Iterative Improvement

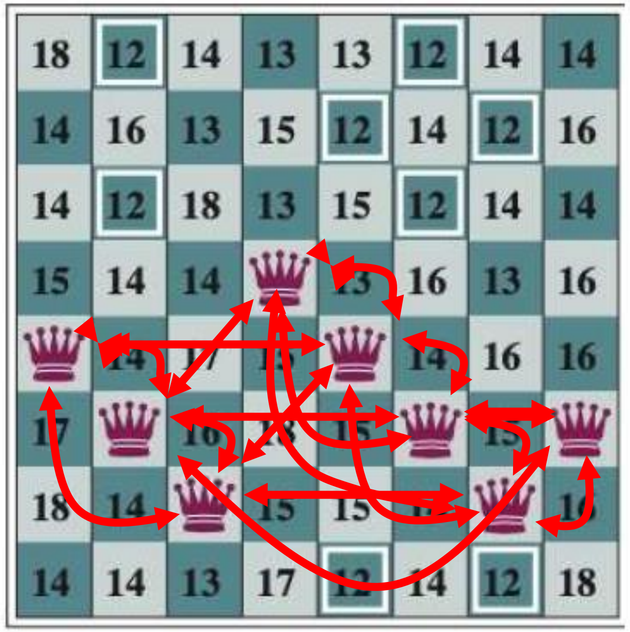
CSCE 420 – Spring 2023

read: Sec. 4.1

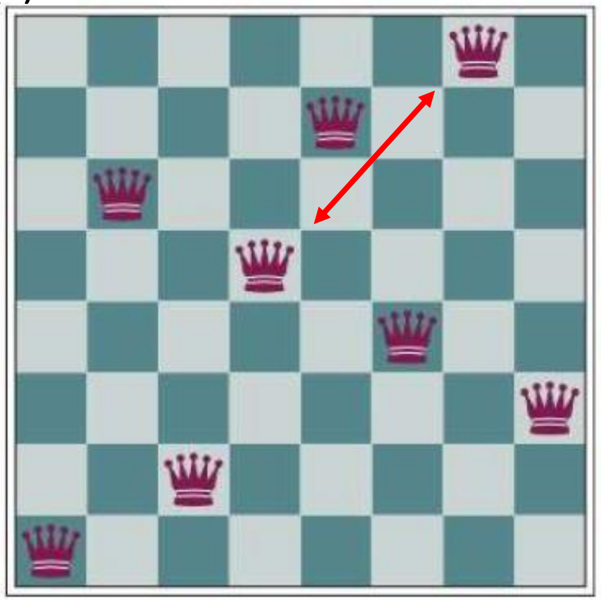
Iterative Improvement Search

- also known as Local Search
- maximize the “quality” of states, q(s) or value(s)
  - note: this is different than path cost
- example: 8-queens
  - can you place 8 queens on chess board such that none can attack each other?
  - initial state: place all 8 queens randomly, one in each column
  - q(s) = -(number of pairs of queens that can attach each other)
    - use negative so higher is better; or modify algorithm to find state with minimum score (gradient descent)

q(s)=-17



q(s)=-1

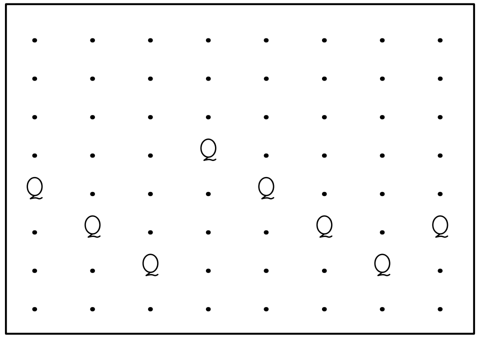


Hill Climbing

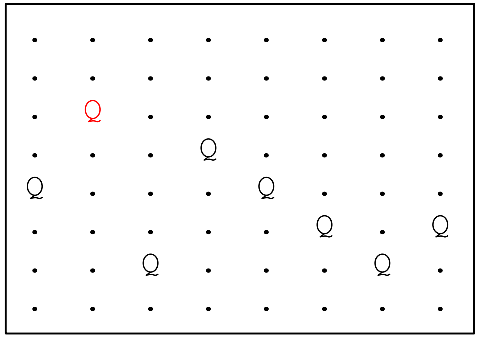
- maintain only a single current state
- generate successors using operator, and pick best

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  current ← problem.INITIAL
  while true do
    neighbor ← a highest-valued successor state of current
    if VALUE(neighbor) ≤ VALUE(current) then return current
    current ← neighbor
```

- operator for 8-queens: move any queen to another row in the same column

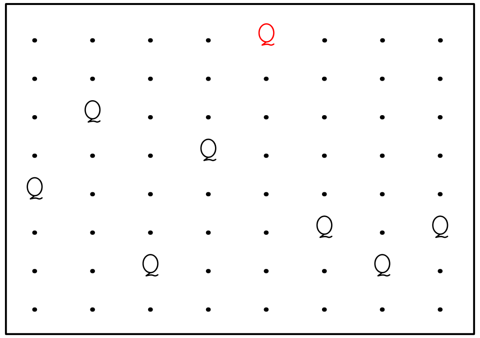


q=17

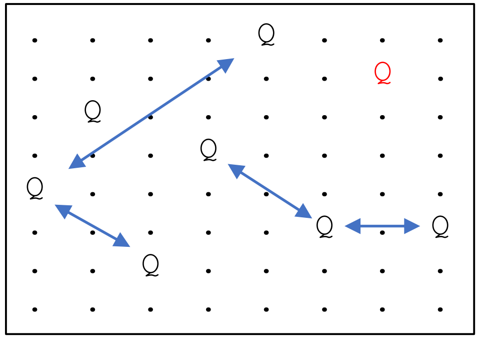


q=17-5+0=12

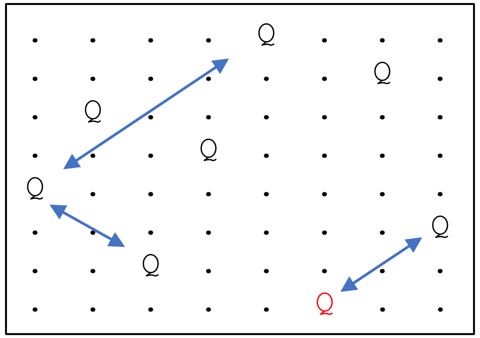
A sequence of iterations, where the best queen is moved to a new position in her column that most reduces the number of overall conflicts.



q=12-5+1=8

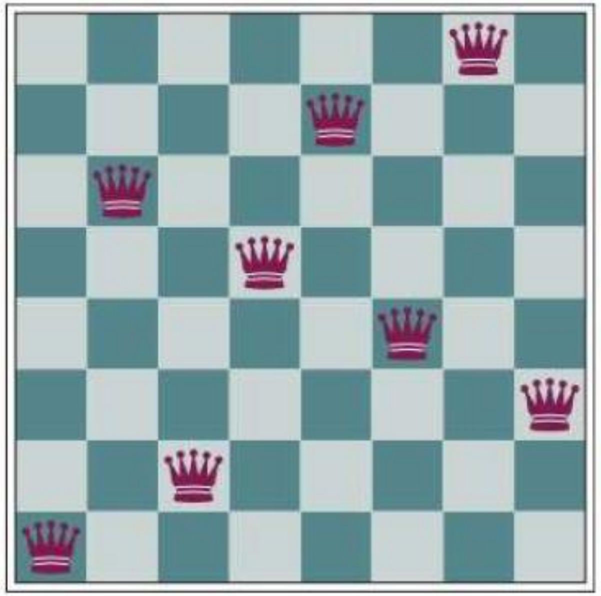


q=8-4+0=4

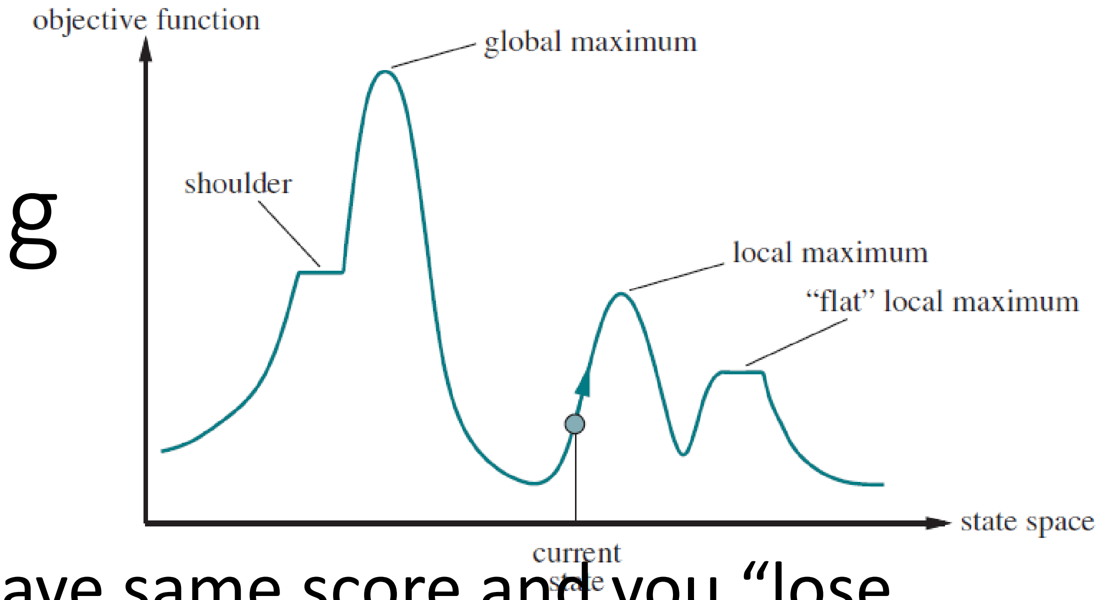


q=4-2+1=3

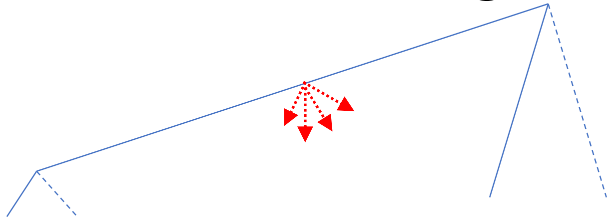
No further improvements can be made.

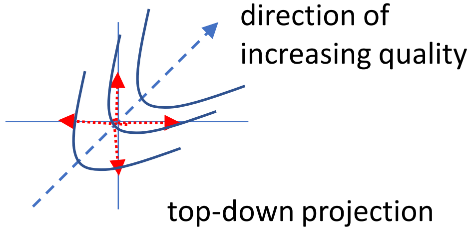


Problems with Hill-Climbing

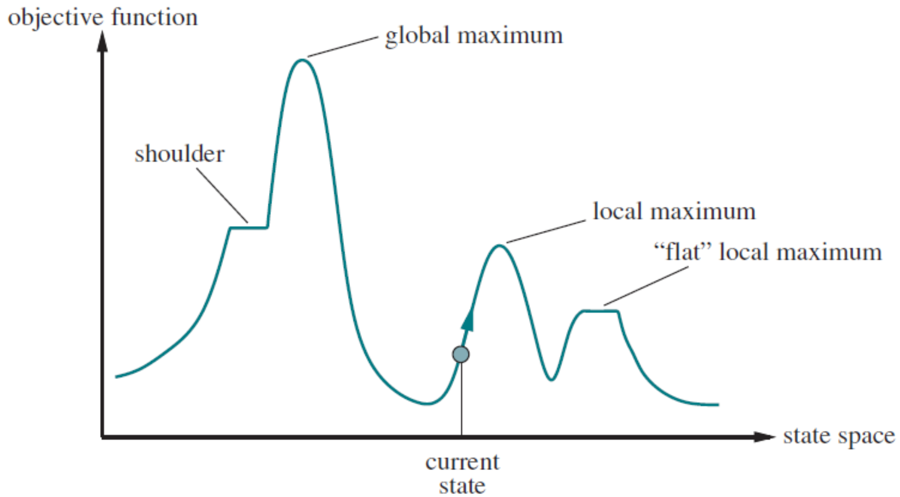


1. **local maxima**
2. **plateau effect** – when all neighbors have same score and you "lose the gradient", even if not at top of hill
3. **ridge effect** – all neighbors have same or lower score, even then there might be other close states that are better
   - suppose only choices are to go N, S, E, or W, but ridge goes up NE; hence all steps go down sides of the ridge
   - often related to limitations of the successor function; consider expanding it to generate more successors in neighborhood (e.g. combinations of 2 steps)





Possible Solutions



- random restart HC
- stochastic HC – choose any successor that is better than current state, not always the best
  - you can't just choose any random successor; must still bias the search upward
  - can this strategy really reduce risk of local minima?
  - this idea leads to **Simulated Annealing**...
- provide memory of previous states
  - HC only maintains 1 state: the current state
  - perhaps we could remember previously expanded-but-not-explored nodes to allow some "back-tracking" if we get stuck at a local maximum
  - this idea leads to **Beam Search**...
- macro operators ("macrops") – create new operators from combinations of 2 or 3 actions, expanding the number of successors

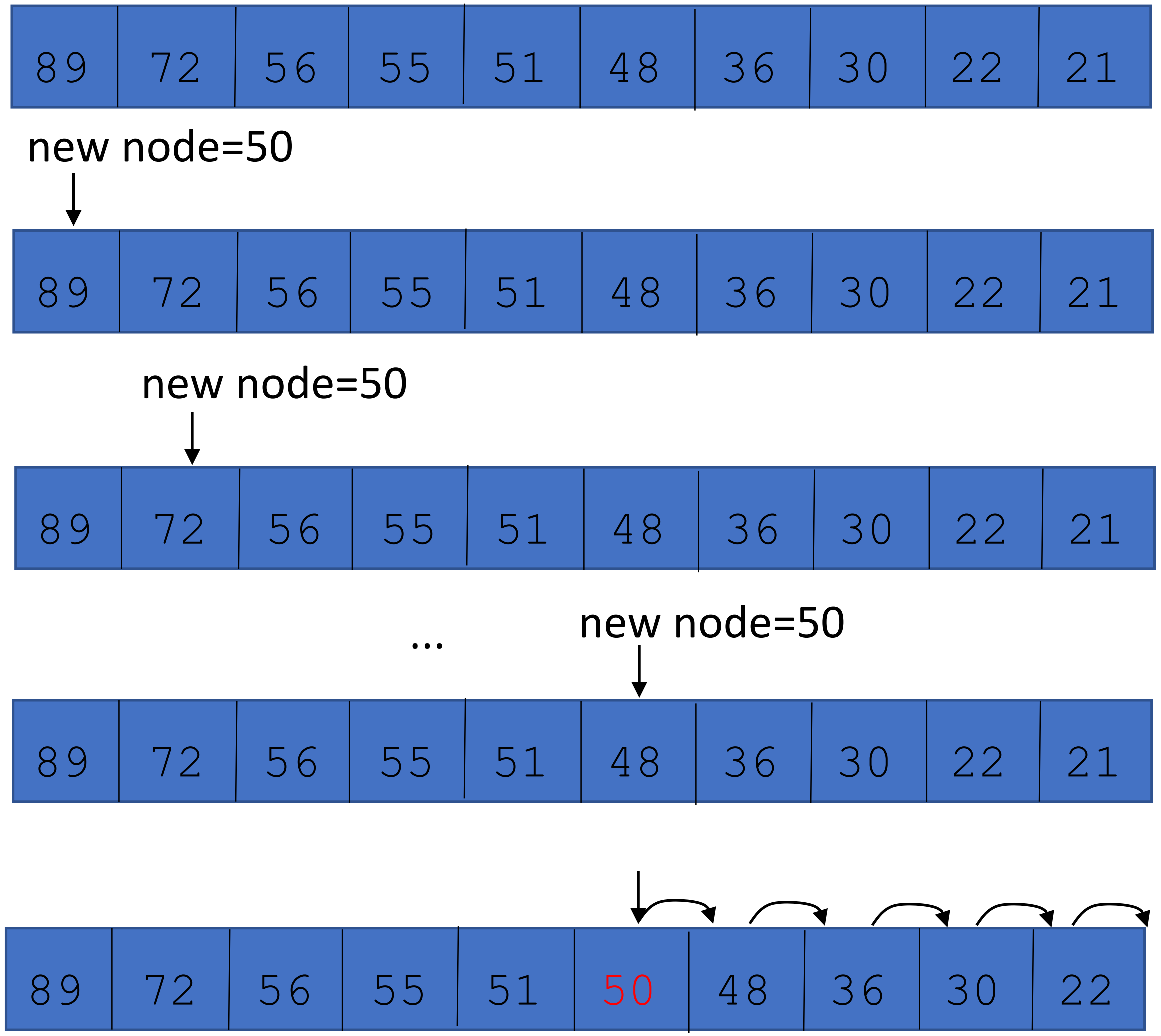
Beam Search

- adding “memory” to Hill-Climbing
  - comparison to Greedy Search: we used a frontier to keep track of *all* previously expanded-but-not-yet-explored states
  - (another difference: Greedy sorts PQ by h(n), and HC chooses successors by q(n))
  - however, this potentially has high space-complexity (exponential frontier size)
  - is there a compromise?
  - yes – keep track of the K best previous nodes (based on state quality q(n))
  - this fixed-size array allows *some* back-tracking, even if not complete enough to explore the whole space (typically, beam size=10-100 nodes)
  - thus, Beam Search could possibly back-track off of one hill and get onto another with a higher local maximum, even though it might fail to find the global max.

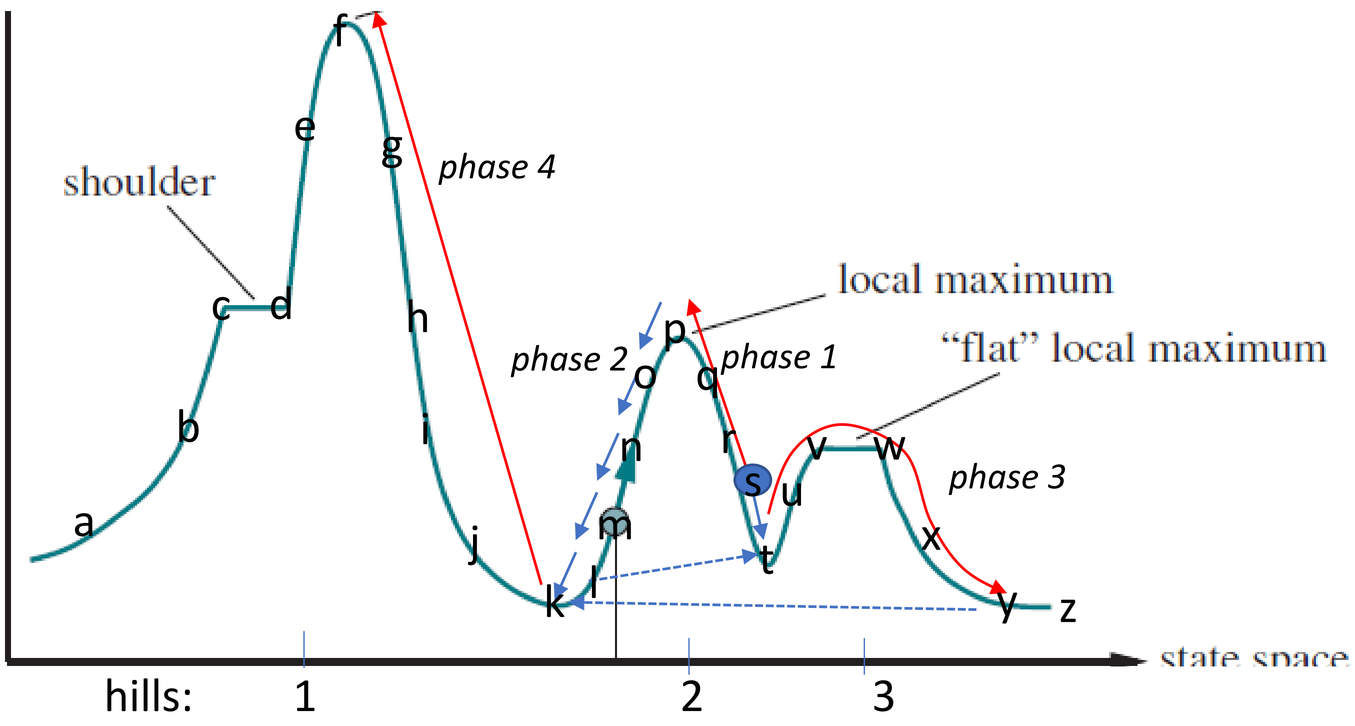
```
BeamSearch(init,k)
    beam←array[k]
    beam.insert(init)
    while True // or until beam empty, or reach max iterations...
         // pop best node in beam (highest score at front)
        curr←beam[0]
        for each child c∈operator(curr):
            beam.insert(c)

beam.insert(node n)
    do insertion sort
    since beam is always presorted, scan the list to find where n
fits, and shift the rest of the nodes down (the last one falls out
of the beam)
```

Quality q(n) of top K nodes:



note: something falls off end of beam (with lowest score) - it could be the k^{th} item in beam, or the new node if it is worse than everything currently in beam (e.g. score<21)



- because this is a simple 1D space, all states have two neighbors
- usually, one of the neighbors has been recently visited, so we discard it
- phase 1: start at ‘s’; climb to local max at ‘p’ (hill 2)
- phase 2: when reach top of hill, the beam remembers these 2 nodes: [o,t]; start descending left slope of hill 2
- phase 3: there is a point where beam has both ‘l’ and ‘t’ in it [l,t]; start ascending hill 3 since ‘t’ is better
- phase 4: eventually, when reach ‘y’, resume search at ‘l’ and climb hill 1

Simulated Annealing

- stochastic search
- choose next child randomly, but “bias it upward”
- always accept better states, and accept worse states probabilistically, proportional to how much lower the quality is

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   current ← problem.INITIAL
   for t = 1 to ∞ do
      T ← schedule(t)
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE(current) – VALUE(next)
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(−ΔE/T)
```

Simulated Annealing

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```

is this correct?

The algorithm in the 4th ed. of the textbook has 2 errors...

AIMA, 4th ed.
(Fig 4.5)

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  current ← *problem*.INITIAL
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE(*current*) – VALUE(*next*)
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{-\Delta E/T}$

(want to accept if next is higher than curr)

(the exponent should be negative, but $-\Delta E/T>0$ since $\Delta E<0$)

AIMA, 3rd ed.
(correct)

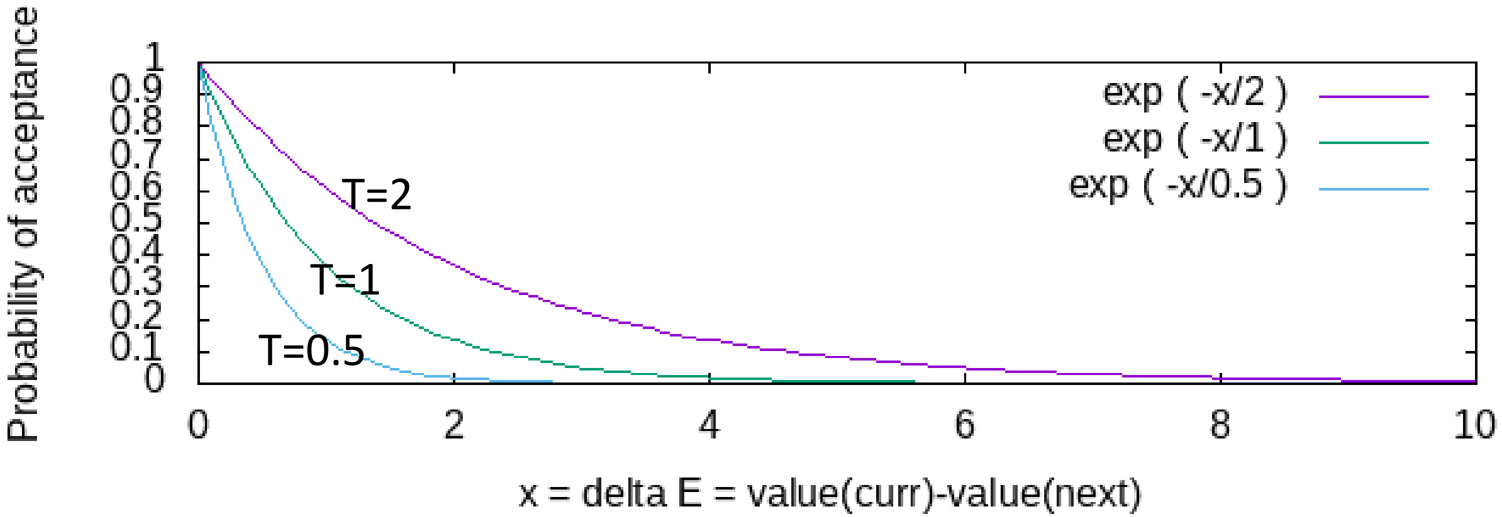
function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
      schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ *next*.VALUE – *current*.VALUE
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Simulated Annealing

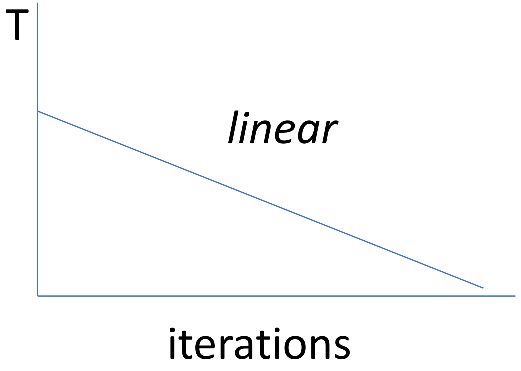
- accept with prob = $e^{-\Delta E/T}$ where ΔE=value(curr)-value(child)
  - if child is only a little worse, ΔE is small, so accept with high prob
  - if child is much worse, ΔE is large, and acceptance is less likely
- T ("temperature" controls) how loose or stringent we are
  - in the limit $T \to \infty$: all backward steps are allowed
  - in the limit T=0: no backward steps are allowed

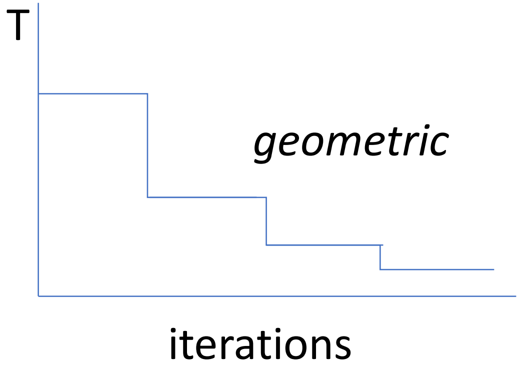
this is analogous to "cooling" in materials like metal; malleable at high temperatures, but gets locked into a lattice structure at low temperatures

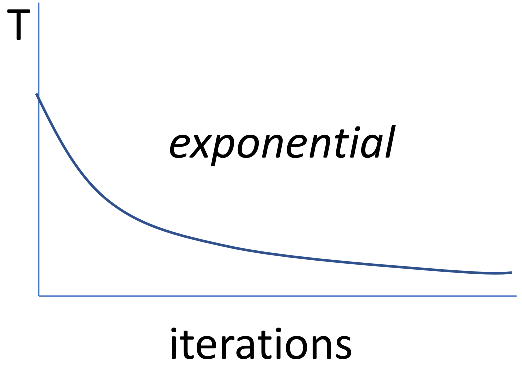


Temperature schedules

- a critical part of SA is to start with a high temperature and gradually lower it
- this allows the search to sample many local maxima initially, but over time, it becomes more selective and climbs up the best hill it it can find
- linear, geometric (e.g. halving every 1000 iterations), exponential...

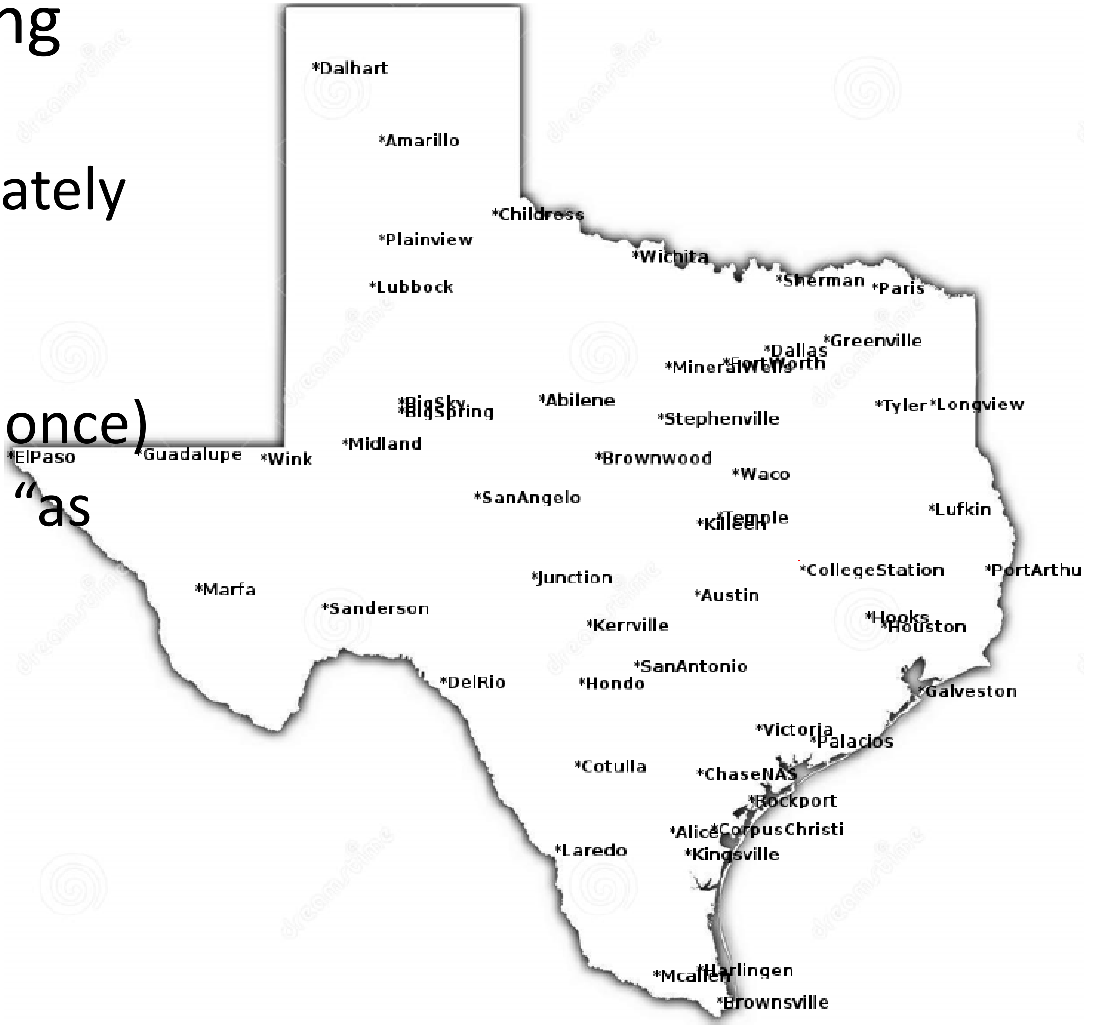


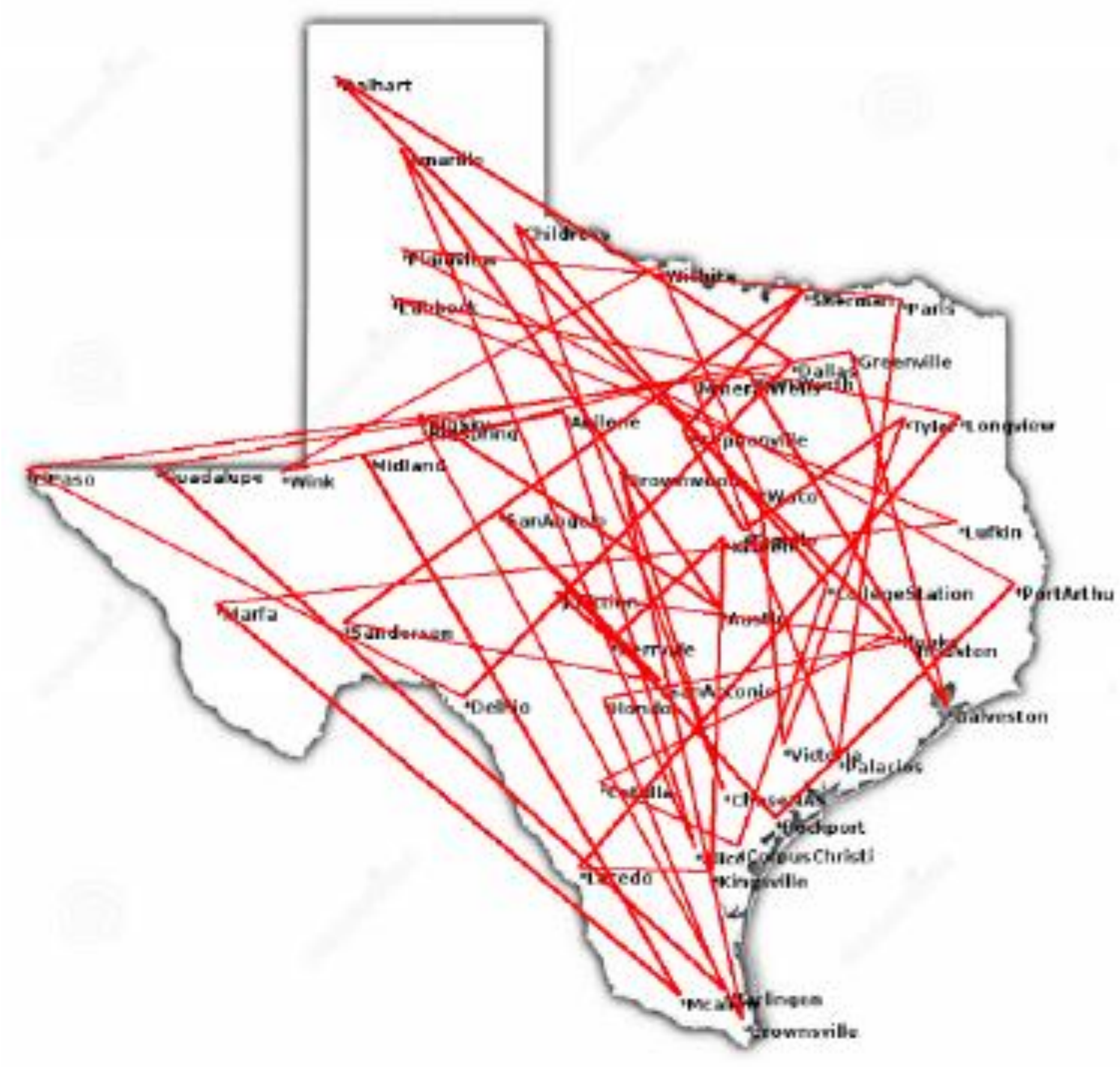




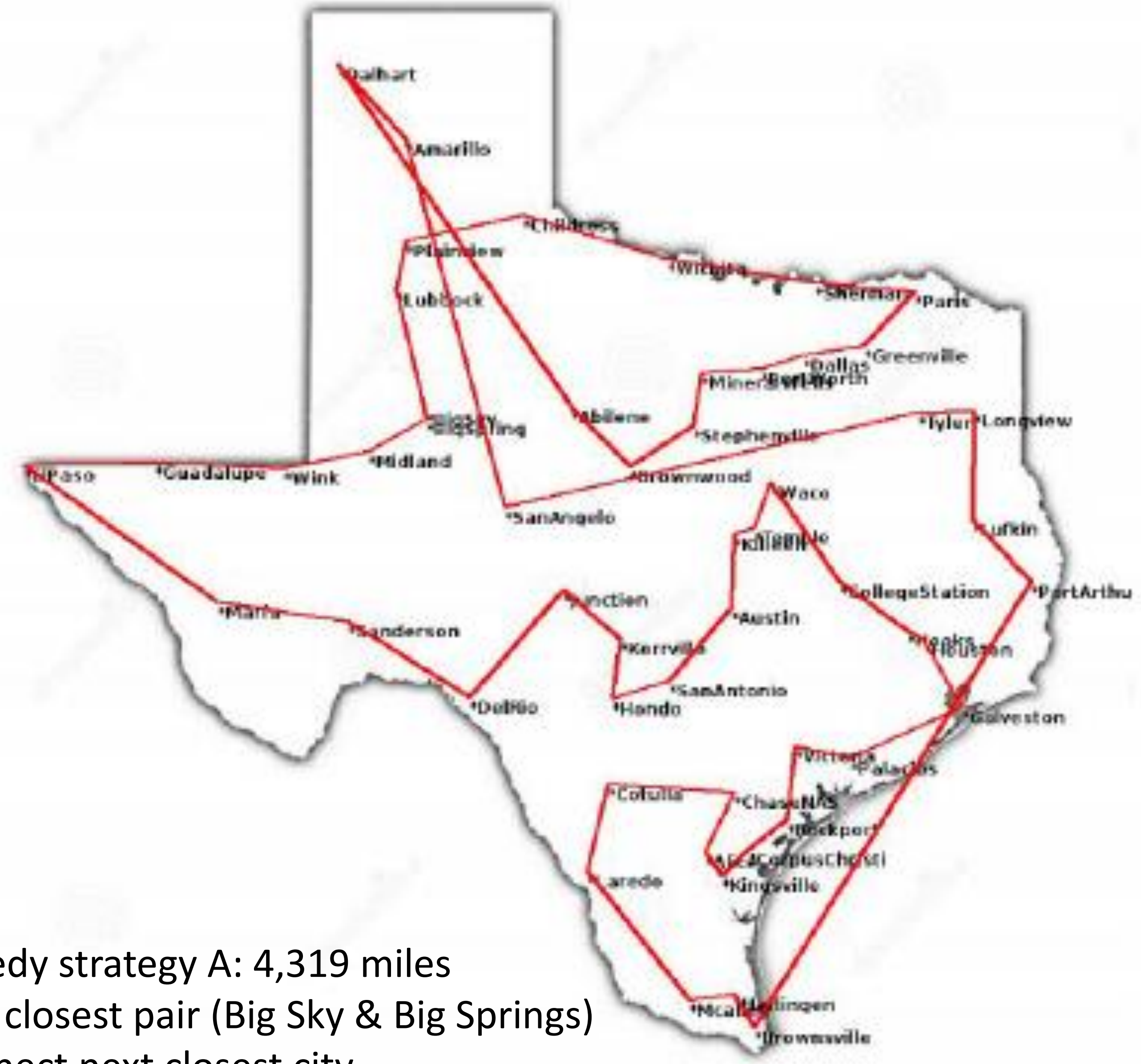
Simulated Annealing

- application of SA to “solving” the Traveling Salesman Problem (TSP)
  - actually, we can only hope to find approximately optimal solutions, because TSP is NP-hard
  - tour with minimum total length
  - (Hamiltonian cycle: visit every node exactly once)
  - example: Texas cities (pairwise connectable “as the crow flies”)





random tour, length=16,697 miles



greedy strategy A: 4,319 miles
join closest pair (Big Sky & Big Springs)
connect next closest city
repeat till all connected

greedy strategy B: 4,032 miles

join closest pair (Big Sky & Big Springs)

connect next closest pair of cities

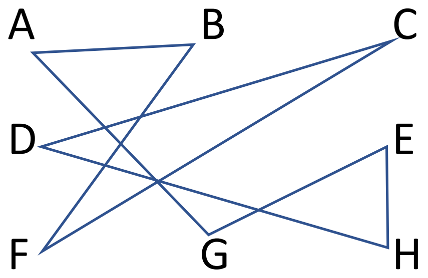
(except cities already connected to 2 others)

repeat till all connected

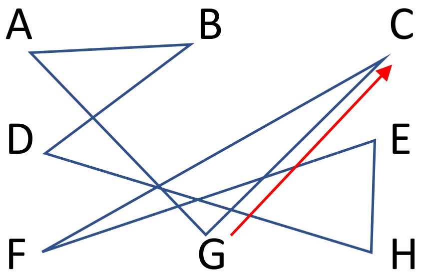
Simulated Annealing

- representation for TSP (for complete graphs): a list of the nodes in any order (permutation)
- operators for TSP (for complete graphs)
  - how to generate “variants” of any given tour (successor states)?
  - there are many ways to do this
  - choose a random subsequence and move it to another position
  - choose a random subsequence and reverse it

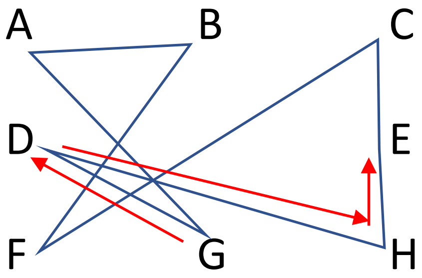
[A G E H D C F B]

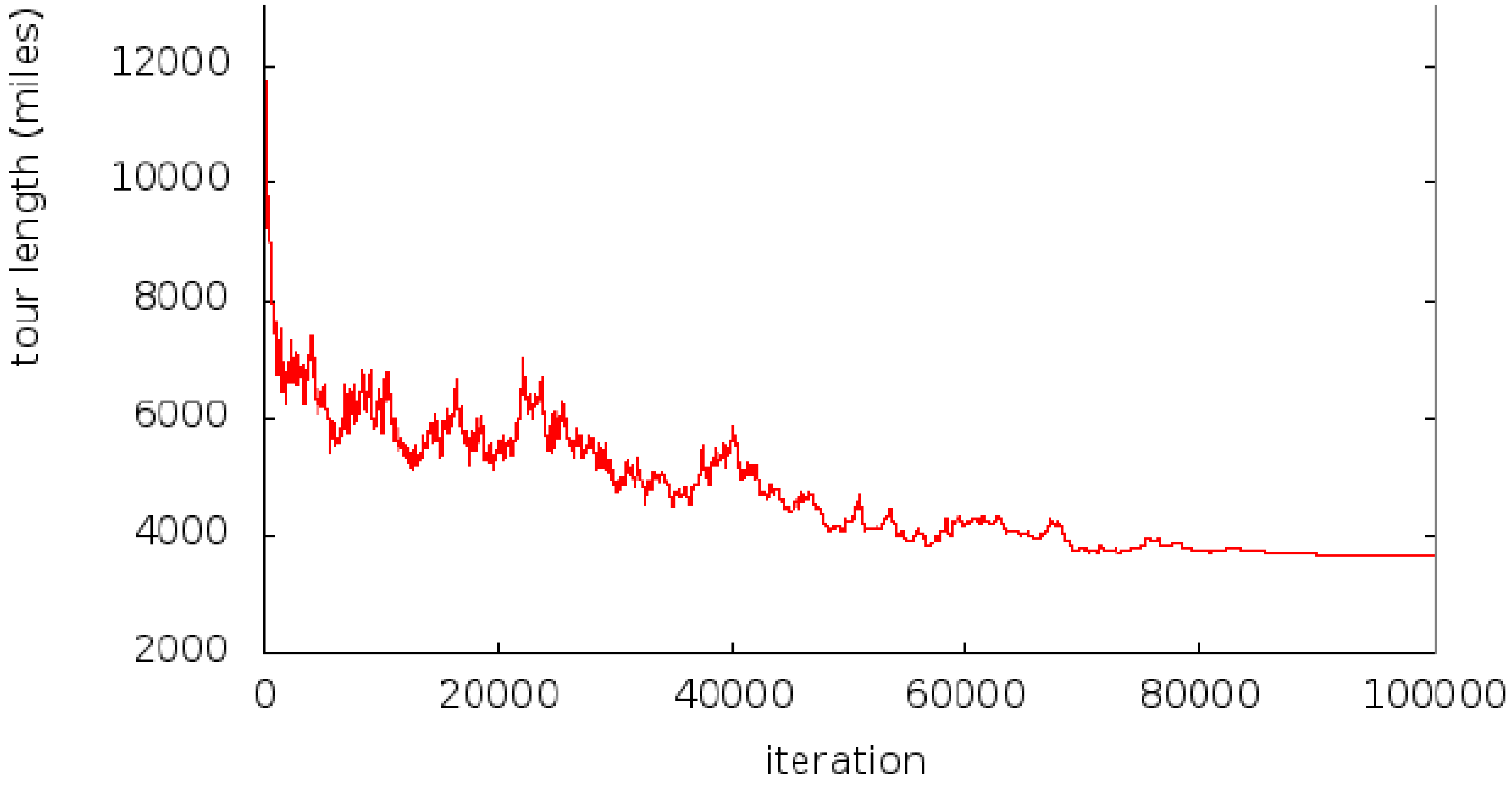


[A G C F E H D B]



[A G D H E C F B]





state = list of cities (complete tour) (state space size = ?)
operators:
  A) pick 2 cities and swap them
  B) pick a subsequence and reverse it
  C) pick a subsequence and move to a new position in list

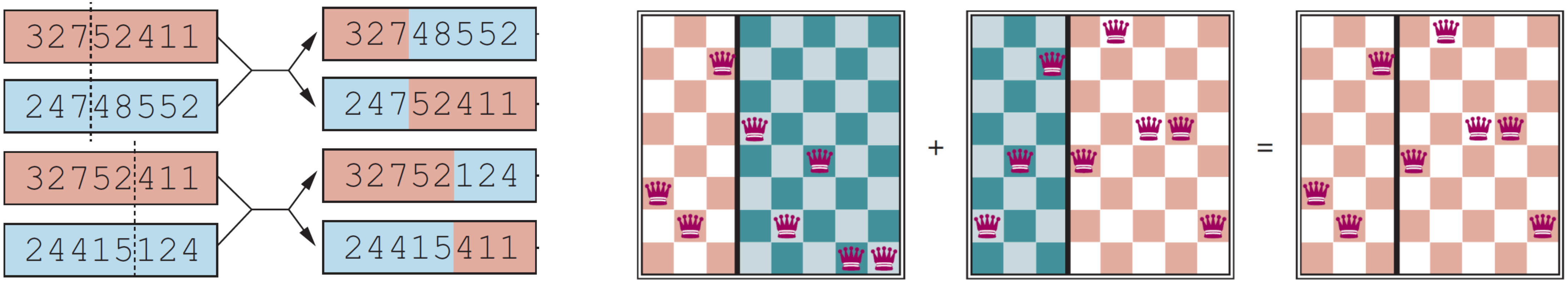
Simulated Annealing: 3,679 miles

Genetic Algorithms

- also known as Evolutionary Programming
- the unique aspects of GA Search are:
  - maintain a *population* of **multiple candidate states** (parallel search, not just curr)
  - mix-and-match states by *recombination*
  - use *fitness* to select winners each round, akin to 'natural selection'
- fitness(state) is a synonym for value(s) or quality(s)
- some GAs use 'chromosomes', which represent state as a bit string
  - example: state of 8-queens is given by a list of 8 integers (0-7), which can be converted to a 24-bit string: 5,1,7,2,3,6,4,0 → 101001111010011110100000
  - but chromosomes are not necessary, as long as states can be recombined

Recombination or ‘cross-over’

- instead of an ‘operator’ to generate successors from states, use ‘recombination’ to combine parts of existing members of population

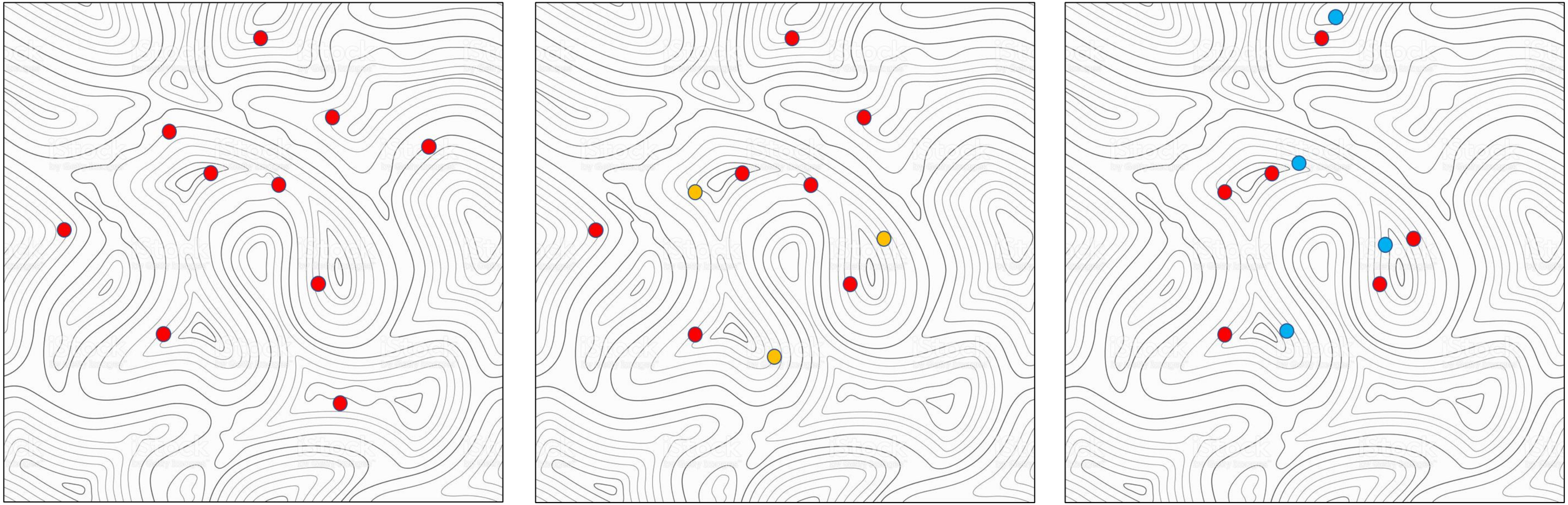


- for chromosomes, splice their strings at a random locations
- for other data types and states representation, use must define ‘cross-over’
- by selecting parents at random and recombining them, you sometimes get the *best of both* and produce and improved state
- food for thought: How would you perform recombination between 2 tours for the TSP to generate a child state?

```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
     weights ← WEIGHTED-BY(population, fitness)
     population2 ← empty list
     for i = 1 to SIZE(population) do
        parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
        child ← REPRODUCE(parent1, parent2)
        if (small random probability) then child ← MUTATE(child)
        add child to population2
     population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

based on fitness



Added some new orange individuals between red ones;
Removed some of the less-fit red individuals.

Genetic Algorithms

- there are many variations on GAs
- some include *mutation*
  - make random changes to state (like operator) at low frequency
- Lamarckian evolution – improvements/adaptation acquired during lifetime of individual can be passed on to offspring
- 'loss of diversity' is a problem for GAs, where population becomes homogeneous (everybody on the same hill)

Genetic Algorithms

- many applications of GAs to search problems,
  - from airfoil design (airplane wings)
  - to automatic program synthesis (random computation trees)
- optimization:
  - the power comes not from mutation, but from **competition**
  - survival of the fittest drives the population as a whole to gradually improve
  - weaker/less fit individuals do not get selected to reproduce and are effectively dropped from the population

Summary of Iterative Improvement Algorithms

- Uninformed (Weak) Search
  - Breadth-first (BFS)
  - Depth-first (DFS)
  - Iterative Deepening (ID)
  - Uniform-cost (UC) – optimal (finds a goal with minimum path cost)
- Informed Search – uses a heuristic h(n)
  - Greedy (Best-first) search
  - A* - optimal (provided heuristic is admissible)
- Iterative Improvement
  - Hill-Climbing
  - Beam search
  - Simulated Annealing – stochastic search
  - Genetic Algorithms – parallel search (with a population of candidate solutions)