Probabilistic Context Free Grammars

Many slides from Michael Collins and Chris Manning

Overview

- Probabilistic Context-Free Grammars (PCFGs)
- The CKY Algorithm for parsing with PCFGs

A Probabilistic Context-Free Grammar (PCFG)

S	$\Rightarrow$	NP	VP	1.0
VP	$\Rightarrow$	Vi		0.4
VP	$\Rightarrow$	Vt	NP	0.4
VP	$\Rightarrow$	VP	PP	0.2
NP	$\Rightarrow$	DT	NN	0.3
NP	$\Rightarrow$	NP	PP	0.7
PP	$\Rightarrow$	P	NP	1.0

Vi	$\Rightarrow$	sleeps	1.0
Vt	$\Rightarrow$	saw	1.0
NN	$\Rightarrow$	man	0.7
NN	$\Rightarrow$	woman	0.2
NN	$\Rightarrow$	telescope	0.1
DT	$\Rightarrow$	the	1.0
IN	$\Rightarrow$	with	0.5
IN	$\Rightarrow$	in	0.5

- Probability of a tree t with rules

$$\alpha_1 \rightarrow \beta_1, \alpha_2 \rightarrow \beta_2, \ldots, \alpha_n \rightarrow \beta_n$$

is $p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$ where $q(\alpha \rightarrow \beta)$ is the probability for rule $\alpha \rightarrow \beta$.

DERIVATION	RULES USED	PROBABILITY
S		

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S		1.0
	S $\rightarrow$ NP VP	
NP VP		

DERIVATION	RULES USED	PROBABILITY
S		1.0
	S $\rightarrow$ NP VP	
NP VP		0.3
	NP $\rightarrow$ DT NN	
DT NN VP		

DERIVATION	RULES USED	PROBABILITY
S		1.0
	$S \rightarrow$ NP VP	
NP VP		0.3
	$NP \rightarrow$ DT NN	
DT NN VP		1.0
	$DT \rightarrow$ the	
the NN VP		

DERIVATION	RULES USED	PROBABILITY
S		1.0
	S $\rightarrow$ NP VP	
NP VP		0.3
	NP $\rightarrow$ DT NN	
DT NN VP		1.0
	DT $\rightarrow$ the	
the NN VP		0.1
	NN $\rightarrow$ dog	
the dog VP		

DERIVATION	RULES USED	PROBABILITY
S	S $\rightarrow$ NP VP	1.0
NP VP	NP $\rightarrow$ DT NN	0.3
DT NN VP	DT $\rightarrow$ the	1.0
the NN VP	NN $\rightarrow$ dog	0.1
the dog VP	VP $\rightarrow$ Vi	0.4
the dog Vi		

DERIVATION	RULES USED	PROBABILITY
S	S $\rightarrow$ NP VP	1.0
NP VP	NP $\rightarrow$ DT NN	0.3
DT NN VP	DT $\rightarrow$ the	1.0
the NN VP	NN $\rightarrow$ dog	0.1
the dog VP	VP $\rightarrow$ Vi	0.4
the dog Vi	Vi $\rightarrow$ laughs	0.5
the dog laughs		

Properties of PCFGs

- Assigns a probability to each *left-most derivation*, or parse-tree, allowed by the underlying CFG

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- Say we have a sentence s, set of derivations for that sentence is $\mathcal{T}(s)$. Then a PCFG assigns a probability $p(t)$ to each member of $\mathcal{T}(s)$. i.e., *we now have a ranking in order of probability*.

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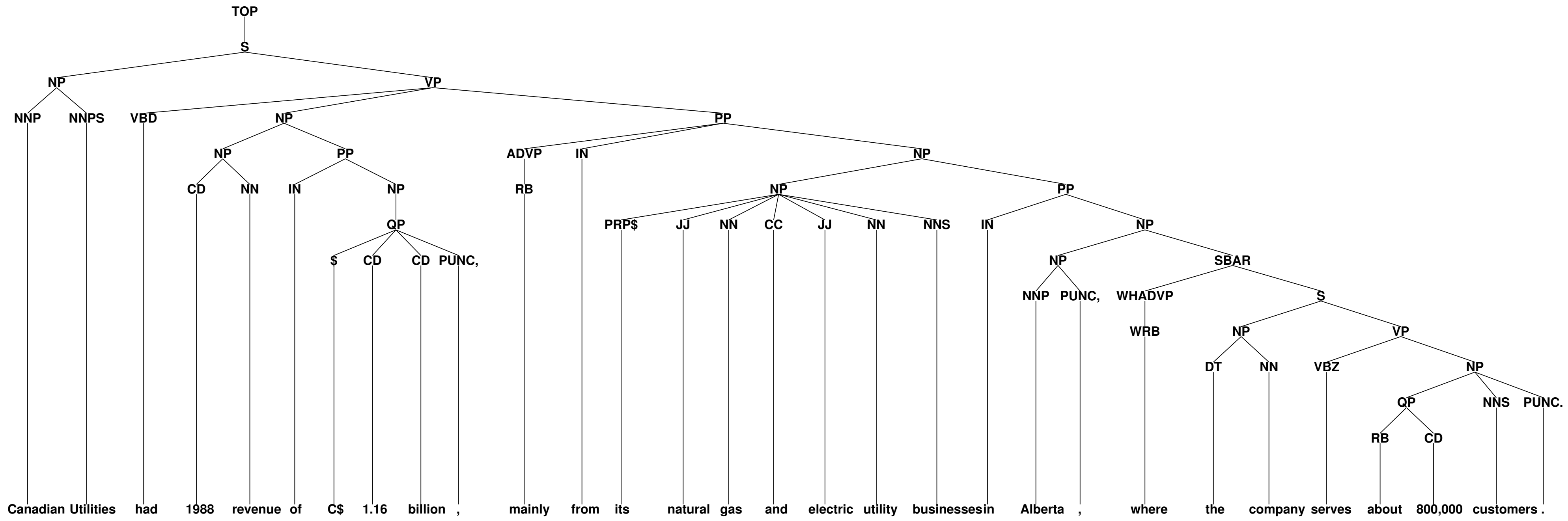
- Assigns a probability to each *left-most derivation*, or parse-tree, allowed by the underlying CFG
- Say we have a sentence s, set of derivations for that sentence is $\mathcal{T}(s)$. Then a PCFG assigns a probability $p(t)$ to each member of $\mathcal{T}(s)$. i.e., *we now have a ranking in order of probability*.
- The most likely parse tree for a sentence s is

$$\arg\max_{t \in \mathcal{T}(s)} p(t)$$

Data for Parsing Experiments: Treebanks

- Penn WSJ Treebank = 50,000 sentences with associated trees
- Usual set-up: 40,000 training sentences, 2400 test sentences

An example tree:



Deriving a PCFG from a Treebank

- Given a set of example trees (a treebank), the underlying CFG can simply be **all rules seen in the corpus**

- Maximum Likelihood estimates:

$$q_{ML}(\alpha \to \beta) = \frac{\text{Count}(\alpha \to \beta)}{\text{Count}(\alpha)}$$

  where the counts are taken from a training set of example trees.

- **If the training data is generated by a PCFG**, then as the training data size goes to infinity, the maximum-likelihood PCFG will converge to the same distribution as the "true" PCFG.

PCFGs

Booth and Thompson (1973) showed that a CFG with rule probabilities correctly defines a distribution over the set of derivations provided that:

1. The rule probabilities define conditional distributions over the different ways of rewriting each non-terminal.
2. A technical condition on the rule probabilities ensuring that the probability of the derivation terminating in a finite number of steps is 1. (This condition is not really a practical concern.)

Parsing with a PCFG

- Given a PCFG and a sentence s, define $\mathcal{T}(s)$ to be the set of trees with s as the yield.
- Given a PCFG and a sentence s, how do we find

$$\arg\max_{t \in \mathcal{T}(s)} p(t)$$

Chomsky Normal Form

A context free grammar $G = (N, \Sigma, R, S)$ in Chomsky Normal Form is as follows

- N is a set of non-terminal symbols
- Σ is a set of terminal symbols
- R is a set of rules which take one of two forms:
  - $X \rightarrow Y_1 Y_2$ for $X \in N$, and $Y_1, Y_2 \in N$
  - $X \rightarrow Y$ for $X \in N$, and $Y \in \Sigma$
- $S \in N$ is a distinguished start symbol

A Dynamic Programming Algorithm

- Given a PCFG and a sentence s, how do we find

$$\max_{t \in \mathcal{T}(s)} p(t)$$

- Notation:

$$n = \text{number of words in the sentence}$$
$$w_i = i\text{'th word in the sentence}$$
$$N = \text{the set of non-terminals in the grammar}$$
$$S = \text{the start symbol in the grammar}$$

- Define a dynamic programming table

$$\pi[i, j, X] = \text{maximum probability of a constituent with non-terminal } X \text{ spanning words } i \dots j \text{ inclusive}$$

- Our goal is to calculate $\max_{t \in \mathcal{T}(s)} p(t) = \pi[1, n, S]$

A Dynamic Programming Algorithm

- Base case definition: for all $i = 1 \ldots n$, for $X \in N$

$$\pi[i, i, X] = q(X \rightarrow w_i)$$

(note: define $q(X \rightarrow w_i) = 0$ if $X \rightarrow w_i$ is not in the grammar)

- Recursive definition: for all $i = 1 \ldots n$, $j = (i+1) \ldots n$, $X \in N$,

$$\pi(i, j, X) = \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z) \right)$$

The Full Dynamic Programming Algorithm

Input: a sentence $s = x_1 \ldots x_n$, a PCFG $G = (N, \Sigma, S, R, q)$.
Initialization:
For all $i \in \{1 \ldots n\}$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} q(X \rightarrow x_i) & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Algorithm:

What's the run time Complexity?

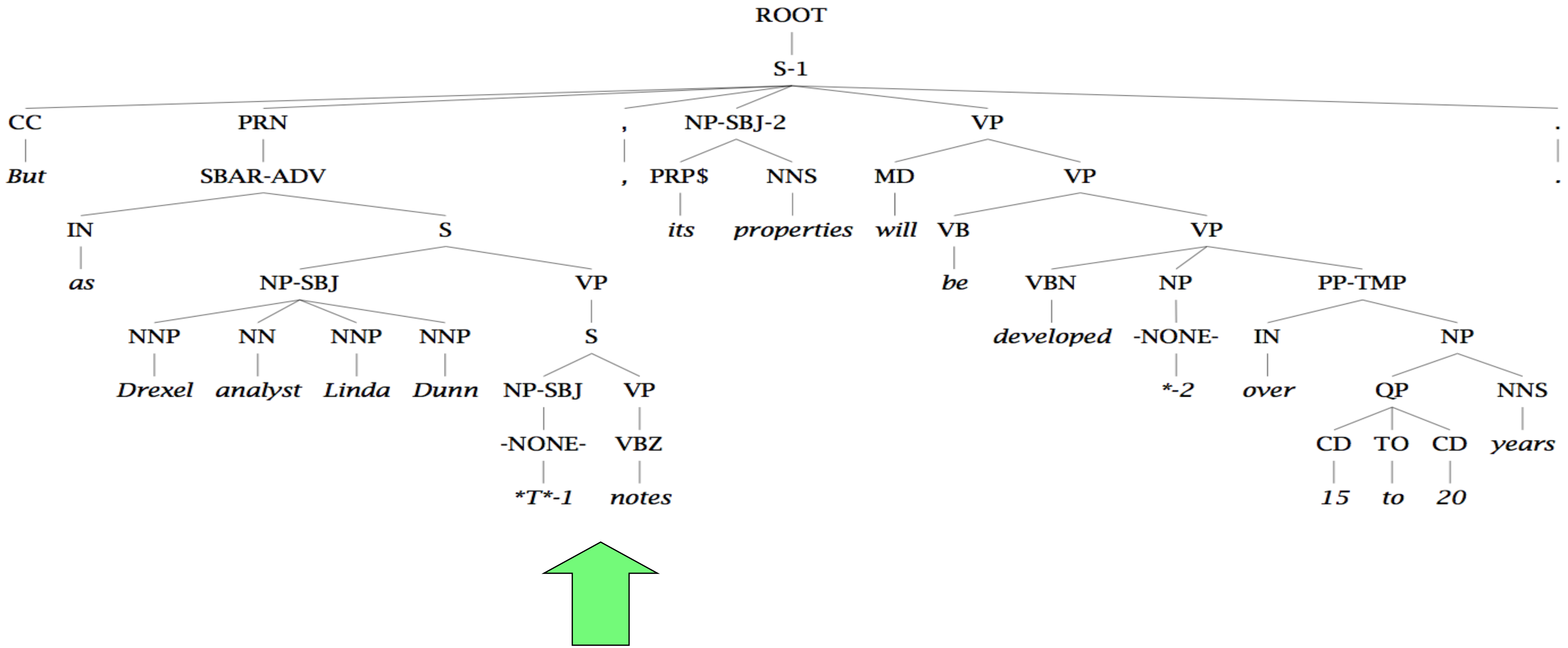
- For $l = 1 \ldots (n-1)$
  - For $i = 1 \ldots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

$$\pi(i, j, X) = \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} (q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z))$$

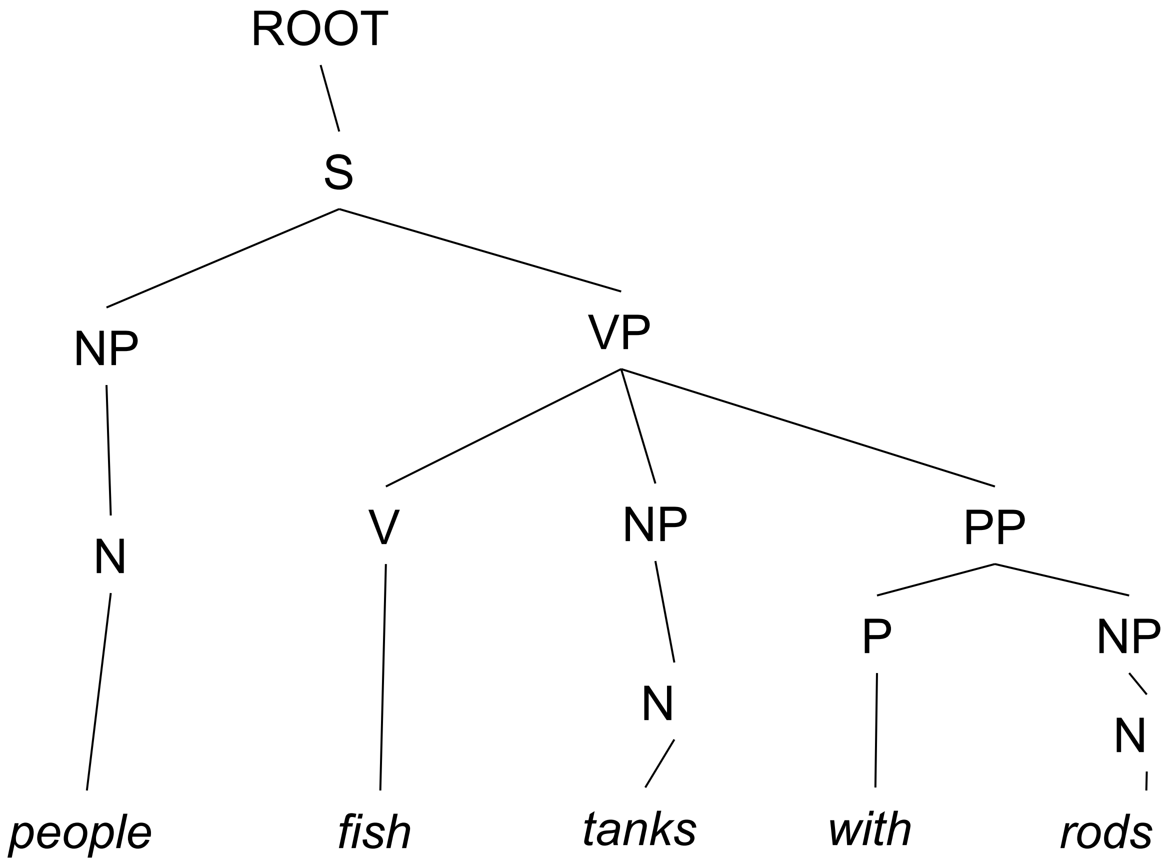
and

$$bp(i, j, X) = \arg \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} (q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z))$$

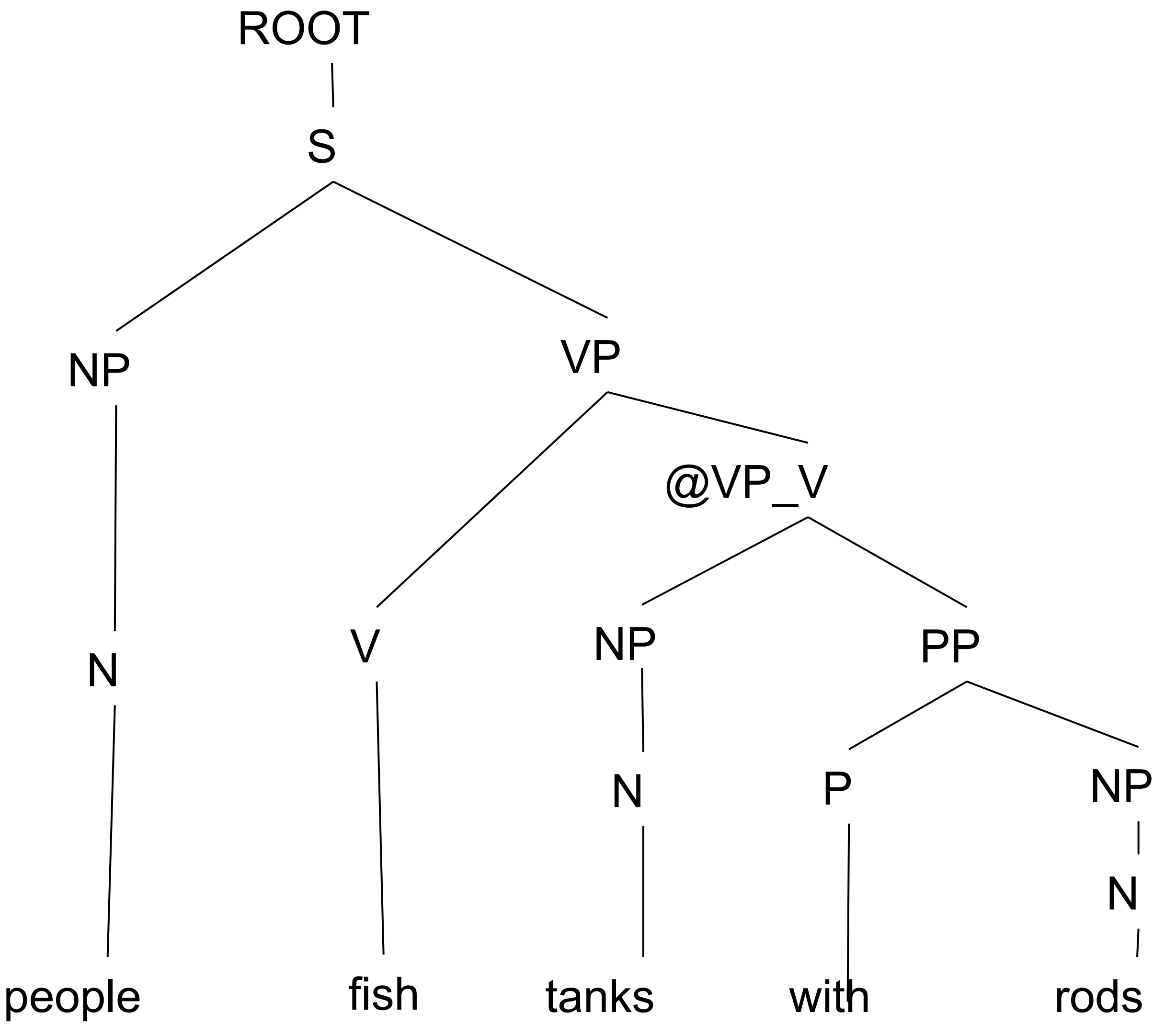
Unary rules:
alchemy in the land of treebanks



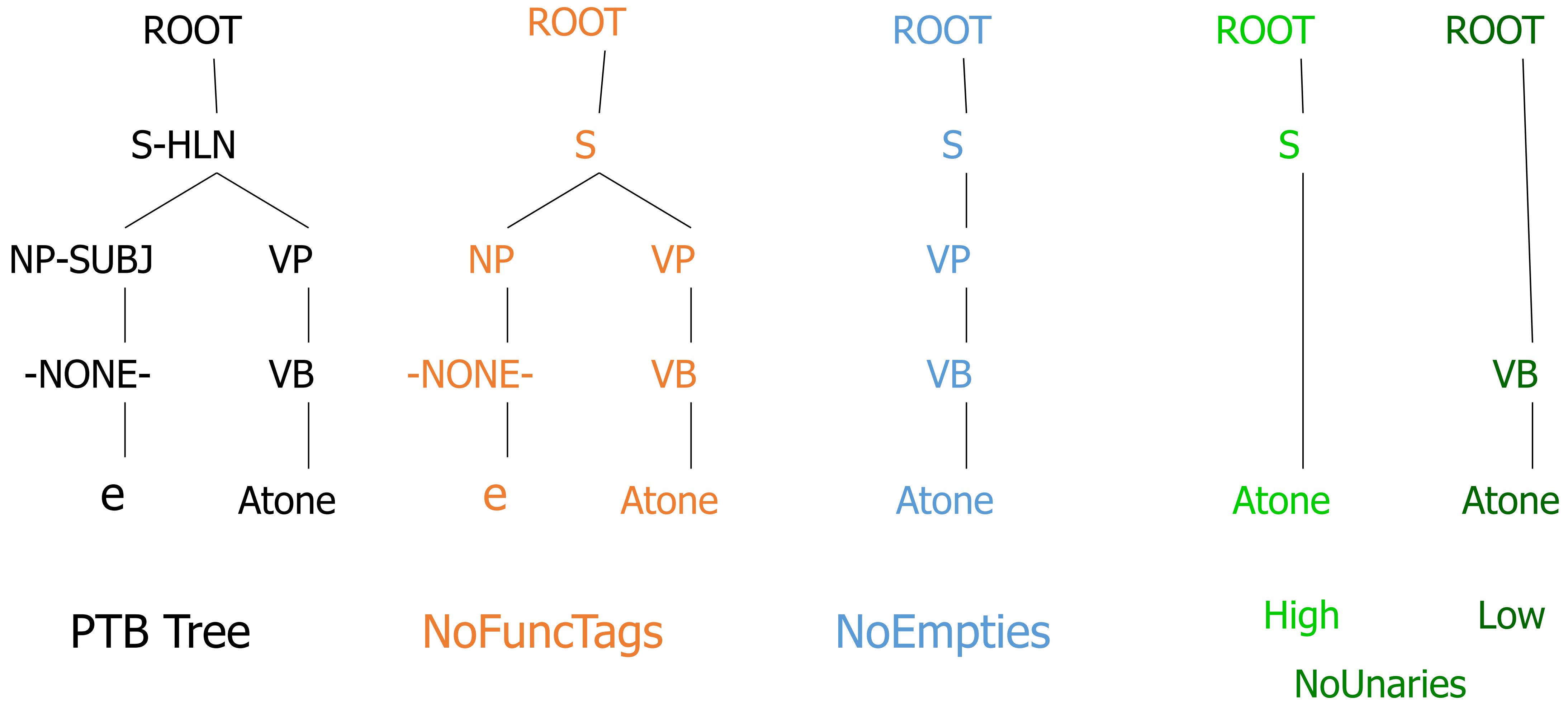
An example: before binarization…



After binarization…



Treebank: empties and unaries



Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts
  - Doesn't increase complexity; essentially like unaries
- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar
    - Binarization may be an explicit transformation or implicit in how the parser works (Early-style dotted rules), but it's always there.

The CKY algorithm (1960/1965)
… extended to unaries

```
function CKY(words, grammar) returns [most_probable_parse,prob]
  score = new double[#(words)+1][#(words)+1][#(nonterms)]
  back = new Pair[#(words)+1][#(words)+1][#nonterms]]
  for i=0; i<#(words); i++
    for A in nonterms
      if A -> words[i] in grammar
        score[i][i+1][A] = P(A -> words[i])

    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
          prob = P(A->B)*score[i][i+1][B]
          if prob > score[i][i+1][A]
            score[i][i+1][A] = prob
            back[i][i+1][A] = B
            added = true
```

The CKY algorithm (1960/1965) … extended to unaries

```
for span = 2 to #(words)
  for begin = 0 to #(words)- span
    end = begin + span
    for split = begin+1 to end-1
      for A,B,C in nonterms
        prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
        if prob > score[begin][end][A]
          score[begin]end][A] = prob
          back[begin][end][A] = new Triple(split,B,C)

    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        prob = P(A->B)*score[begin][end][B];
        if prob > score[begin][end][A]
          score[begin][end][A] = prob
          back[begin][end][A] = B
          added = true
return buildTree(score, back)
```

CKY Parsing

A worked example

The grammar:
Binary, Unaries, no epsilons,

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

0	fish 1	people 2	fish 3	tanks 4
1	score[0][1]	score[0][2]	score[0][3]	score[0][4]
2		score[1][2]	score[1][3]	score[1][4]
3			score[2][3]	score[2][4]
4				score[3][4]

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
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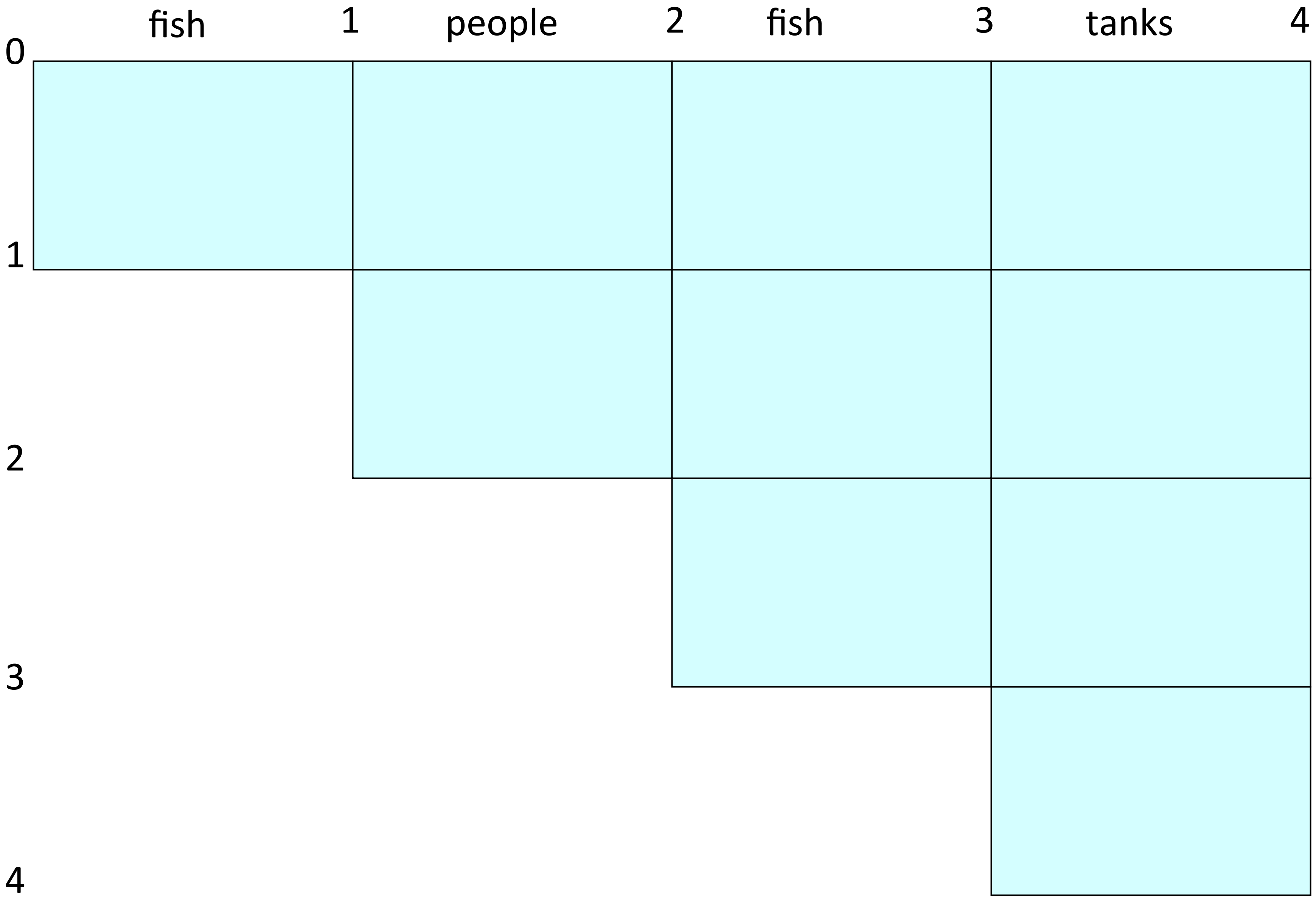
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0
N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0



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N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

```
for i=0; i<#(words); i++
   for A in nonterms
     if A -> words[i] in grammar
       score[i][i+1][A] = P(A -> words[i]);
```

	fish (0–1)	people (1–2)	fish (2–3)	tanks (3–4)
0	N → fish 0.2 V → fish 0.6			
1		N → people 0.5 V → people 0.1		
2			N → fish 0.2 V → fish 0.6	
3				N → tanks 0.2 V → tanks 0.3

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	0 fish 1	1 people 2	2 fish 3	3 tanks 4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006			
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		
2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	
3				N → tanks 0.2 V → tanks 0.3 NP → N 0.14 VP → V 0.03 S → VP 0.003

```
// handle unaries
boolean added = true
  while added
    added = false
    for A, B in nonterms
      if score[i][i+1][B] > 0 && A->B in grammar
        prob = P(A->B)*score[i][i+1][B]
        if(prob > score[i][i+1][A])
          score[i][i+1][A] = prob
          back[i][i+1][A] = B
          added = true
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0
N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish (0–1)	people (1–2)	fish (2–3)	tanks (3–4)
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → NP VP 0.00126		
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	
2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → NP VP 0.00378
3				N → tanks 0.2 V → tanks 0.3 NP → N 0.14 VP → V 0.03 S → VP 0.003
4				

```
prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
if (prob > score[begin][end][A])
   score[begin]end][A] = prob
   back[begin][end][A] = new Triple(split,B,C)
```

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S → VP	0.1
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VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish 1	people 2	fish 3	tanks 4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105		
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	
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```
//handle unaries
boolean added = true
while added
  added = false
  for A, B in nonterms
    prob = P(A->B)*score[begin][end][B];
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = B
      added = true
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	fish (0–1)	people (1–2)	fish (2–3)	tanks (3–4)
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105	NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882	
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	
2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042
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4				

```
for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin]end][A] = prob
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	fish (0–1)	people (1–2)	fish (2–3)	tanks (3–4)
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2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042
3				N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003
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NP → N		0.7
PP → P NP		1.0
N → *people*		0.5
N → *fish*		0.2
N → *tanks*		0.2
N → *rods*		0.1
V → *people*		0.1
V → *fish*		0.6
V → *tanks*		0.3
P → *with*		1.0

	0 fish 1	1 people 2	2 fish 3	3 tanks 4
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1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323
2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042
3				N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003
4				

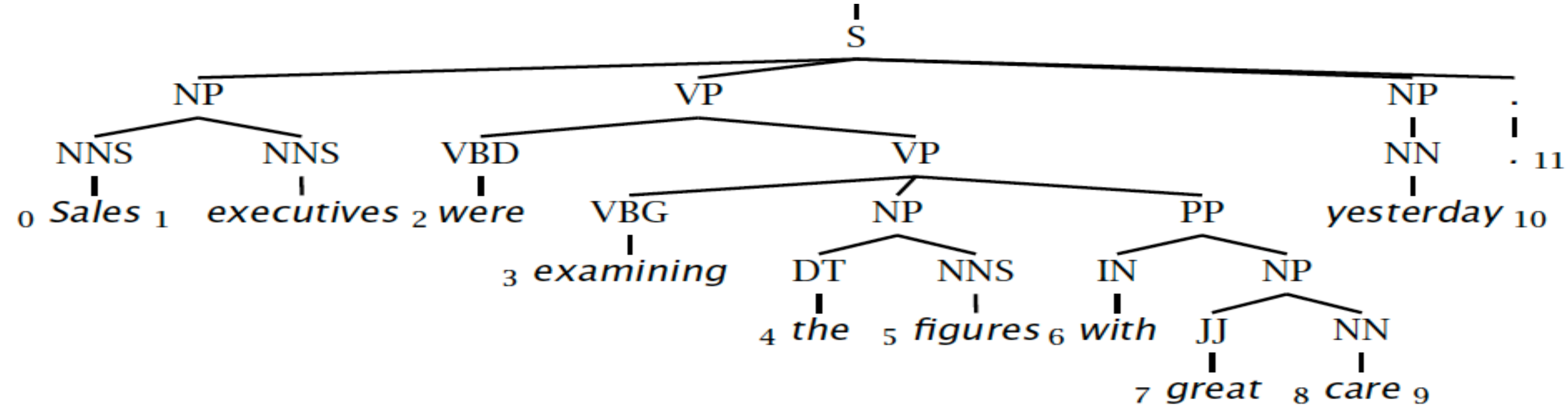
```
for split = begin+1 to end-1
   for A,B,C in nonterms
      prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
      if prob > score[begin][end][A]
         score[begin]end][A] = prob
         back[begin][end][A] = new Triple(split,B,C)
```

Call buildTree(score, back) to get the best parse

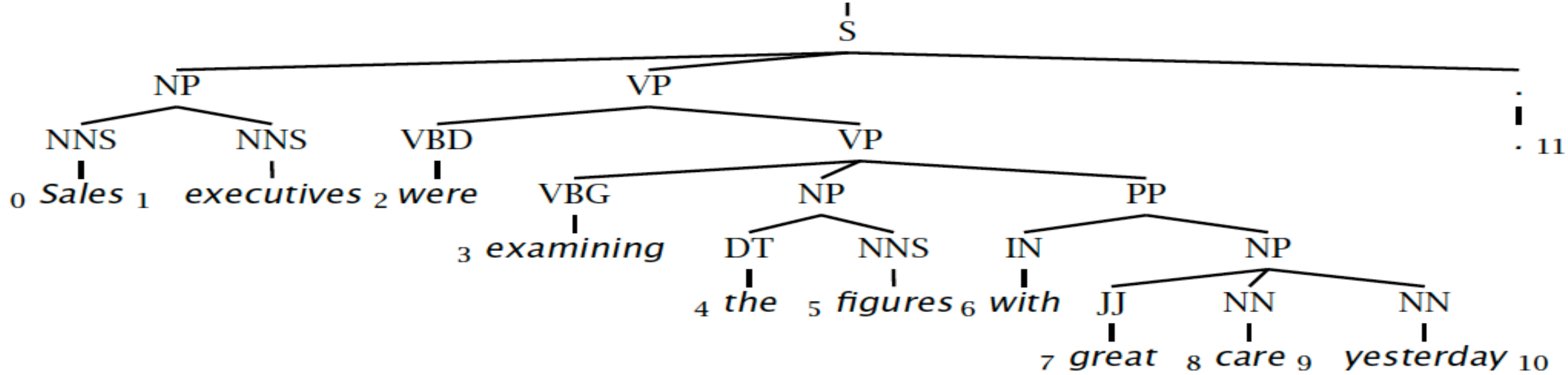
Constituency Parser Evaluation

Evaluating constituency parsing

Gold standard brackets: **S-(0:11), NP-(0:2)**, VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: **S-(0:11), NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)



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Labeled Precision	3/7 = 42.9%
Labeled Recall	3/8 = 37.5%
LP/LR F1	40.0%
Tagging Accuracy	11/11 = 100.0%

Summary

- PCFGs augments CFGs by including a probability for each rule in the grammar.
- The probability for a parse tree is the product of probabilities for the rules in the tree
- To build a PCFG-parsed parser:
  1. Learn a PCFG from a treebank
  2. Given a test data sentence, use the CKY algorithm to compute the highest probability tree for the sentence under the PCFG

How good are PCFGs?

- Penn WSJ parsing accuracy: about 73% LP/LR F1
- Robust
  - Usually admit everything, but with low probability
- Partial solution for grammar ambiguity
  - A PCFG gives some idea of the plausibility of a parse
  - But not so good because the independence assumptions are too strong
- Give a probabilistic language model
  - But in the simple case it performs worse than a trigram model
- The problem seems to be that PCFGs lack the lexicalization of a trigram model