- Hidden Markov Models (HMM)

Many slides from Michael Collins

Overview and HMMs

- The Tagging Problem
- Generative models, and the noisy-channel model, for supervised learning
- Hidden Markov Model (HMM) taggers
  - Basic definitions
  - Parameter estimation
  - The Viterbi algorithm

Part-of-Speech Tagging

INPUT:
Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT:
Profits/N soared/V at/P Boeing/N Co./N ,/, easily/ADV topping/V forecasts/N on/P Wall/N Street/N ,/, as/P their/POSS CEO/N Alan/N Mulally/N announced/V first/ADJ quarter/N results/N ./.

N = Noun
V = Verb
P = Preposition
Adv = Adverb
Adj = Adjective
...

Named Entity Recognition

INPUT: Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT: Profits soared at [Company Boeing Co.], easily topping forecasts on [Location Wall Street], as their CEO [Person Alan Mulally] announced first quarter results.

Named Entity Extraction as Tagging

INPUT:

Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT:

Profits/NA soared/NA at/NA Boeing/SC Co./CC ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

NA = No entity
SC = Start Company
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...

Our Goal

Training set:

1 Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.

2 Mr./NNP Vinken/NNP is/VBZ chairman/NN of/IN Elsevier/NNP N.V./NNP ,/, the/DT Dutch/NNP publishing/VBG group/NN ./.

3 Rudolph/NNP Agnew/NNP ,/, 55/CD years/NNS old/JJ and/CC chairman/NN of/IN Consolidated/NNP Gold/NNP Fields/NNP PLC/NNP ,/, was/VBD named/VBN a/DT nonexecutive/JJ director/NN of/IN this/DT British/JJ industrial/JJ conglomerate/NN ./.

. . .

38,219 It/PRP is/VBZ also/RB pulling/VBG 20/CD people/NNS out/IN of/IN Puerto/NNP Rico/NNP ,/, who/WP were/VBD helping/VBG Huricane/NNP Hugo/NNP victims/NNS ,/, and/CC sending/VBG them/PRP to/TO San/NNP Francisco/NNP instead/RB ./.

- From the training set, induce a function/algorithm that maps new sentences to their tag sequences.

Two Types of Constraints

Influential/JJ members/NNS of/IN the/DT House/NNP Ways/NNP and/CC Means/NNP Committee/NNP introduced/VBD legislation/NN that/WDT would/MD restrict/VB how/WRB the/DT new/JJ savings-and-loan/NN bailout/NN agency/NN can/MD raise/VB capital/NN ./.

- "Local": e.g., *can* is more likely to be a modal verb MD rather than a noun NN
- "Contextual": e.g., a noun is much more likely than a verb to follow a determiner
- Sometimes these preferences are in conflict:

  The trash can is in the garage

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Supervised Learning Problems

- We have training examples $x^{(i)}, y^{(i)}$ for $i = 1 \ldots m$. Each $x^{(i)}$ is an input, each $y^{(i)}$ is a label.
- Task is to learn a function f mapping inputs x to labels $f(x)$

Supervised Learning Problems

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- Task is to learn a function f mapping inputs x to labels $f(x)$
- Conditional models:
  - Learn a distribution $p(y|x)$ from training examples
  - For any test input x, define $f(x) = \arg\max_y p(y|x)$

Generative Models

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- Generative models:
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  - Often we have $p(x, y) = p(y)p(x|y)$
- Note: we then have

$$p(y|x) = \frac{p(y)p(x|y)}{p(x)}$$

where $p(x) = \sum_y p(y)p(x|y)$

Decoding with Generative Models

- We have training examples $x^{(i)}, y^{(i)}$ for $i = 1 \ldots m$. Task is to learn a function f mapping inputs x to labels $f(x)$.
- Generative models:
  - Learn a distribution $p(x, y)$ from training examples
  - Often we have $p(x, y) = p(y)p(x|y)$
- Output from the model:

$$
\begin{aligned}
f(x) &= \arg\max_y p(y|x) \\
&= \arg\max_y \frac{p(y)p(x|y)}{p(x)} \\
&= \arg\max_y p(y)p(x|y)
\end{aligned}
$$

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Hidden Markov Models

- We have an input sentence $x = x_1, x_2, \ldots, x_n$ (x_i is the i'th word in the sentence)
- We have a tag sequence $y = y_1, y_2, \ldots, y_n$ (y_i is the i'th tag in the sentence)
- We'll use an HMM to define

$$p(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n)$$

  for any sentence $x_1 \ldots x_n$ and tag sequence $y_1 \ldots y_n$ of the same length.

- Then the most likely tag sequence for x is

$$\arg\max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1, y_2, \ldots, y_n)$$

Trigram Hidden Markov Models (Trigram HMMs)

For any sentence $x_1 \ldots x_n$ where $x_i \in \mathcal{V}$ for $i = 1 \ldots n$, and any tag sequence $y_1 \ldots y_{n+1}$ where $y_i \in \mathcal{S}$ for $i = 1 \ldots n$, and $y_{n+1} =$ STOP, the joint probability of the sentence and tag sequence is

$$p(x_1 \ldots x_n, y_1 \ldots y_{n+1}) = \prod_{i=1}^{n+1} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{n} e(x_i | y_i)$$

where we have assumed that y_0 = y_-1 = *.

Parameters of the model:

- $q(s|u, v)$ for any $s \in \mathcal{S} \cup \{\text{STOP}\}$, $u, v \in \mathcal{S} \cup \{*\}$
- $e(x|s)$ for any $s \in \mathcal{S}$, $x \in \mathcal{V}$

An Example

If we have $n = 3$, $x_1 \dots x_3$ equal to the sentence *the dog laughs*, and $y_1 \dots y_4$ equal to the tag sequence D N V STOP, then

$$\begin{aligned} & p(x_1 \dots x_n, y_1 \dots y_{n+1}) \\ = \; & q(\mathtt{D}|*,*) \times q(\mathtt{N}|*,\mathtt{D}) \times q(\mathtt{V}|\mathtt{D},\mathtt{N}) \times q(\mathtt{STOP}|\mathtt{N},\mathtt{V}) \\ & \times e(\textit{the}|\mathtt{D}) \times e(\textit{dog}|\mathtt{N}) \times e(\textit{laughs}|\mathtt{V}) \end{aligned}$$

- STOP is a special tag that terminates the sequence
- We take $y_0 = y_{-1} = *$, where * is a special "padding" symbol

Why the Name?

$$p(x_1 \ldots x_n, y_1 \ldots y_n) = \underbrace{q(\mathsf{STOP}|y_{n-1}, y_n) \prod_{j=1}^{n} q(y_j \mid y_{j-2}, y_{j-1})}_{\text{Markov Chain}}$$

$$\times \underbrace{\prod_{j=1}^{n} e(x_j \mid y_j)}_{x_j\text{'s are observed}}$$

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Smoothed Estimation

$$\begin{aligned} q(\textsf{Vt} \mid \textsf{DT, JJ}) &= \lambda_1 \times \frac{\textsf{Count(Dt, JJ, Vt)}}{\textsf{Count(Dt, JJ)}} \\ &+\lambda_2 \times \frac{\textsf{Count(JJ, Vt)}}{\textsf{Count(JJ)}} \\ &+\lambda_3 \times \frac{\textsf{Count(Vt)}}{\textsf{Count()}} \end{aligned}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1, \quad \text{and for all } i,\ \lambda_i \geq 0$$

$$e(\textsf{base} \mid \textsf{Vt}) = \frac{\textsf{Count(Vt, base)}}{\textsf{Count(Vt)}}$$

Dealing with Low-Frequency Words: An Example

Profits soared at Boeing Co. , easily topping forecasts on Wall Street , as their CEO Alan Mulally announced first quarter results .

Dealing with Low-Frequency Words

A common method is as follows:

- **Step 1**: Split vocabulary into two sets

Frequent words	= words occurring $\geq$ 5 times in training
Low frequency words	= all other words

- **Step 2**: Map low frequency words into a small, finite set, depending on prefixes, suffixes etc.

Dealing with Low-Frequency Words: An Example

[Bikel et. al 1999] **(named-entity recognition)**

Word class	Example	Intuition
twoDigitNum	90	Two digit year
fourDigitNum	1990	Four digit year
containsDigitAndAlpha	A8956-67	Product code
containsDigitAndDash	09-96	Date
containsDigitAndSlash	11/9/89	Date
containsDigitAndComma	23,000.00	Monetary amount
containsDigitAndPeriod	1.00	Monetary amount, percentage
othernum	456789	Other number
allCaps	BBN	Organization
capPeriod	M.	Person name initial
firstWord	first word of sentence	no useful capitalization information
initCap	Sally	Capitalized word
lowercase	can	Uncapitalized word
other	,	Punctuation marks, all other words

Dealing with Low-Frequency Words: An Example

Profits/NA soared/NA at/NA Boeing/SC Co./CC ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

$\Downarrow$

`firstword`/NA soared/NA at/NA `initCap`/SC Co./CC ,/NA easily/NA `lowercase`/NA forecasts/NA on/NA `initCap`/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP `initCap`/CP announced/NA first/NA quarter/NA results/NA ./NA

NA = No entity
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- Inference and the Viterbi Algorithm

The Viterbi Algorithm

Problem: for an input $x_1 \dots x_n$, find

$$\arg \max_{y_1 \dots y_{n+1}} p(x_1 \dots x_n, y_1 \dots y_{n+1})$$

where the $\arg\max$ is taken over all sequences $y_1 \dots y_{n+1}$ such that $y_i \in \mathcal{S}$ for $i = 1 \dots n$, and $y_{n+1} =$ STOP.

We assume that p again takes the form

$$p(x_1 \dots x_n, y_1 \dots y_{n+1}) = \prod_{i=1}^{n+1} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{n} e(x_i | y_i)$$

Recall that we have assumed in this definition that $y_0 = y_{-1} =$ *, and $y_{n+1} =$ STOP.

Brute Force Search is Hopelessly Inefficient

Problem: for an input $x_1 \ldots x_n$, find

$$\arg \max_{y_1 \ldots y_{n+1}} p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

where the $\arg\max$ is taken over all sequences $y_1 \ldots y_{n+1}$ such that $y_i \in \mathcal{S}$ for $i = 1 \ldots n$, and $y_{n+1} =$ STOP.

The Viterbi Algorithm

- Define n to be the length of the sentence
- Define S_k for $k = -1 \ldots n$ to be the set of possible tags at position k:

$$S_{-1} = S_0 = \{*\}$$
$$S_k = S \quad \text{for } k \in \{1 \ldots n\}$$

- Define

$$r(y_{-1}, y_0, y_1, \ldots, y_k) = \prod_{i=1}^{k} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{k} e(x_i | y_i)$$

- Define a dynamic programming table

$$\pi(k, u, v) \quad = \quad \text{maximum probability of a tag sequence ending in tags } u, v \text{ at position } k$$

that is,
$\pi(k, u, v) = \max_{\langle y_{-1}, y_0, y_1, \ldots, y_k \rangle : y_{k-1}=u, y_k=v} r(y_{-1}, y_0, y_1 \ldots y_k)$

A Recursive Definition

Base case:

$$\pi(0, *, *) = 1$$

Recursive definition:

For any $k \in \{1 \ldots n\}$, for any $u \in \mathcal{S}_{k-1}$ and $v \in \mathcal{S}_k$:

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v)\right)$$

The Viterbi Algorithm

Input: a sentence $x_1 \ldots x_n$, parameters $q(s|u,v)$ and $e(x|s)$.

Initialization: Set $\pi(0, *, *) = 1$

Definition: $\mathcal{S}_{-1} = \mathcal{S}_0 = \{*\}$, $\mathcal{S}_k = \mathcal{S}$ for $k \in \{1 \ldots n\}$

Algorithm:

- For $k = 1 \ldots n$,
  - For $u \in \mathcal{S}_{k-1}$, $v \in \mathcal{S}_k$,

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1, w, u) \times q(v|w,u) \times e(x_k|v) \right)$$

- **Return** $\max_{u \in \mathcal{S}_{n-1}, v \in \mathcal{S}_n} \left(\pi(n, u, v) \times q(\text{STOP}|u,v) \right)$

The Viterbi Algorithm with Backpointers

Input: a sentence $x_1 \ldots x_n$, parameters $q(s|u,v)$ and $e(x|s)$.

Initialization: Set $\pi(0, *, *) = 1$

Definition: $\mathcal{S}_{-1} = \mathcal{S}_0 = \{*\}$, $\mathcal{S}_k = \mathcal{S}$ for $k \in \{1 \ldots n\}$
Algorithm:

- For $k = 1 \ldots n$,
  - For $u \in \mathcal{S}_{k-1}$, $v \in \mathcal{S}_k$,

$$\begin{aligned}\pi(k,u,v) &= \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1,w,u) \times q(v|w,u) \times e(x_k|v)\right) \\ bp(k,u,v) &= \arg\max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1,w,u) \times q(v|w,u) \times e(x_k|v)\right)\end{aligned}$$

- Set $(y_{n-1}, y_n) = \arg\max_{(u,v)} (\pi(n,u,v) \times q(\mathsf{STOP}|u,v))$
- For $k = (n-2) \ldots 1$, $y_k = bp(k+2, y_{k+1}, y_{k+2})$
- **Return** the tag sequence $y_1 \ldots y_n$

The Viterbi Algorithm: Running Time

- $O(n|\mathcal{S}|^3)$ time to calculate $q(s|u,v) \times e(x_k|s)$ for all k, s, u, v.
- $n|\mathcal{S}|^2$ entries in π to be filled in.
- $O(|\mathcal{S}|)$ time to fill in one entry
- $\Rightarrow O(n|\mathcal{S}|^3)$ time in total

A Simple Bi-gram Example:

(X, Y): P(X/Y),   POS tags for “bears fish” ?

• noun * .80	bears noun .02
• Verb * .10	bears verb .02
• STOP noun .50	fish verb .07
• STOP verb .50	fish noun .08
• noun verb .77	
• verb noun .65	
• noun noun .0001	
• nerb verb .0001	

Answer

- bears: noun
- fish: verb

The Forward Algorithm

Input: a sentence $x_1 \ldots x_n$, parameters $q(s|u,v)$ and $e(x|s)$.

Initialization: Set $\pi(0, *, *) = 1$

Definition: $\mathcal{S}_{-1} = \mathcal{S}_0 = \{*\}$, $\mathcal{S}_k = \mathcal{S}$ for $k \in \{1 \ldots n\}$

Algorithm:

- For $k = 1 \ldots n$,
  - For $u \in \mathcal{S}_{k-1}$, $v \in \mathcal{S}_k$,

$$\pi(k,u,v) = \underset{w \in \mathcal{S}_{k-2}}{\text{Sum}} \left(\pi(k-1, w, u) \times q(v|w,u) \times e(x_k|v)\right)$$

- **Return** $\text{Sum}_{u \in \mathcal{S}_{n-1}, v \in \mathcal{S}_n} \left(\pi(n,u,v) \times q(\text{STOP}|u,v)\right)$

Pros and Cons

- Hidden markov model taggers are very simple to train (just need to compile counts from the training corpus) If you already have a labeled training set.

  Use forward-backward algorithms in the unsupervised setting.
- Perform relatively well (over 90% performance on named entity recognition)
- Main difficulty is modeling

$$e(word \mid tag)$$

can be very difficult if "words" are complex