Discriminative Estimation (Maxent models and perceptron)

Generative vs. Discriminative models

Many slides are adapted from slides by Christopher Manning and perceptron slides by Alan Ritter

Introduction

- So far we’ve looked at “generative models”
  - Naive Bayes
- But there is now much use of conditional or discriminative probabilistic models in NLP, Speech, IR (and ML generally)
- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features
  - They allow automatic building of language independent, retargetable NLP modules

Joint vs. Conditional Models

- We have some data {(*d*, *c*)} of paired observations *d* and hidden classes *c*.
- Joint (generative) models place probabilities over both observed data and the hidden stuff (generate the observed data from hidden stuff):
  - All the classic StatNLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars, IBM machine translation alignment models

$P(c,d)$

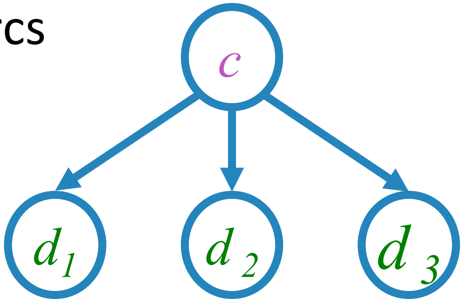
Joint vs. Conditional Models

- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data:
  - Logistic regression, conditional loglinear or maximum entropy models, conditional random fields
  - Also, SVMs, (averaged) perceptron, etc. are discriminative classifiers (but not directly probabilistic)

$P(c|d)$

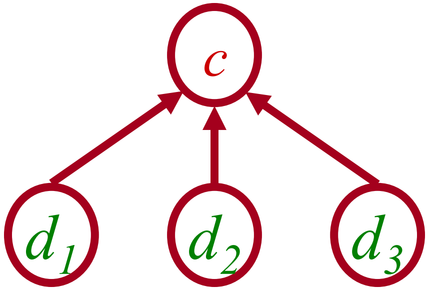
Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs



Naive Bayes

Generative



Logistic Regression

Discriminative

Conditional vs. Joint Likelihood

- A *joint* model gives probabilities $P(d,c)$ and tries to maximize this joint likelihood.
  - It turns out to be trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities $P(c|d)$. It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

Maxent Models and Discriminative Estimation

Generative vs. Discriminative models

Discriminative Model Features

Making features from text for discriminative NLP models

Features

- In these slides and most maxent work: *features* f are elementary pieces of evidence that link aspects of what we observe *d* with a category *c* that we want to predict
- A feature is a function with a bounded real value: $f\colon C \times D \rightarrow \mathbb{R}$

A Belief: to create a data partition

Features

- In NLP uses, usually a feature specifies
  1. an indicator function – a yes/no boolean matching function – of properties of the input and
  2. a particular class

$$f_i(c, d) \equiv [\Phi(d) \wedge c = c_j]$$

[Value is 0 or 1]

- Each feature picks out a data subset and suggests a label for it

Example features

- $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$
- $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$
- $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$

- Models will assign to each feature a *weight:*
  - A positive weight votes that this configuration is likely correct
  - A negative weight votes that this configuration is likely incorrect

Feature-Based Models

- The decision about a data point is based only on the features active at that point.

Text Categorization	Word-Sense Disambiguation	POS Tagging
Data BUSINESS: Stocks hit a yearly low …	Data … to restructure bank:MONEY debt.	Data DT JJ NN … The previous fall …
Label: BUSINESS Features {…, stocks, hit, a, yearly, low, …}	Label: MONEY Features {…, w_{-1}=restructure, w_{+1}=debt, …}	Label: NN Features {w=fall, t_{-1}=JJ w_{-1}=previous}

Example: Text Categorization

(Zhang and Oles 2001)

- Features are presence of each word in a document and the document class (they do feature selection to use reliable indicator words)
- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% F_1
  - Linear regression: 86.0%
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%
- Paper emphasizes the importance of *regularization* (smoothing) for successful use of discriminative methods (not used in much early NLP/IR work)

Other Maxent Classifier Examples

- You can use a maxent classifier whenever you want to assign data points to one of a number of classes:
  - Sentence boundary detection (Mikheev 2000)
    - Is a period end of sentence or abbreviation?
  - Sentiment analysis (Pang and Lee 2002)
    - Word unigrams, bigrams, POS counts, …
  - PP attachment (Ratnaparkhi 1998)
    - Attach to verb or noun? Features of head noun, preposition, etc.
  - Parsing decisions in general (Ratnaparkhi 1997; Johnson et al. 1999, etc.)

Discriminative Model Features

Making features from text for discriminative NLP models

Feature-based Linear Classifiers

How to put features into a classifier

Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - We consider each class for an observed datum d
  - For a pair (c,d), features vote with their weights:
    - vote(c) = $\Sigma\lambda_i f_i(c,d)$

PERSON *in Québec*

LOCATION *in Québec*

DRUG *in Québec*

  - Choose the class c which maximizes $\Sigma\lambda_i f_i(c,d)$

Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - We consider each class for an observed datum d
  - For a pair (c,d), features vote with their weights:
    - $\text{vote}(c) = \Sigma\lambda_i f_i(c,d)$

  - Choose the class c which maximizes $\Sigma\lambda_i f_i(c,d)$ = LOCATION

Feature-Based Linear Classifiers

There are many ways to chose weights for features

With different loss functions as the optimization goal

- Perceptron: find a currently misclassified example, and nudge weights in the direction of its correct classification

- Margin-based methods (Support Vector Machines)

Feature-Based Linear Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Make a probabilistic model from the linear combination $\Sigma\lambda_i f_i(c,d)$

$$P(c \mid d,\lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Makes votes positive (numerator)

Normalizes votes (denominator)

- P(LOCATION|*in Québec*) = $e^{1.8}e^{-0.6}/(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.586$
- P(DRUG|*in Québec*) = $e^{0.3} /(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.238$
- P(PERSON|*in Québec*) = $e^{0} /(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.176$

- The weights are the parameters of the probability model, combined via a “soft max” function

Aside: logistic regression

- Maxent models in NLP are essentially the same as multiclass logistic regression models in statistics (or machine learning)
  - The key role of feature functions in NLP and in this presentation
    - The features are more general, with f also being a function of the class

Quiz Question

- Assuming exactly the same set up (3 class decision: LOCATION, PERSON, or DRUG; 3 features as before, maxent), what are:
  - P(PERSON | *by Goéric*) =
  - P(LOCATION | *by Goéric*) =
  - P(DRUG | *by Goéric*) =

- 1.8 $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$
- -0.6 $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$
- 0.3 $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$

PERSON
by Goéric

LOCATION
by Goéric

DRUG
by Goéric

$$P(c \mid d, \lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Feature-based Linear Classifiers

How to put features into a classifier

Building a Maxent Model

The nuts and bolts

Building a Maxent Model

- We define features (indicator functions) over data points
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
    - Words, but also "word contains number", "word ends with *ing*", etc.
- We will simply encode each Φ feature as a unique String (index)
  - A datum will give rise to a set of Strings: the active Φ features
  - Each feature $f_i(c, d) \equiv [\Phi(d) \wedge c = c_j]$ gets a real number weight
- We concentrate on Φ features but the math uses i indices of f_i

Building a Maxent Model

- Features are often added during model development to target errors
  - Often, the easiest thing to think of are features that mark bad combinations
- Then, for any given feature weights, we want to be able to calculate:
  - Data conditional likelihood
  - Derivative of the likelihood wrt each feature weight
    - Uses expectations of each feature according to the model
- We can then find the optimum feature weights (discussed later).

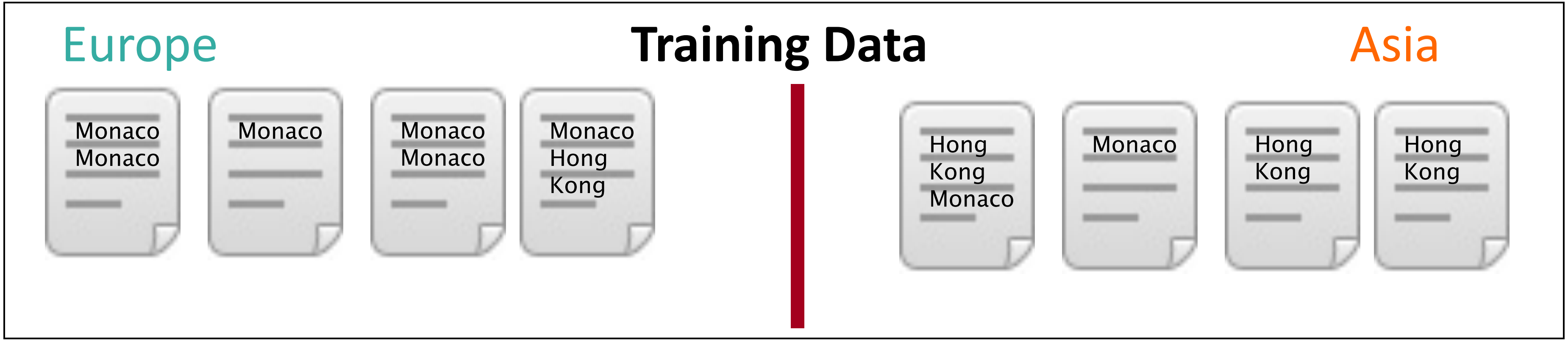
Building a Maxent Model

The nuts and bolts

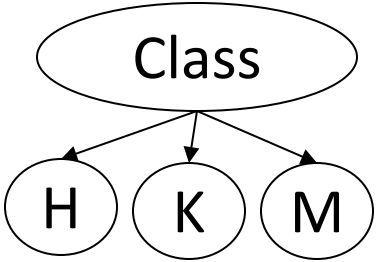
Naive Bayes vs. Maxent models

Generative vs. Discriminative models: The problem of overcounting evidence

Text classification: Asia or Europe



NB Model



NB FACTORS:

- P(A) = P(E) =
- P(M|A) =
- P(M|E) =
- P(H|A) = P(K|A) =
- P(H|E) = PK|E) =

PREDICTIONS:

- P(A,H,K,M) =
- P(E,H,K,M) =
- P(A|H,K,M) =
- P(E|H,K,M) =

Naive Bayes vs. Maxent Models

- Naive Bayes models multi-count correlated evidence
  - Each feature is multiplied in, even when you have multiple features telling you the same thing
- Maximum Entropy models (pretty much) solve this problem
  - As we will see, this is done by weighting features so that model expectations match the observed (empirical) expectations

Naive Bayes vs. Maxent models

Generative vs. Discriminative models: The problem of overcounting evidence

Maxent Models and Discriminative Estimation

Maximizing the likelihood

Feature Expectations

- We will crucially make use of two *expectations*
  - actual or predicted counts of a feature firing:

- Empirical count (expectation) of a feature:

Goal: well fit the data

$$\text{empirical } E(f_i) = \sum_{(c,d)\in \text{observed}(C,D)} f_i(c,d)$$

- Model expectation of a feature:

$$E(f_i) = \sum_{(c,d)\in (C,D)} P(c,d) f_i(c,d)$$

Exponential Model Likelihood

- Maximum (Conditional) Likelihood Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.

$$\log P(C \mid D, \lambda) = \sum_{(c,d) \in (C,D)} \log P(c \mid d, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

The Likelihood Value

- The (log) conditional likelihood of iid data (*C,D*) according to maxent model is a function of the data and the parameters λ:

$$\log P(C \mid D, \lambda) = \log \prod_{(c,d)\in(C,D)} P(c \mid d, \lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d, \lambda)$$

- If there aren't many values of *c*, it's easy to calculate:

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

The Likelihood Value

- We can separate this into two components:

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)$$

$$\log P(C \mid D, \lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component

The Derivative I: Numerator

$$\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} f_i(c,d)$$

Derivative of the numerator is: the empirical count(f_i, c)

The Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{1} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} P(c'|d,\lambda) f_i(c',d)$$

= predicted count(f_i, λ)

The Derivative III

$$\frac{\partial \log P(C \mid D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).
- These models are also called maximum entropy models because we find the model having maximum entropy and satisfying the constraints: $E_p(f_j) = E_{\tilde{p}}(f_j), \forall j$

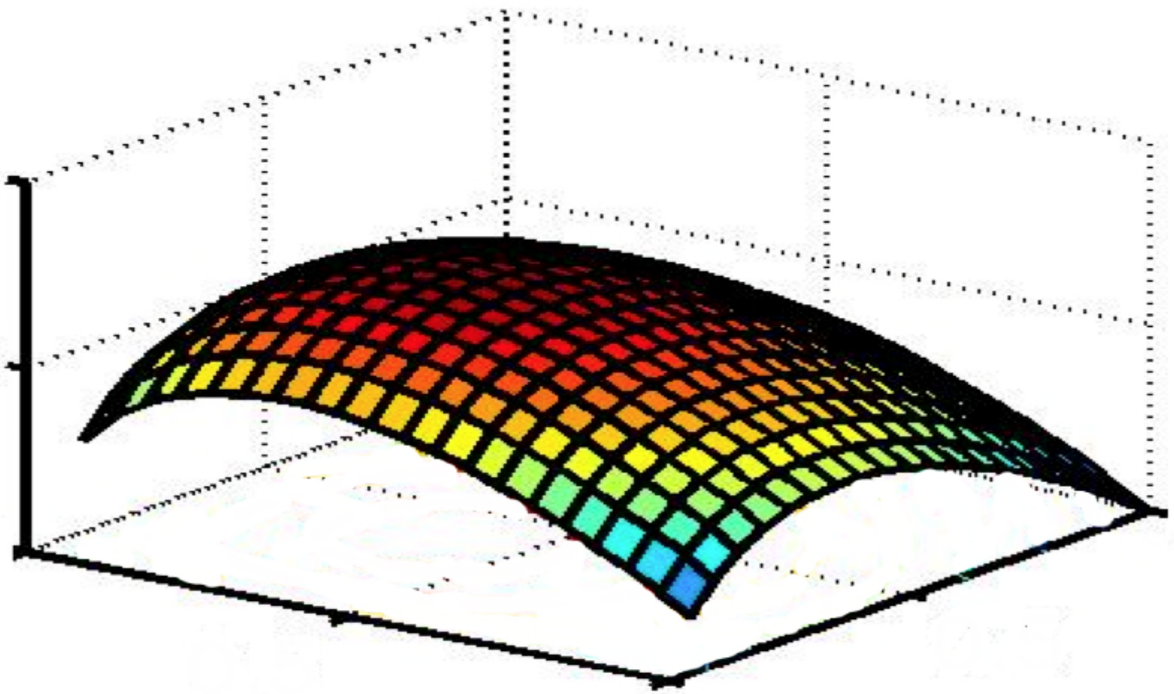
Finding the optimal parameters

- We want to choose parameters $\lambda_1, \lambda_2, \lambda_3, \ldots$ that maximize the conditional log-likelihood of the training data

$$CLogLik(D) = \sum_{i=1}^{n} \log P(c_i \mid d_i)$$

- To be able to do that, we've worked out how to calculate the function value and its partial derivatives (its gradient)

A likelihood surface



Finding the optimal parameters

- Use your favorite numerical optimization package….
    - Commonly, you **minimize** the negative of *CLogLik*

1. Gradient descent (GD); Stochastic gradient descent (SGD)
2. Iterative proportional fitting methods: Generalized Iterative Scaling (GIS) and Improved Iterative Scaling (IIS)
3. Conjugate gradient (CG), perhaps with preconditioning
4. Quasi-Newton methods – limited memory variable metric (LMVM) methods, in particular, L-BFGS

Gradient Descent (GD)

Gradient ascent algorithm: iterate until change < ε

$$w_0^{(t+1)} \leftarrow w_0^{(t)} + \eta \sum_j [y^j - \hat{P}(Y^j = 1 \mid \mathbf{x}^j, \mathbf{w}^{(t)})]$$

For *i* = 1,…,*d*,

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta \sum_j x_i^j [y^j - \hat{P}(Y^j = 1 \mid \mathbf{x}^j, \mathbf{w}^{(t)})]$$

repeat

Maxent Models and Discriminative Estimation

Maximizing the likelihood

Feature Sparsity Regularization

Combating overfitting

Smoothing: Issues of Scale

- Lots of features:
  - NLP maxent models can have well over a million features.
  - Even storing a single array of parameter values can have a substantial memory cost.
- Lots of sparsity:
  - Overfitting very easy – we need smoothing!
  - Many features seen in training will never occur again at test time.
- Optimization problems:
  - Feature weights can be infinite, and iterative solvers can take a long time to get to those infinities.

Smoothing/Priors/ Regularization

- Combating over fitting
- Intuition: don't let the weights get very large

$$w_{\text{MLE}} = \text{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w)$$

$$\text{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w) - \delta \sum_{i=1}^{V} w_i^2$$

Standard vs. Regularized Updates

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta \sum_j x_i^j [y^j - \hat{P}(Y^j = 1 \mid \mathbf{x}^j, \mathbf{w}^{(t)})]$$

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta \left\{ -\lambda w_i^{(t)} + \sum_j x_i^j [y^j - \hat{P}(Y^j = 1 \mid \mathbf{x}^j, \mathbf{w}^{(t)})] \right\}$$

Feature Sparsity Regularization

Combating overfitting

Batch vs. Online Learning

GD vs. SGD

Stochastic Gradient Decent (SGD)

Batch vs. Online learning:

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta \left\{ -\lambda w_i^{(t)} + \sum_j x_i^j [y^j - \widehat{P}(Y^j = 1 \mid \mathbf{x}^j, \mathbf{w}^{(t)})] \right\}$$

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

Batch vs. Online Learning

GD vs. SGD

Perceptron

Another Online Learning algorithem

Perceptron Algorithm

- Algorithm is Very similar to logistic regression
- Not exactly computing gradients

```
Initalize weight vector w = 0
Loop for K iterations
    Loop For all training examples x_i
        if sign(w * x_i) != y_i
            w += (y_i - sign(w * x_i)) * x_i
```

MaxEnt v.s Perceptron

- Perceptron doesn’t always make updates
- Probabilities v.s scores

Regularization in the Perceptron Algorithm

- No gradient computed, so can't directly include a regularizer in an object function.
- Instead run different numbers of iterations
- Use parameter averaging, for instance, average of all parameters after seeing each data point