*Hint:* here you need to prove "*if and only if*".)
6. Prove that given a nonnegative integer n, there is a unique nonnegative integer m such that $m^2 \leq n < (m + 1)^2$. (*Hint:* here you also need to prove the uniqueness.)
7. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.