Probabilistic Context Free Grammars

Many slides from Michael Collins and Chris Manning

Overview

- Probabilistic Context-Free Grammars (PCFGs)
- The CKY Algorithm for parsing with PCFGs

A Probabilistic Context-Free Grammar (PCFG)

S	$\Rightarrow$	NP	VP	1.0
VP	$\Rightarrow$	Vi		0.4
VP	$\Rightarrow$	Vt	NP	0.4
VP	$\Rightarrow$	VP	PP	0.2
NP	$\Rightarrow$	DT	NN	0.3
NP	$\Rightarrow$	NP	PP	0.7
PP	$\Rightarrow$	P	NP	1.0

Vi	$\Rightarrow$	sleeps	1.0
Vt	$\Rightarrow$	saw	1.0
NN	$\Rightarrow$	man	0.7
NN	$\Rightarrow$	woman	0.2
NN	$\Rightarrow$	telescope	0.1
DT	$\Rightarrow$	the	1.0
IN	$\Rightarrow$	with	0.5
IN	$\Rightarrow$	in	0.5

- Probability of a tree t with rules

$$\alpha_1 \to \beta_1, \alpha_2 \to \beta_2, \ldots, \alpha_n \to \beta_n$$

is $p(t) = \prod_{i=1}^{n} q(\alpha_i \to \beta_i)$ where $q(\alpha \to \beta)$ is the probability for rule $\alpha \to \beta$.

DERIVATION	RULES USED	PROBABILITY
S	S $\rightarrow$ NP VP	1.0
NP VP	NP $\rightarrow$ DT NN	0.3
DT NN VP	DT $\rightarrow$ the	1.0
the NN VP	NN $\rightarrow$ dog	0.1
the dog VP	VP $\rightarrow$ Vi	0.4
the dog Vi	Vi $\rightarrow$ laughs	0.5
the dog laughs		

Properties of PCFGs

- Assigns a probability to each *left-most derivation*, or parse-tree, allowed by the underlying CFG

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- Say we have a sentence s, set of derivations for that sentence is $\mathcal{T}(s)$. Then a PCFG assigns a probability $p(t)$ to each member of $\mathcal{T}(s)$. i.e., *we now have a ranking in order of probability*.

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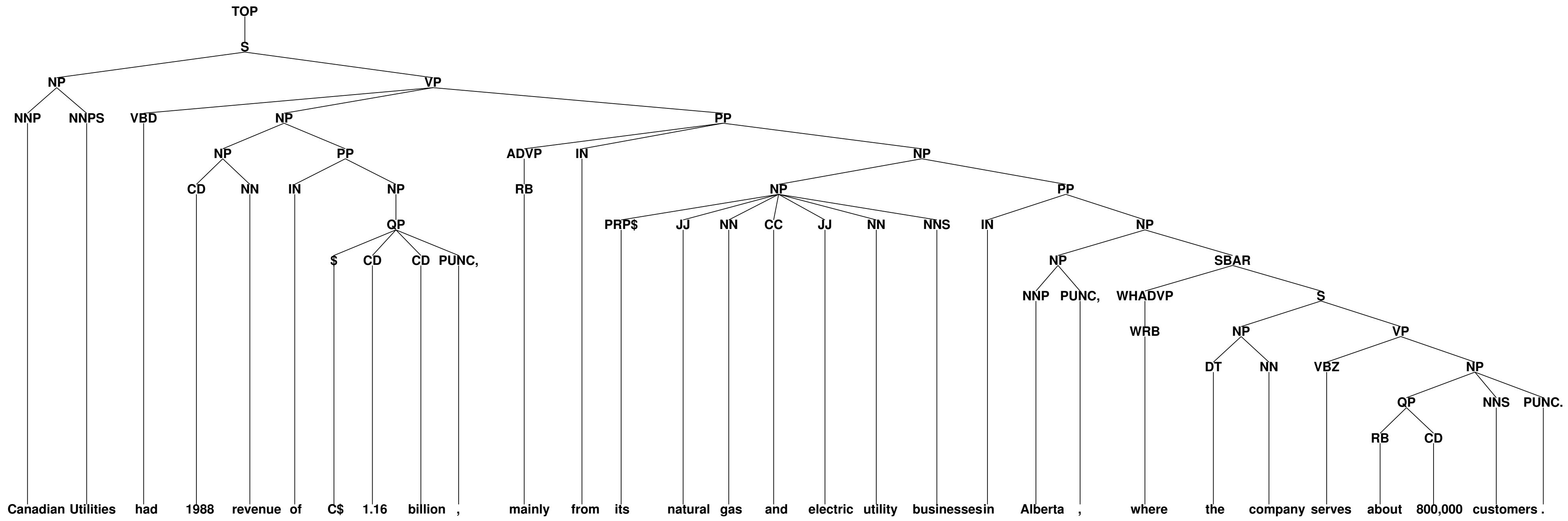
- Say we have a sentence s, set of derivations for that sentence is $\mathcal{T}(s)$. Then a PCFG assigns a probability $p(t)$ to each member of $\mathcal{T}(s)$. i.e., *we now have a ranking in order of probability*.
- The most likely parse tree for a sentence s is

$$\arg\max_{t \in \mathcal{T}(s)} p(t)$$

Data for Parsing Experiments: Treebanks

- Penn WSJ Treebank = 50,000 sentences with associated trees
- Usual set-up: 40,000 training sentences, 2400 test sentences

An example tree:



Deriving a PCFG from a Treebank

- Given a set of example trees (a treebank), the underlying CFG can simply be **all rules seen in the corpus**

- Maximum Likelihood estimates:

$$q_{ML}(\alpha \rightarrow \beta) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$$

  where the counts are taken from a training set of example trees.

- **If the training data is generated by a PCFG**, then as the training data size goes to infinity, the maximum-likelihood PCFG will converge to the same distribution as the "true" PCFG.

Parsing with a PCFG

- Given a PCFG and a sentence s, define $\mathcal{T}(s)$ to be the set of trees with s as the yield.
- Given a PCFG and a sentence s, how do we find

$$\arg\max_{t \in \mathcal{T}(s)} p(t)$$

Chomsky Normal Form

A context free grammar $G = (N, \Sigma, R, S)$ in Chomsky Normal Form is as follows

- N is a set of non-terminal symbols
- Σ is a set of terminal symbols
- R is a set of rules which take one of two forms:
  - $X \rightarrow Y_1 Y_2$ for $X \in N$, and $Y_1, Y_2 \in N$
  - $X \rightarrow Y$ for $X \in N$, and $Y \in \Sigma$
- $S \in N$ is a distinguished start symbol

A Dynamic Programming Algorithm

- Given a PCFG and a sentence s, how do we find

$$\max_{t \in \mathcal{T}(s)} p(t)$$

- Notation:

$$n = \text{number of words in the sentence}$$

$$w_i = i\text{'th word in the sentence}$$

$$N = \text{the set of non-terminals in the grammar}$$

$$S = \text{the start symbol in the grammar}$$

- Define a dynamic programming table

$$\pi[i, j, X] \quad = \quad \text{maximum probability of a constituent with non-terminal } X \text{ spanning words } i \ldots j \text{ inclusive}$$

- Our goal is to calculate $\max_{t \in \mathcal{T}(s)} p(t) = \pi[1, n, S]$

A Dynamic Programming Algorithm

- Base case definition: for all $i = 1 \ldots n$, for $X \in N$

  $$\pi[i, i, X] = q(X \rightarrow w_i)$$

  (note: define $q(X \rightarrow w_i) = 0$ if $X \rightarrow w_i$ is not in the grammar)

- Recursive definition: for all $i = 1 \ldots n$, $j = (i+1) \ldots n$, $X \in N$,

  $$\pi(i, j, X) = \max_{\substack{X \rightarrow YZ \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \rightarrow YZ) \times \pi(i, s, Y) \times \pi(s+1, j, Z)\right)$$

The Full Dynamic Programming Algorithm

Input: a sentence $s = x_1 \ldots x_n$, a PCFG $G = (N, \Sigma, S, R, q)$.

Initialization:

For all $i \in \{1 \ldots n\}$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} q(X \rightarrow x_i) & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Algorithm:

What's the run time Complexity?

- For $l = 1 \ldots (n-1)$
  - For $i = 1 \ldots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

      $$\pi(i, j, X) = \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z) \right)$$

      and

      $$bp(i, j, X) = \arg \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z) \right)$$

CKY Parsing

A worked example

Sample Grammar

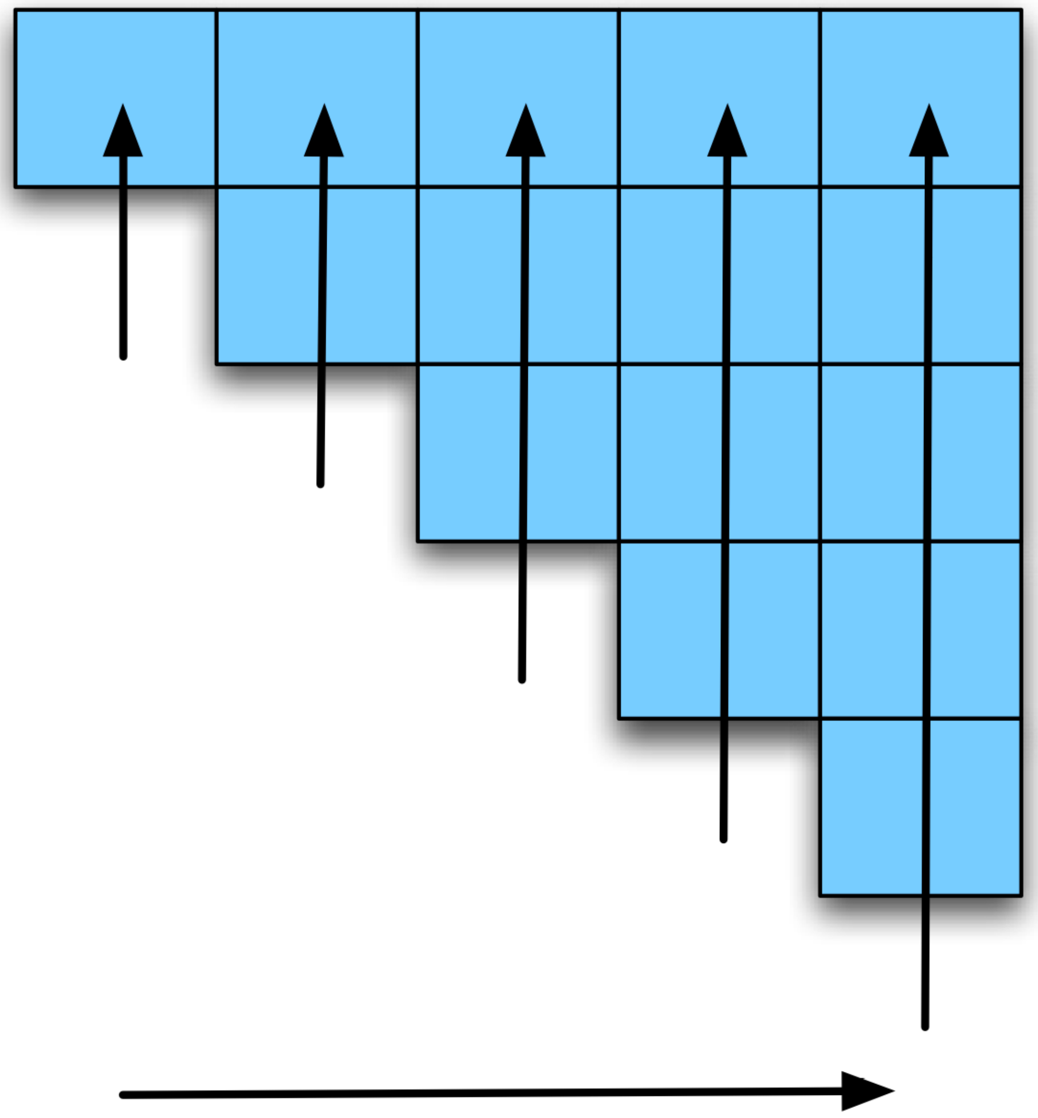
Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal\ Noun$	
$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	
$VP \rightarrow Verb\ NP$	
$VP \rightarrow Verb\ NP\ PP$	
$VP \rightarrow Verb\ PP$	
$VP \rightarrow VP\ PP$	
$PP \rightarrow Preposition\ NP$	

CNF Conversion

$\mathscr{L}_1$ **Grammar**	$\mathscr{L}_1$ **in CNF**
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

CKY Parsing: table filling illustrated

Book	*the*	*flight*	*through*	*Houston*
S, VP, Verb Nominal, Noun [0,1]	[0,2]	S,VP,X2 [0,3]	[0,4]	S,VP,X2 [0,5]
	Det [1,2]	NP [1,3]	[1,4]	NP [1,5]
		Nominal, Noun [2,3]	[2,4]	Nominal [2,5]
			Prep [3,4]	PP [3,5]
				NP, Proper-Noun [4,5]



$\mathscr{L}_1$ **in CNF**
$S \rightarrow NP\ VP$
$S \rightarrow X1\ VP$
$X1 \rightarrow Aux\ NP$
$S \rightarrow book \mid include \mid prefer$
$S \rightarrow Verb\ NP$
$S \rightarrow X2\ PP$
$S \rightarrow Verb\ PP$
$S \rightarrow VP\ PP$
$NP \rightarrow I \mid she \mid me$
$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$
$VP \rightarrow X2\ PP$
$X2 \rightarrow Verb\ NP$
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$VP \rightarrow VP\ PP$
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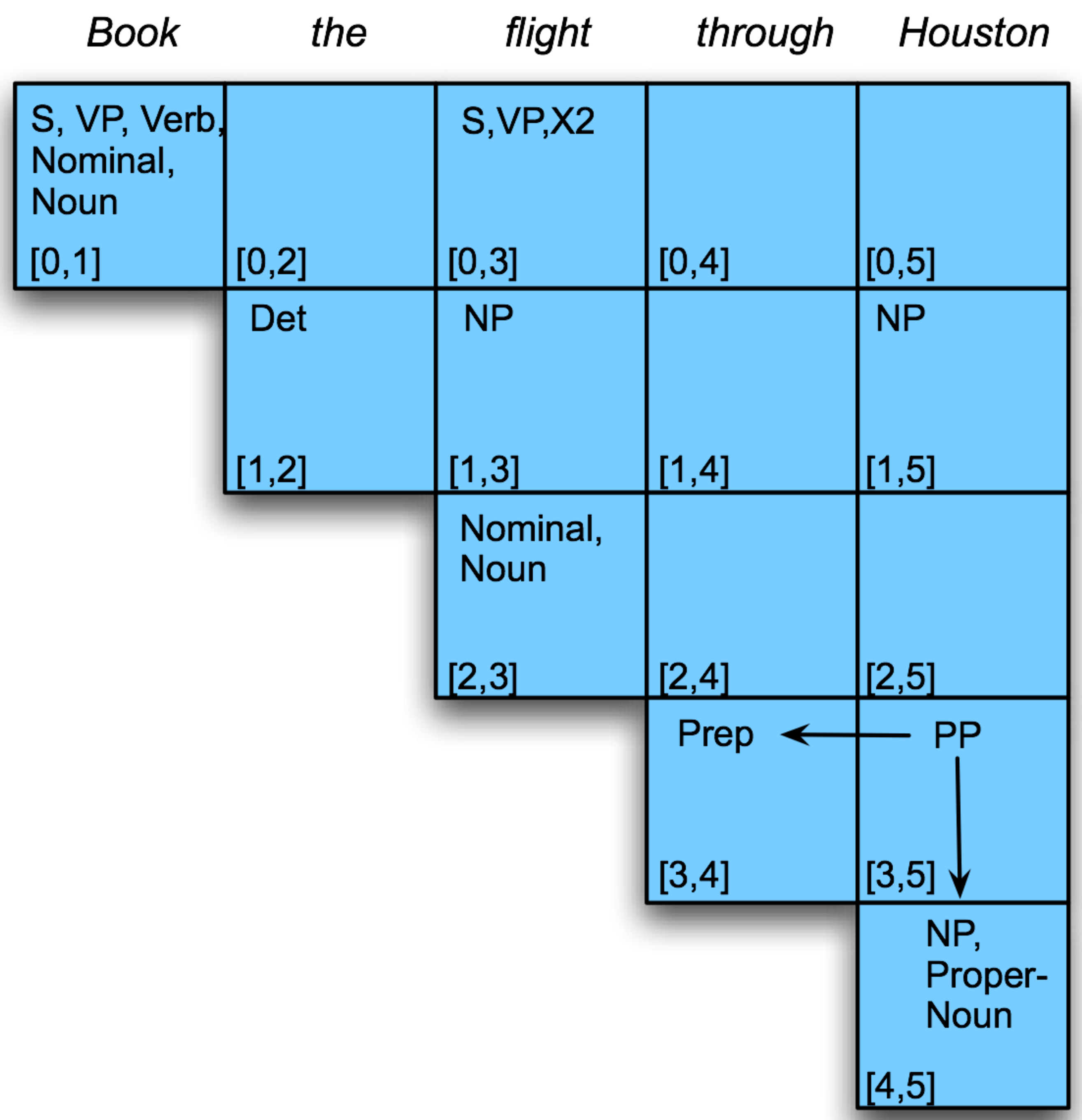

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$S \rightarrow Verb\ PP$
$S \rightarrow VP\ PP$
$NP \rightarrow I \mid she \mid me$
$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$
$VP \rightarrow X2\ PP$
$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$

Book	*the*	*flight*	*through*	*Houston*
S, VP, Verb, Nominal, Noun [0,1]	[0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]
	Det [1,2]	NP [1,3]	[1,4]	NP [1,5]
		Nominal, Noun [2,3]	[2,4]	Nominal [2,5]
			Prep [3,4]	PP [3,5]
				NP, Proper-Noun [4,5]

$\mathscr{L}_1$ in CNF

$S \rightarrow NP\ VP$
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$NP \rightarrow Det\ Nominal$
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$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow book \mid include \mid prefer$
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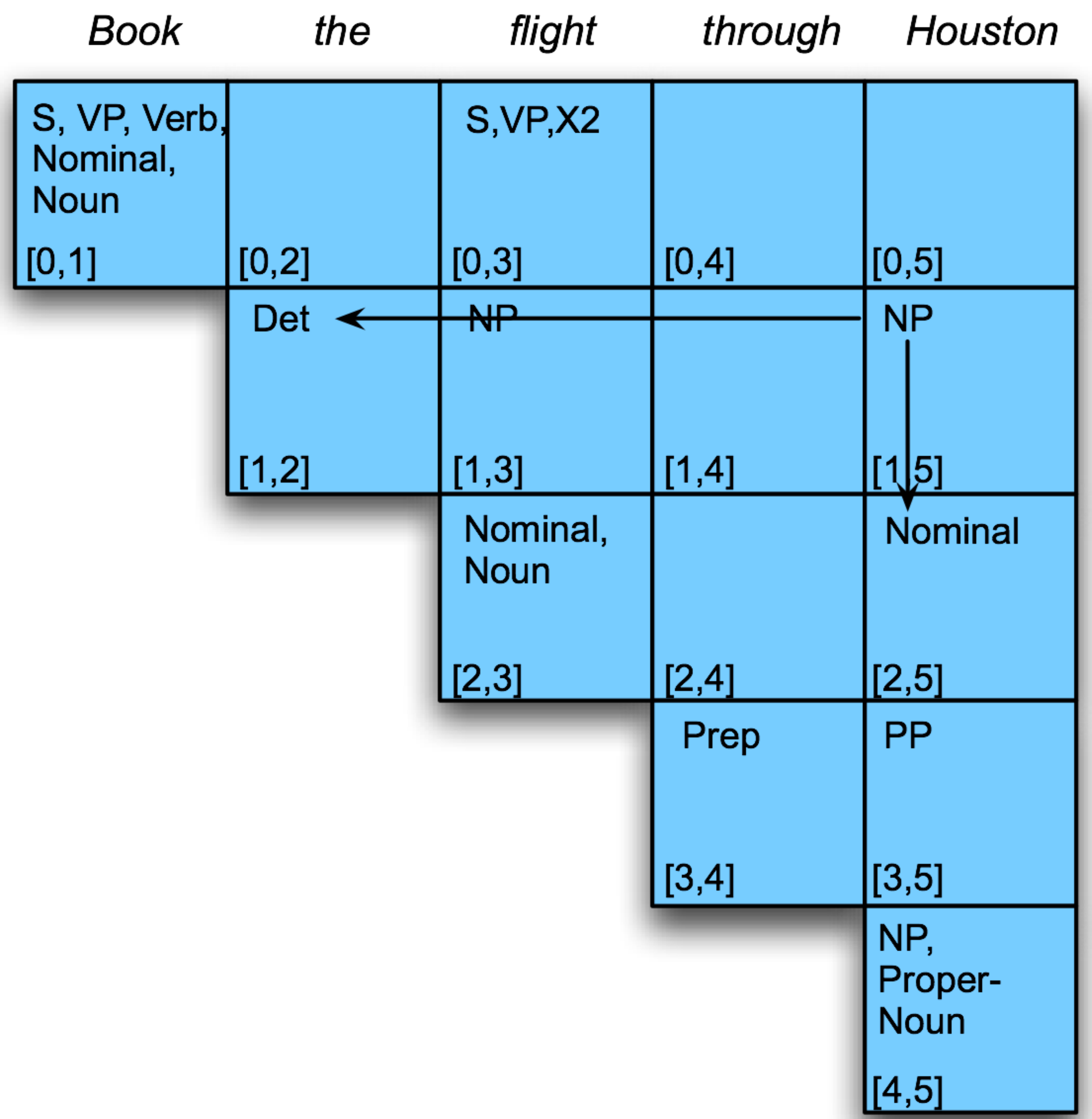
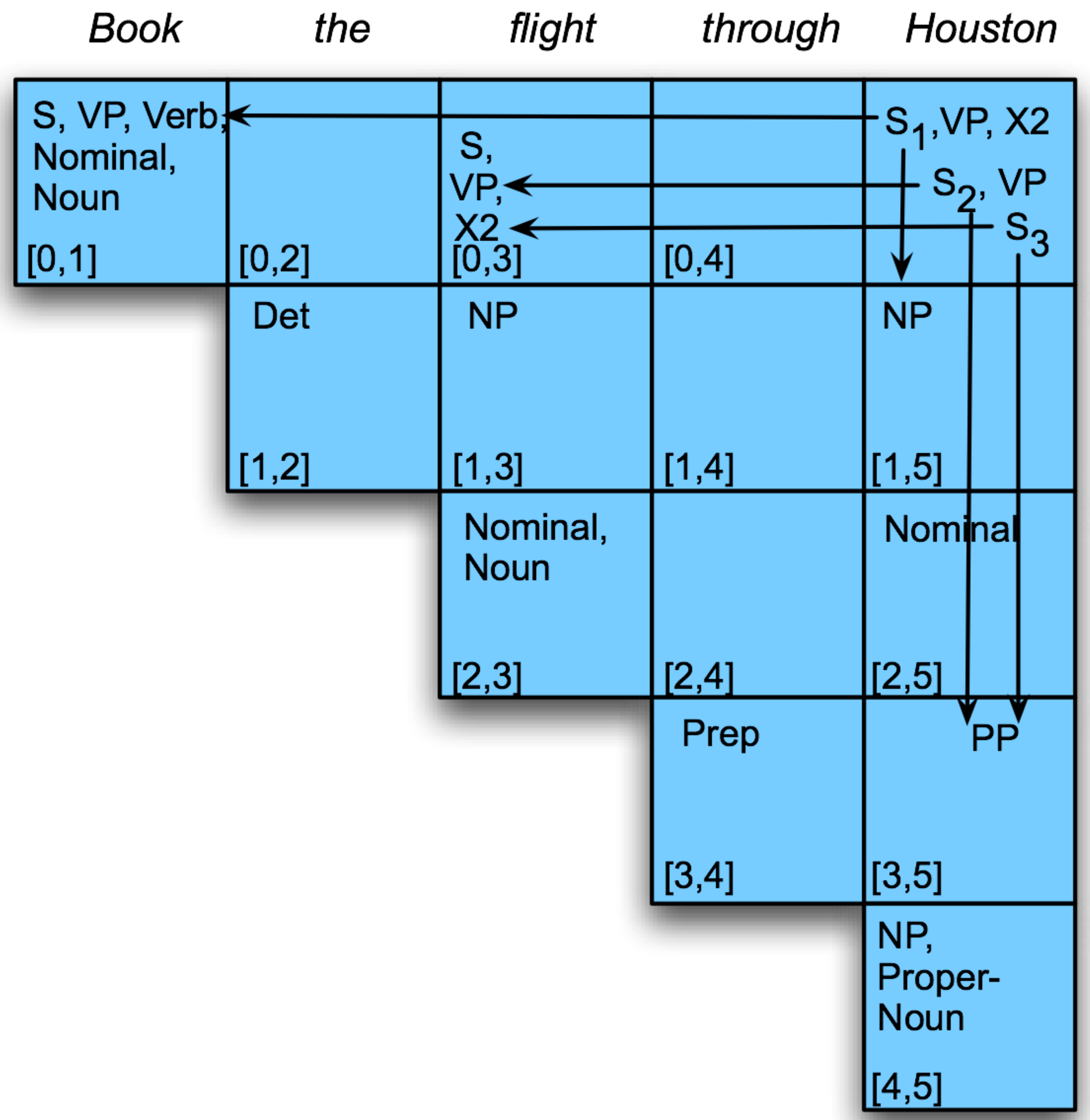


$\mathscr{L}_1$ in CNF

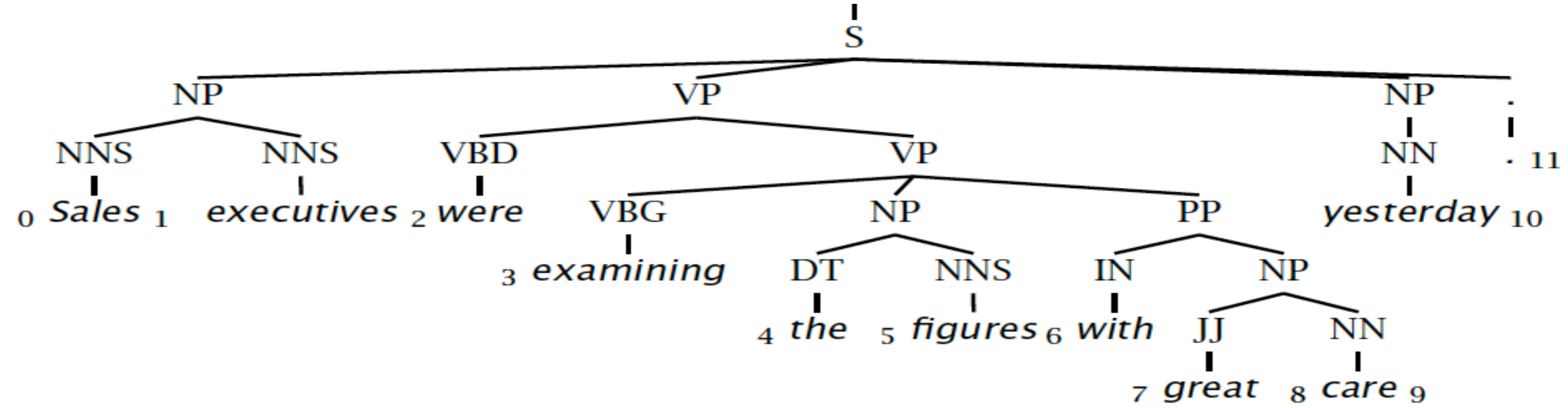
S $\rightarrow$ *NP VP*
S $\rightarrow$ *X1 VP*
X1 $\rightarrow$ *Aux NP*
S $\rightarrow$ *book* | *include* | *prefer*
S $\rightarrow$ *Verb NP*
S $\rightarrow$ *X2 PP*
S $\rightarrow$ *Verb PP*
S $\rightarrow$ *VP PP*
NP $\rightarrow$ *I* | *she* | *me*
NP $\rightarrow$ *TWA* | *Houston*
NP $\rightarrow$ *Det Nominal*
Nominal $\rightarrow$ *book* | *flight* | *meal* | *money*
Nominal $\rightarrow$ *Nominal Noun*
Nominal $\rightarrow$ *Nominal PP*
VP $\rightarrow$ *book* | *include* | *prefer*
VP $\rightarrow$ *Verb NP*
VP $\rightarrow$ *X2 PP*
X2 $\rightarrow$ *Verb NP*
VP $\rightarrow$ *Verb PP*
VP $\rightarrow$ *VP PP*
PP $\rightarrow$ *Preposition NP*



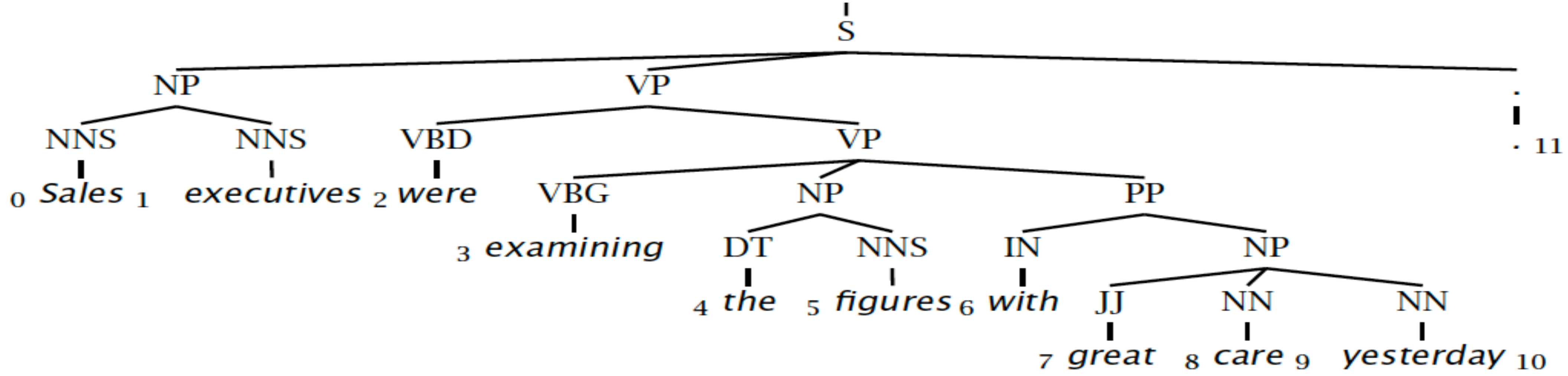
Constituency Parser Evaluation

Evaluating constituency parsing

Gold standard brackets: **S-(0:11), NP-(0:2)**, VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: **S-(0:11), NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)



Evaluating constituency parsing

Gold standard brackets:

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)

Candidate brackets:

S-(0:11), **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)

Labeled Precision	3/7 = 42.9%
Labeled Recall	3/8 = 37.5%
LP/LR F1	40.0%
Tagging Accuracy	11/11 = 100.0%

Summary

- PCFGs augments CFGs by including a probability for each rule in the grammar.
- The probability for a parse tree is the product of probabilities for the rules in the tree
- To build a PCFG-parsed parser:
  1. Learn a PCFG from a treebank
  2. Given a test data sentence, use the CKY algorithm to compute the highest probability tree for the sentence under the PCFG

How good are PCFGs?

- Penn WSJ parsing accuracy: about 73% LP/LR F1
- Robust but not so accurate
  - Usually admit everything, but with low probability
  - A PCFG gives some idea of the plausibility of a parse
  - But not so good because the independence assumptions are too strong
- Give a probabilistic language model
  - But in the simple case it performs worse than a trigram model
- The problem seems to be that PCFGs lack the lexicalization of a trigram model