### ELEN 689:Special Topics Advanced Mixed-Signal Interfaces

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### A Lot of New Names for Future Broadband Communication Systems

Isolator

ADC

DAC

Power amplifier Digital

processing

#### The Names

- Software Defined Radios
- Multi-Standard Radios
- Cognitive Radios
- Universal Radios

#### Common Features

 Very wideband systems, multiband channels, opportunistic frequency allocation, bandwidth reuse, intensely digital, scalable/reconfigurable RF/analog.

#### Challenges

 Conflicting requirements, large bandwidth/dynamic range but still want low power/small area.



## **Receiver Topologies**

### The Receiver Design Problem in Broadband Communications



- How much RF processing do I do before the ADC?
- How do I take advantage of technology scaling in this RF pre-procesing?
- How do I make the front-end scalable and configurable to fit multiple standards?

## **Conventional Receivers**

- Superheterodyne Receiver
- Single Conversion Receiver
- > Upconversion
- Dual Conversion
- Image-Reject Receiver (Complex I&Q mixing)
- Direct Conversion Receiver

## **Image Rejection**

➢ In high-IF RF receivers, RF LC or SAW filters are used to suppress the image before the down-conversion. Larger IFs are preferable to relax the filter Q factor.



Fig. 1. Conventional receiver with RF image filter.

Ideally zero IF does not require the RF filter but still suffers from gain and phase mismatches.



Fig. 2. Direct-conversion receiver.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

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## Image Rejection Ratio

$$I_{LO} = \text{Bcos}(\omega_{LO}t)$$

$$Q_{LO} = A\sin(\omega_{LO}t)$$

$$Cos(x)cos(y) = \frac{1}{2}[cos(x-y) + cos(x+y)]$$

$$I_{LO} * x_{RF} = \frac{A}{2}[cos((\omega_{RF} + \omega_{LO})t) + cos((\omega_{RF} - \omega_{LO})t)] \xrightarrow{LPF} + \frac{B}{2}cos(\omega_{IF}t) + \frac{A}{2}cos(\omega_{IF}t) + \frac{A}{2}cos(\omega_{IF}t)$$

$$Q_{LO} * x_{RF} = \frac{A}{2}[cos((\omega_{LO} - \omega_{IF} + \omega_{LO})t) - sin((\omega_{RF} - \omega_{LO})t)] \xrightarrow{LPF} + \frac{A}{2}cos(\omega_{IF}t) + \frac{A}{2}cos(\omega_{IF}t) + \frac{A}{2}cos(\omega_{IF}t)$$

$$Q_{LO} * x_{IMAGE} = cos((\omega_{LO} - \omega_{IF} + \omega_{LO})t) + cos((\omega_{LO} - \omega_{IF} - \omega_{LO})t)] \xrightarrow{LPF} + \frac{B}{2}cos(\omega_{IF}t) + \frac{B}{2}cos(\omega_{IF}t) - \frac{A}{2}cos(\omega_{IF}t)$$

$$Q_{LO} * x_{IMAGE} = \frac{A}{2}[sin((\omega_{LO} - \omega_{IF} + \omega_{LO})t) - sin((\omega_{LO} - \omega_{IF} - \omega_{LO})t)] \xrightarrow{LPF} + \frac{A}{2}sin(\omega_{IF}t) \rightarrow \frac{B}{2}cos(\omega_{IF}t) - \frac{A}{2}cos(\omega_{IF}t)$$

$$RR_{gsin} = \left[\frac{A + B}{A - B}\right]^2 = \left[\frac{1 + B/A}{1 - B/A}\right]^2 = \left[\frac{1 + (1 + \epsilon)}{1 - (1 + \epsilon)}\right]^2 \approx \frac{4}{\epsilon^2}$$

$$RR_{phase} = 1 + 4 * (cot \Delta\phi)^2 = \frac{4}{(\Delta\phi)^2}$$

$$> 0.1\% gain error and 1° phase error leads to IRR of about 41 dB.$$

The design of CMOS Radio-Frequency Integrated Circuits, Thomas H. Lee.Spring 2009S. Hoyos - Advanced Mixed-Signal Interfaces

## **Image Rejection Ratio**

> HW # 4: Find the IRR for the case the input comes with a quadrature component as well, i.e.,  $x_{RF} = I_D \cos(\omega_{RF}t) + Q_D \sin(\omega_{RF}t)$  and a direct zero-IF receiver is used.

## **Basic Equations of Image Rejection**

> Gain mismatch  $\alpha$  and phase mismatch  $\theta$  :

$$I' = \left(1 + \frac{\alpha}{2}\right) \cos(\omega t + \frac{\theta}{2})$$

$$= \left(1 + \frac{\alpha}{2}\right) \left\{\frac{e^{j(\omega t + \frac{\theta}{2})} + e^{-j(\omega t + \frac{\theta}{2})}}{2}\right\}$$
(1)
$$Q' = \left(1 - \frac{\alpha}{2}\right) \sin\left(\omega t - \frac{\theta}{2}\right)$$

$$= \left(1 - \frac{\alpha}{2}\right) \left\{\frac{e^{j(\omega t - \frac{\theta}{2})} - e^{-j(\omega t - \frac{\theta}{2})}}{2j}\right\}.$$
(2)

Then, assuming  $\alpha$  and  $\theta$  are small, the non-ideal complex signal I' + jQ' and the non-ideal complex image I' - jQ' can be approximated using Taylor series.

$$I' + jQ' \approx e^{j\omega t} + \left(\frac{\alpha - j\theta}{2}\right)e^{-j\omega t} \tag{3}$$

$$I' - jQ' \approx \left(\frac{\alpha + j\theta}{2}\right) e^{j\omega t} + e^{-j\omega t}.$$
 (4)

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

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## **Matrix Formulation**



Fig. 5. Proposed image rejection concept.

Matrix formulation of the non-ideal mixing :

> Matrix formulation of the nonidealities correction :

$$\begin{bmatrix} I'\\Q' \end{bmatrix} = \begin{bmatrix} 1 + \frac{\alpha}{2} & -\frac{\theta}{2} \\ -\frac{\theta}{2} & 1 - \frac{\alpha}{2} \end{bmatrix} \begin{bmatrix} I\\Q \end{bmatrix}.$$

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

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### LMS algorithm for the estimation of $\alpha$ and $\theta$

$$\alpha'[k+1] = \alpha'[k] + \mu_{\alpha} \operatorname{sgn}[\operatorname{LPF}\{\operatorname{sgn}(I+Q) \\ \cdot \operatorname{sgn}(I-Q)\}]$$
$$\theta'[k+1] = \theta'[k] - \mu_{\theta} \operatorname{sgn}[\operatorname{LPF}\{\operatorname{sgn}(I) \cdot \operatorname{sgn}(Q)\}]$$

Fully-digital implementation



Fig. 12. Image rejection system.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

## **Analog Implementation**



Fig. 13. (a) Complex baseband S/H. (b) Gain-boosted telescopic cascoded operational amplifier.



Fig. 17. Chip die photo.



Fig. 14. Nine-bit trim capacitor.



Fig. 15. (a) Analog comparator. (b) Preamplifier. (c) Latch.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

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## 256 QAM Spectra Before and After Image Rejection



Fig. 18. Measured 256-QAM spectra before and after image rejection.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

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## 256 QAM Constellation Before and After Image Rejection



Before, IRR=26dB



Fig. 19. Effect of ADC resolution on IRR for 256-QAM.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

## Effect of IRR on Error Probability



Fig. 22. Effect of IRR on error probability of 64-QAM and 256-QAM.

Supisa Lerstaveesin, and Bang-Sup Song: "A Complex Image Rejection Circuit With Sign Detection Only," IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 12, DECEMBER 2006

## **Non-linearities**

## Improvement of Mixer Nonlinearities (IIP2) for Active Mixers



Fig. 1. Simplified block diagram of a direct-conversion WCDMA receiver.

Detailed circuit analysis of nonlinearities in diff pairs and doubled balanced mixers.

The proposed approach is analog,; trimming of current bias.

Liwei Sheng; Larson, L.E.;"An Si-SiGe BiCMOS direct-conversion mixer with second-order and third-order nonlinearity cancellation for WCDMA applications," <u>Microwave Theory and Techniques, IEEE Transactions on</u> Volume 51, <u>Issue</u> <u>11</u>, Nov. 2003 Page(s):2211 - 2220

## Improvement of Mixer Nonlinearities (IIP2) for Active Mixers



Fig. 8. Proposed even-order distortion cancellation scheme of the double-balanced mixer.

Uses PN sequences and correlation to estimate the nonlinearities. The LO bias is tuned to minimize distortion.

Liwei Sheng; Larson, L.E.;"An Si-SiGe BiCMOS direct-conversion mixer with second-order and third-order nonlinearity cancellation for WCDMA applications," Microwave Theory and Techniques, IEEE Transactions on Volume 51, Issue 11, Nov. 2003 Page(s):2211 - 2220

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### Some of the New Approaches to Broadband Receivers

#### □ A high-frequency software defined radio

N. C. Davies, "A high performance HF software radio," in *Proc.* 8<sup>th</sup> Int. Conf. HF Radio Systems and Techniques, Guildford, U.K., 2000, pp. 249–256.

#### □ Frequency channelizers

D. R. Zahirniak, D. L. Sharpin, and T. W. Fields, "A hardware-efficient, multirate, digital channelized receiver architecture," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, no. 1, pp. 137–152, Jan. 1998.

#### □ Selectable RF filters and downconversion

H. Yoshida, T. Kato, T. Tomizawa, S. Otaka, and H. Tsurumi, "Multimode software defined radio receiver using direct conversion and low-IF principle: Implementation and evaluation," *Electr. Commun. In Japan (Part I: Communications)*, vol. 86, pp. 55–65, 2003.

#### Subsampling and undersampling

#### Analog decimation

D. Jakonis, K. Folkesson, J. Dabrowski, P. Eriksson, and C. Svensson, "A 2.4-GHz RF sampling receiver front-end in 0.18-mCMOS," *IEEE J. Solid-State Circuits*, vol. 40, no. 6, pp. 1265–1277, Jun. 2005.

### Some of the New Approaches to Broadband Receivers (cont...)

#### Sampling with built-in anti-aliasing

Y. S. Poberezhskiy and G. Y. Poberezhskiy, "Sampling and signal reconstruction circuits performing internal antialiasing filtering and their influence on the design of digital receivers and transmitters," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 1, pp. 118–129, Jan. 2004.

#### Sample rate, downsampling and filtering

R. Crochiere and L. Rabiner, Multirate Digital Signal Processing. Englewood Cliffs, NJ: Prentice Hall, 1983.

#### A discrete-time RF sampling receiver

R. B. Staszewski, et. al. "All-digital TX frequency synthesizer and discrete-time receiver for Bluetooth radio in 130-nm CMOS," *IEEE J. Solid-State Circuits*, vol. 39, no. 12, pp. 2278–2291, Dec. 2004.

#### UCLA SDR receiver

- Bagheri, R.; Mirzaei, A.; Heidari, M.E.; Chehrazi, S.; Minjae Lee; Mikhemar, M.;Tang, W.K.; Abidi, A.A.; Softwaredefined radio receiver: dream to reality, <u>Communications Magazine, IEEE</u>, Volume 44, <u>Issue 8</u>, Aug. 2006 Page(s):111 - 118
- Abidi, "The path to software-defined radio receiver", IEEE JSSC, May 2007

#### Frequency-domain-sampling receivers

- S. Hoyos, B. M. Sadler, and G. R. Arce, "Broadband Multicarrier Communications Receiver Based on Analog to Digital Conversion in the Frequency Domain," *IEEE Transactions on Wireless Communications*, March 2006.
- S. Hoyos and B. M. Sadler, "Ultra-wideband analog to digital conversion via signal expansion," IEEE Transactions on Vehicular Technology, Vol. 54, No. 5, Sept. 2006, Pages: 1609-1622. Invited
- S. Hoyos, B. M. Sadler "UWB Mixed-Signal Transform-Domain Direct-Sequence Receiver," Accepted for publication in IEEE Transactions on Wireless Communications, 2007.

### Sampling with built-in anti-aliasing



- Sinc(x) anti-aliasing provided by windowing and integration. The sidelobes decay at 20 dB/decade with zeros at fs, 2fs, ..
- More general mixing waveforms can be used, although complexity goes up.





## A simple integrator



- Assume a low noise transconductance amplifier
- LO=2.4 GHz
- Pseudo-differential architecture (b) is preferable.
- This is just mixing followed by integration which provides down-conversion and filtering in a single stage.
- How do you read the voltage out of the caps?

## **Cyclic Read-Out**

- Cyclic change and discharge of caps. every N cycles.

- This can be modeled as a moving average

$$w_i = \sum_{l=0}^{N-1} u_{i-l}$$

- If modeled as MA, the filter has a sinc frequency response whose lobes width and nulls positions depend on N.



## High-Rate IIR Filtering

- History capacitor C<sub>H</sub> added
- LNTA sees constant capacitance C<sub>s</sub>

- Let  $a_1 = C_H / (C_H + C_R)$ 

- At switching time,  $C_H$  retains  $a_1$ portion of its total charge and shares (1- $a_1$ ) to the discharged  $C_R$  cap. At sampling time j, the system charge  $s_i$  is:

$$s_j = a_1 s_{j-1} + w_j$$

-The output charge x<sub>i</sub> is

$$x_{j} = (1 - a_{1})s_{j-1}$$

This is a IIR filter with sampling frequency of  $f_0/N$  and single pole at



$$f_{c1} = \frac{1}{2\pi} \frac{f_0}{N} (1 - a_1) = \frac{1}{2\pi} \frac{f_0}{N} \frac{C_R}{C_H + C_R}$$

## Example

- CR= 0.5pF, CH=15.425pF, a1=0.9686
- fo/N=2.4GHz/4=300MHz
- -Additional zeros with M=4

Additional IIR filtering during read-out process can also be introduced



## Adding more zeros to the FIR



- Redundant switched caps. Introduce more zeros in the transfer function when adding up their charges during read out:

$$y_k = \sum_{l=0}^{M-1} u_{k-l}$$

-Illustrated is the case with M=4



## A discrete-time RF sampling receiver



 Bluetooth and GSM receivers from TI use integrate and dump sampling followed by down sampling and filtering.

Programable filtering and decimation to achieve the anti-aliasing needed.

R. B. Staszewski, et. al. "All-digital TX frequency synthesizer and discrete-time receiver for Bluetooth radio in 130-nm CMOS, "*IEEE J. Solid-State Circuits*, vol. 39, no. 12, pp. 2278–2291, Dec. 2004.

### UCLA SDR receiver



• Direct conversion with tunable LO in the freq. range 800 MHz to 6 GHz.

 Cascade of sinc<sup>N</sup> filters followed by decimation to achieve the initializing needed.

 Good for narrowband signals as a single ADC can handle the bandwidth. But SDR should also be good for wideband and ultra-wideband signals. Need parallel ADC to sample at a fraction of Nyquist rate. Parallelization of the front-end will be needed if want to keep the ADC sampling rate down.

A. Abidi, "The path to software-defined radio receiver", IEEE JSSC, May 2007 Spring 2009 S. Hoyos - Advanced Mixed-Signal Interfaces

# SDR for narrowband, wideband and ultra-wideband signals

 Assume we have a tunable front-end that provides the downconversion and the antialiasing filtering needed for a wide range of standards.

• The problem now is that the signal bandwidth will have > 10X range. Example : 802.11g ( $\Sigma\Delta$  ADC @ 50 Ms/s and 8 bits), UWB (ADC @ 500 Ms/s and 5 bits). Say you can run the  $\Sigma\Delta$  ADC @ 100Ms/s and 5 bits, i.e. exchange OSR by DR). Can we use 5 of these  $\Sigma\Delta$  ADCs to cope with UWB ?

• Note that the same  $\Sigma\Delta$  ADC could operate @ 200 KHz and 14 bits for GSM and @ 1MHz and 12 bits for Bluetooth.

• How do you parallelize the ADCs and even the RF front-end to create a SDR for narrowband, wideband and ultra-wideband signals?

## Motivation

Digital intensive RF receivers -> ADCs with wide bandwidths and large dynamic range. Solution ? -> Parallelization

#### **Parallelized ADCs**

Time-interleaved ADC



#### Drawbacks

□SHA has stringent tracking bandwidth requirements □Each ADC sees full input signal bandwidth (nonlinearity and aliasing)





#### Drawbacks

Filters with very tough specs (aliasing)Signal reconstruction increases complexity

### **Parallel-Path Sampling**



#### **Salient Features**

- □ Simple mixers and integrators in the front end
- Windowed integration provides inherent antialiasing. Relaxed Filter design.
- □ Relaxed sample and hold requirements
- No signal reconstruction. Direct digital processing of Frequency samples.
- Relaxed ADC design with lower speeds



Drawback

Area overhead associated with parallelization



#### time

[Ref] P. K. Prakasam, M. Kulkarni, X. Chen, Z. Yu, S. Hoyos, J.,Silva-Martinez and E. Sanchez-Sinencio, "Applications of Multi-Path Transform-Domain Charge-Sampling Wideband Receivers", *IEEE Transactions on Circuits and Systems II, pp309-313, Vol. 55,Issue 4, April 2008.* 

#### **Circuit Implementation of the Gm Stage**



- 1. High Linear Gm stage
- □ Flicker noise is removed by the degeneration resistor
- □ IIP3 is boost to almost 30 dBm
- □ Large Vdd is required.
- □ Used in 10 bits full scale input receiver.

#### **Circuit Implementation of the Gm Stage**



[Ref] X. Chen, J., Silva-Martinez and S. Hoyos, "A CMOS differential noise cancelling low noise transconductance amplifier ", Circuits and Systems Workshop: System-on-Chip - Design, Applications, Integration, and Software, page(s): 1-4, 2008 IEEE Dallas

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#### **Circuit Implementation of the Mixer**

#### **Double Balanced Passive Mixer**

- □ Minimum signal and clock feed-through
- □ Even order harmonics are cancelled
- □ Almost noise free





## **Overlap in windows**



## **1 Path Circuit**

Shooting for 10 bits 2.5 Gs/s ADC

- □ High linear Gm stage to accommodate fullscale input
- Overlapping windowing



The whole front-end: 10 path ( 5 lo frequencies I/Q )

## **Chip Layout**

□ Area: 2.5mm\*2.5mm

□ Core Power: 320 mW

□ 64 mW / Path ( I and Q )

Overall Power consumption of the ADC: 320 mW + N \* P\_path,adc 45 nm (TI technology)



## System level issues of FD receiver

#### **Noise Amplification**

- Out-of-band noise folds back creating dips in performance
- Overlap improves the filter
- Overlap results in over-sampling which reduces aliasing.
- □ Additional carriers can be detected.



#### **Effect of Jitter**

- □ Jitter sources: LO signal, Sampling clocks
- □ Jitter from LO signal dominant
- □ Filter mitigates noise from LO jitter.
- Long integration windows reduces jitter from sampling clocks.



## **Least Squares Data Estimation**

Input symbols modulated on carriers Output sampled basis coefficients

$$\vec{a} = [a_i(0), a_q(0), a_i(1), a_q(1), \dots a_i(S-1), a_q(S)]$$
$$\vec{r} = [R_{0,0}, R_{0,1} \dots R_{0,N-1}, R_{1,0}, R_{1,1}, \dots R_{M-1,N-1}]^T$$

Entire system represented as a linear transformation from data symbols (a) to output samples of multi-path receiver (r)

$$G\cdot ec{a}=ec{r}$$
  
Least Squares (LS) solution for the system ->  $H=(G^HG)^{-1}G^H$   
Estimate Equation ->  $\hat{a}=H\cdot ec{r}$ 

#### **Need for Calibration ?**

H is sensitive to mismatches, offsets and imperfections in the system

H must match the circuit implementation accurately for good SNR

## Mismatches in the system



## **Complete System Calibration**



## **LMS** Calibration

Initialization of G matrix	$G \cdot \vec{a} = \vec{r}$
Input -> a1 a2 a3aS	Output -> r1 r2 r3rS
a1 -> [1 0 0 0 0] <sup>T</sup>	r1 forms 1 <sup>st</sup> row of G matrix
a2 -> [0 1 0 0 0] <sup>T</sup>	r2 forms 2 <sup>nd</sup> row of G matrix
a3 -> [0 0 1 0 0] <sup>T</sup>	r3 forms 3 <sup>rd</sup> row of G matrix

 $aS \rightarrow [0 \ 0 \ 0 \ 0 \ \dots \ 1]^{T}$ 

#### LMS calibration

Two methods:

#### **1.Forward Problem Calibration**

rS forms S<sup>th</sup> row of G matrix

#### 2. Reverse Problem Calibration



## Simulations

#### Mean squared error convergence 1. With arbitrary H matrix



2. With H matrix initialized from 'r' vector

### Simulations

SNDR post calibration

Input SNR = 100 dB Frequency offset in carriers is not included

1. With arbitrary H matrix





All static mismatches are calibrated in both cases.

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### **Frequency offset Estimation**

The sampled basis coefficients in block L,

$$R_{m,n,L} = \int_{mT_s + \Delta T}^{mT_s + T_c + \Delta T} x_L(t) \Phi_n^*(t) dt$$

where,

$$\Phi_{n}(t) = e^{-j[2\pi f_{LO}(n)t + \phi_{LO}(n)]} - \frac{1}{3}e^{j[3 \cdot 2\pi f_{LO}(n)t + 3 \cdot \phi_{LO}(n)]} + \frac{1}{5}e^{-j[5 \cdot 2\pi f_{LO}(n)t + 5 \cdot \phi_{LO}(n)]} - \dots$$
 LO signal

$$\begin{aligned} x_L(t) &= \sum_{s=1}^{S} \left[ a_i(s) cos \left( 2\pi F'_c(s) t + \phi_c(s) + 2\pi \Delta F_c(L-1)T \right) \right. \\ &+ a_q(s) sin \left( 2\pi F'_c(s) t + \phi_c(s) + 2\pi \Delta F_c(L-1)T \right) \right] & \text{Input signal} \\ F'_c(s) &= F_c(s) + \Delta F_c \end{aligned}$$

**Frequency offset in carriers** 

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### **Frequency offset Estimation**

After a few steps of simplification,  

$$R_{m,n,L} = e^{2\pi j \Delta F_c(L-1)T} \int_{mT_s}^{mT_s+T_c} \sum_{s=1}^{S} A_n e^{j\theta_n} \times \text{Term inside integral is independent of}$$

$$\begin{bmatrix} \frac{a_i(s)}{2} e^{j[2\pi F_c'(s)t + \phi_c(s) + 2\pi F_c'(s)\Delta T - 2\pi f_{LO}(n)t + \phi_{LO}'(n)]} \\ + \frac{a_q(s)}{2j} e^{j[2\pi F_c'(s)t + \phi_c(s) + 2\pi F_c'(s)\Delta T - 2\pi f_{LO}(n)t + \phi_{LO}'(n)]} \end{bmatrix} dt$$

If 
$$R_{m,n,L} = \alpha_{m,n} e^{j\beta_{m,n}}$$
 then  $R_{m,n,L+1} = e^{2\pi j\Delta F_c T} \times \alpha_{m,n} e^{j\beta_{m,n}}$ 

Including noise in these terms, the samples in the L<sup>th</sup> and (L+1)<sup>th</sup> block are,

$$R_{m,n,L} = \alpha_{m,n} \, e^{\, j\beta_{m,n}} + W_{m,n,L}$$

$$R_{m,n,L+1} = e^{2\pi j \Delta F_c T} \times \alpha_{m,n} e^{j\beta_{m,n}} + W_{m,n,L+1}$$

#### Maximum-Likelihood Estimation of Frequency offset,

$$\Delta \hat{F}_{c} = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \text{Correction to} \\ \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right] \quad \vec{r_{L}}(update) = \frac{1}{2\pi T} \tan^{-1} \left[ \frac{\sum_{L=1}^{K} Im(R_{m,n,L+1}R_{m,n,L}^{*})}{\sum_{L=1}^{K} Re(R_{m,n,L+1}R_{m,n,L}^{*})} \right]$$

#### **Correction to the 'r' vector**

$$\vec{r_L}(update) = \vec{r_L} \cdot e^{-j2\pi\Delta F_c(L-1)T}$$

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> About 20 dB improvement in performance with frequency offset estimation.

Performance limited by the accuracy of the estimate.



#### **Analog Complexity**

Sinc Filter Bank	Continuous integrator filter bank	
f <sub>-3dB</sub> ≈ 0.44 / Ts	f <sub>-3dB</sub> ≈ 1 / 2πR <sub>f</sub> C <sub>f</sub>	
DC gain = Gm Ts / C	DC gain = GmR <sub>f</sub>	
Int. noise = KT/C (2GmTs/C) + KT/C	Int. noise = GmR <sub>f</sub> .KT/C <sub>f</sub> + KT/C <sub>f</sub> + KT/C <sub>s</sub>	
GBW (op-amp 1,2) >> 1/ $2\pi R_o C$ GBW (1,2) > 7/(settling time) ( $\beta$ ~1) (10bits)	GBW (1) >> 1/ 2πR <sub>f</sub> C <sub>f</sub> GBW (2) > 7/(settling time) (β~1) (10bits)	
Example: Assuming Gm = 1mA/V, Ts = 4ns,		
DC gain = 4, f <sub>-3dB</sub> ≈ 110MHz	For DC gain = 4, Rf = 4K For $f_{-3dB} \approx 110$ MHz, $C_f \approx C/3$	
Noise = 9KT/C	Noise = $13KT/C + KT/C_s$	
GBW (1,2) ≈ 1.75 GHz	GBW (1) $\approx$ 1.5 GHz (as C <sub>f</sub> $\approx$ C/3) GBW (2) $\approx$ 3.5 GHz (for settling time = 2ns)	

### **Digital Complexity**

Step 1:  $\vec{p} = G^H \cdot \vec{r}$  $\hat{a} = H \cdot \vec{r}$ **Step 2:**  $\hat{a} = (G^H G)^{-1} \vec{p}$  $= (G^H G)^{-1} G^H \cdot \vec{r}$ Each element in G is given by,  $G_{m,n,s} = \int_{mT_s}^{mT_s+T_c} e^{-j2\pi F_c(s)t} \Phi_n(t) dt$  $= e^{-j2\pi F_c(s)mT_s} \int_{0}^{T_c} e^{-j2\pi F_c(s)t} \Phi_{m,n}(t) dt$  $f_{IO}(n)$ .  $T_s$  is an integer. So,  $\Phi_{m,n}(t)$  is periodic repetition of  $\Phi_{0,n}(t)$  $G_{m,n,s} = e^{-j2\pi F_c(s)mT_s} \int_0^{T_c} e^{-j2\pi F_c(s)t} \Phi_{0,n}(t)dt \qquad p_s = \sum_{\substack{m=0\\N-1}}^{m=0} \sum_{\substack{n=0\\M-1}}^{m=0} G_{m,n,s}^* R_{m,n}$  $= e^{-j2\pi F_c(s)mT_s} Q_{s,n}$  $= \sum Q_{s,n}^* \sum R_{m,n} e^{j2\pi sm/M}$ n=0 = 0N=1 m=0 $F_{a}(s)$ .  $T_{a} = s/M + integer$  $= \sum Q_{s,n}^* T_{s,n}$  $G_{m,n,s} = e^{-j2\pi sm/M} Q_{s,n}$ 

Complexity of computation of p:  $o(4NM \log M) + o(4NS) \approx o(4S(\log M + N))$ 

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## **Digital Complexity**

#### Sparsity of (G<sup>H</sup>G)<sup>-1</sup>

 $G^H G$  is denoted by  $X = [X_{i,j}]_{S \times S}$ 

$$X_{i,j} = \sum_{\substack{m=0\\N-1}}^{M-1} \sum_{n=0}^{N-1} e^{-j2\pi(i-j)m/M} Q_{i,n} Q_{j,n}^*$$
$$= \sum_{n=0}^{M-1} Q_{i,n} Q_{j,n}^* \sum_{m=0}^{M-1} e^{-j2\pi(i-j)m/M}$$
$$X_{i,j} = \begin{cases} M \sum_{n=0}^{N-1} Q_{i,n} Q_{j,n}^* & (i-j)mod M = 0\\ 0 & otherwise \end{cases}$$

 $X_{i,i}$  is non-zero only when (i-j) mod M = 0

- G<sup>H</sup>G has only 2N non-zero elements in each row
- Inverse of G<sup>H</sup>G also has the same sparsity.

Complexity of computation of

- G<sup>H</sup>G -> o(2N x 2N x 2S) = o(8N<sup>2</sup>S)
- (G<sup>H</sup>G)<sup>-1</sup> -> o(2N x 2N x 2S) = o(8N<sup>2</sup>S)



## **Digital Complexity**

Step 1: $\vec{p} = G^H \cdot \vec{r}$ Complexity:  $o(4S(\log M + N))$ Step 2: $\hat{a} = (G^H G)^{-1} \vec{p}$ Complexity: o(4NS)

#### Total Complexity of LS estimation : o(4S( logM + N)) + o(4NS)

Example: S = 128, M = 32, N = 5

Complexity of FFT: o(4SlogS) = **o(28S)** 

Complexity of LS estimate,

Sinc filter bank: o(4S(logM + N)) + o(4NS) = o(60S)

Analog filter bank: o(4NMS) = o(640S)

#### **Complexity of estimation during LMS calibration**

Forward Problem:Reverse Problem:Example:S = 128, M = 32, N = 5 $\hat{a} = H \cdot \vec{r}$  $\hat{a} = H \cdot \vec{r}$ Forward Problem: $= (G^H G)^{-1} G^H \cdot \vec{r}$  $\hat{a} = H \cdot \vec{r}$  $o(16N^2S) + o(4S (logM+N)) + o(4NS) = o(460S)$  $o(16N^2S) - o(4S (logM+N))$ o(4NMS)Reverse Problem:o(4NMS)o(4NMS)o(4NMS) = o(640S)

Comparative Study	Sinc Filter Bank	Analog Filter Bank
Analog Front end	Larger capacitors	Smaller capacitors
complexity	No resistor required. Reset	Resistor required for finite
	ensures finite DC gain.	DC gain.
	Lesser noise	Noise is high.
	Smaller GBW for op-amps.	Larger GBW for op-amps.
Analog Power consumption	Less	4 times higher
Digital complexity	o (4S (N + logM) ) + o(	o ( 4NMS )
(Estimation)	4NS)	Example: o (640S)
	Example: o (60S)	
Digital complexity	o(16N <sup>2</sup> S) + o(4S (logM+N)) +	o (4NMS)
(Estimation @ calibration)	o(4NS)	
	Example: o(460S)	Example: o (640S)
Digital power consumption	Low	High
	Example: About 10% of	Example: 10 times more

#### Multi-Standard reconfigurable FD receiver



S. Hoyos - AdvaFfced Mixed-Signal Interfaces

#### **Decentralized TD Sensor Network**

