# ECEN474: (Analog) VLSI Circuit Design
# Fall 2011

## Lecture 9: Frequency Response

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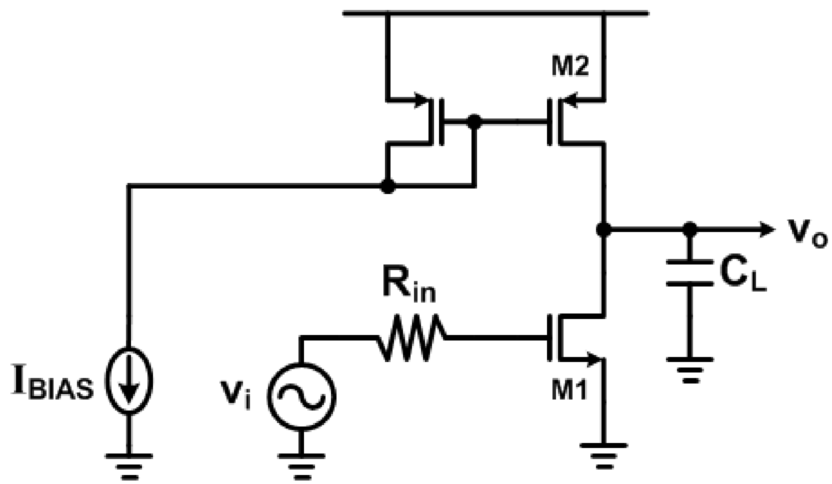
# Announcements

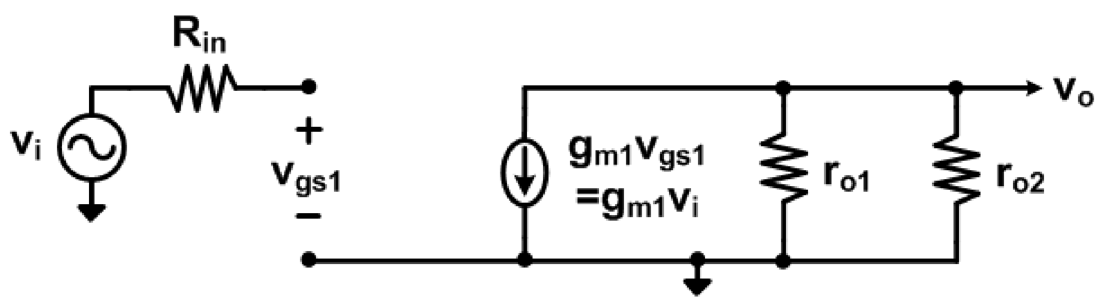
- Reading
  - Razavi’s CMOS book chapters 3 & 6

# Agenda

- Common-Source Amp Frequency Response
- Open-Circuit Time Constants (OC$\tau$) Bandwidth Estimation Technique

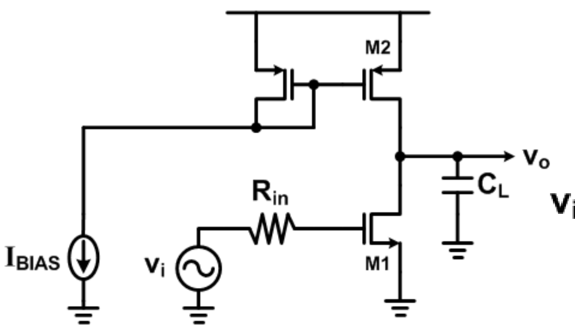
# Common-Source Amplifier: Low Frequency Response



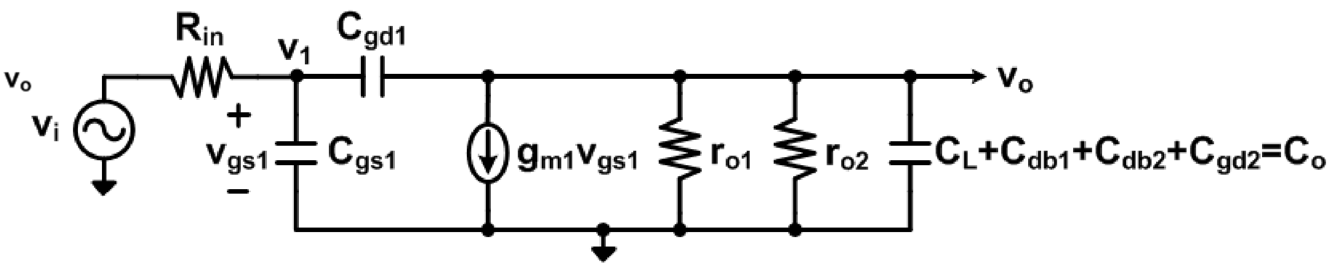




$$\frac{v_o}{v_i} = -\frac{g_{m1}}{g_{o1} + g_{o2}}$$

# Common-Source Amplifier: High Frequency Response




Small-Signal Model (Assuming $V_{G2}$ is AC gnd)




$$\textbf{KCL @ Node } \mathbf{v_1}: (v_1 - v_i)G_{in} + v_1 sC_{gs1} + (v_1 - v_o)sC_{gd1} = 0$$

$$\textbf{KCL @ Node } \mathbf{v_o}: (v_o - v_1)sC_{gd1} + g_{m1}v_1 + v_o(g_o + sC_o) = 0$$

**where** $g_o = g_{o1} + g_{o2}$

After some algebra, we get the exact transfer function:

$$\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\dfrac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

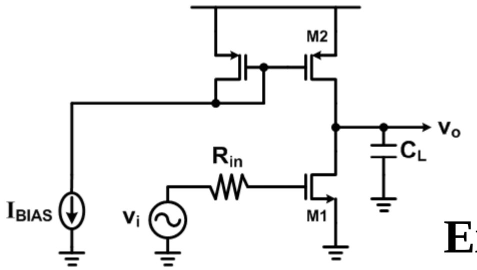
**where**

$$a = R_{in}\left[C_{gs1} + C_{gd1}(1 + g_m r_o)\right] + r_o\left(C_{gd1} + C_o\right)$$

**and**

$$b = R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

# Common-Source Amp Frequency Response




**Exact Transfer Function :** $\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$

**For the common case when the two poles are real and far apart**

**Denominator** $D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$

**Thus,** $\omega_{p1} = -\frac{1}{a} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right] + r_o(C_{gd1} + C_o)}$

**and the transfer function can be approximated as a single pole system**

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1}r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right] + r_o(C_{gd1} + C_o)\right)}$$

# Open-Circuit Time Constants (OC$\tau$)

- Open-circuit time constants technique can be used to estimate bandwidth
  - Much easier than deriving transfer function
  - Accurate for systems with one dominant pole

$$\textbf{All-Pole Transfer Function:}\ \frac{v_o(s)}{v_i(s)} = \frac{a_0}{(\tau_1 s+1)(\tau_2 s+1)\ldots(\tau_n s+1)}$$

$$\textbf{Denominator:}\ b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1$$

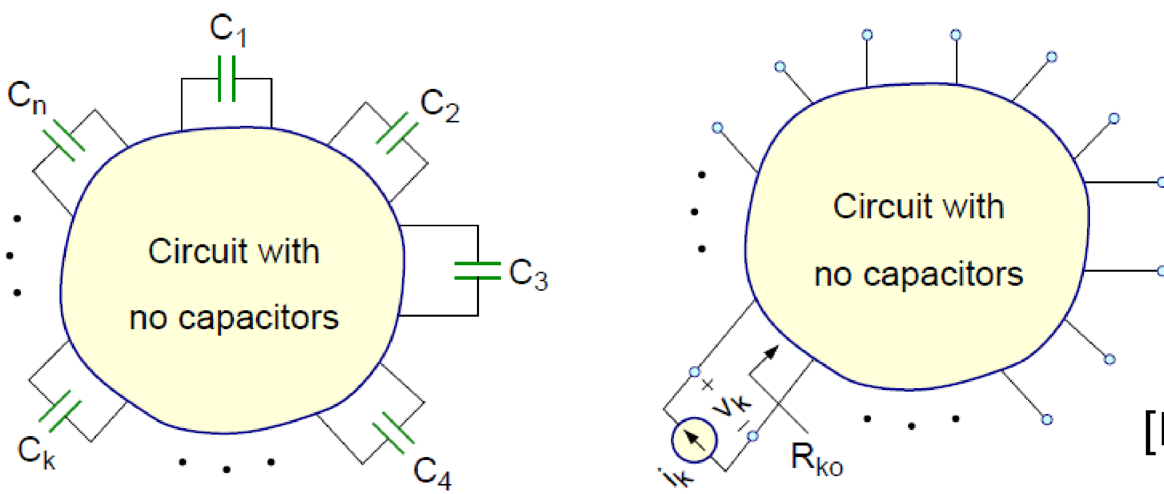
$$\textbf{Here}\quad b_n = \prod_{i=1}^{n} \tau_i \quad \textbf{and} \quad b_1 = \sum_{i=1}^{n} \tau_i$$

**A Dominant-Pole System can be approximated as**

$$\frac{v_o(s)}{v_i(s)} \cong \frac{a_0}{b_1 s + 1} = \frac{a_0}{\left(\sum_{i=1}^{n} \tau_i\right) s + 1}$$

$$\boxed{\textbf{Bandwidth}\quad \omega_h \cong \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \omega_{h,est}}$$

# Open-Circuit Time Constants (OCτ)

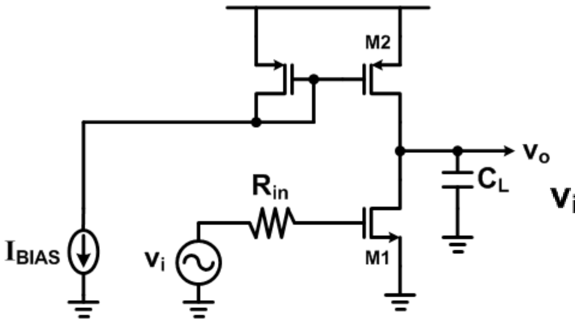



[Karsilayan]

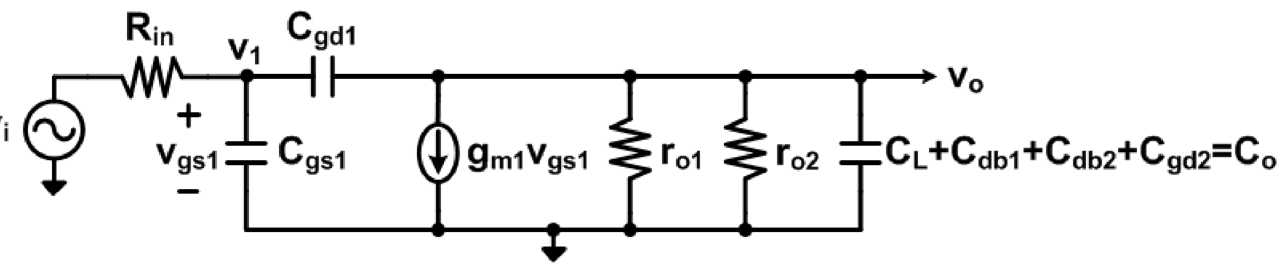
- To compute time-constants
  1. Compute effective resistance $R_{ko}$ facing each kth capacitor with all other caps open-circuited
  2. Form the product $\tau_{ko}=R_{ko}C_k$
  3. Sum all n "open-circuit" time constants

$$\omega_{h,est} = \frac{1}{\sum_{k=1}^{n} R_{ko}C_k}$$

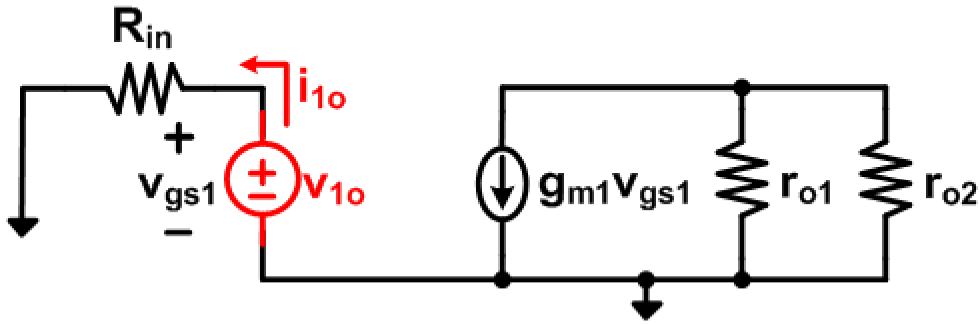
# Common-Source Amp w/ OCτ




Small-Signal Model (Assuming $V_{G2}$ is AC gnd)



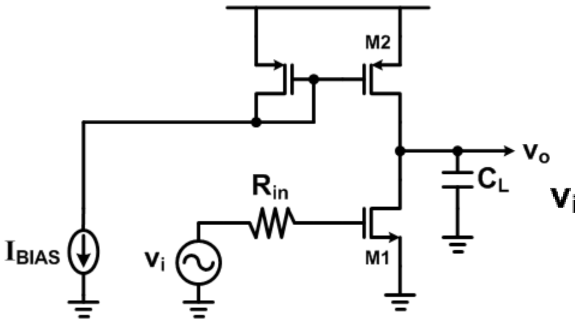

- For $C_{gs1}$



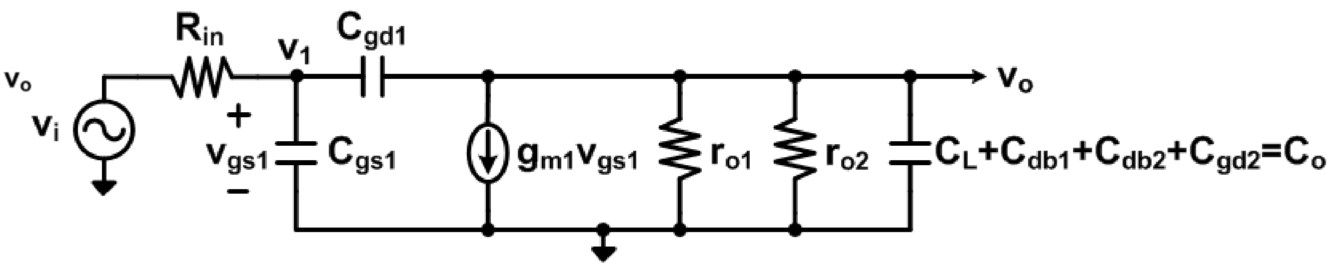

$$R_{1o} = \frac{v_{1o}}{i_{1o}} = \frac{v_{1o}}{\left(\frac{v_{1o}}{R_{in}}\right)} = R_{in}$$

$$\tau_{1o} = R_{in} C_{gs1}$$

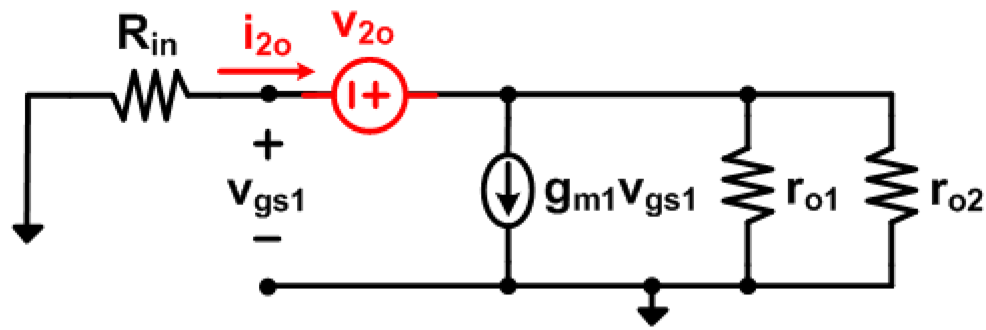
# Common-Source Amp w/ OCτ




Small-Signal Model (Assuming $V_{G2}$ is AC gnd)




- For $C_{gd1}$




**(1)** $i_{2o} = g_m v_{gs1} + \frac{(v_{2o} + v_{gs1})}{r_o}$

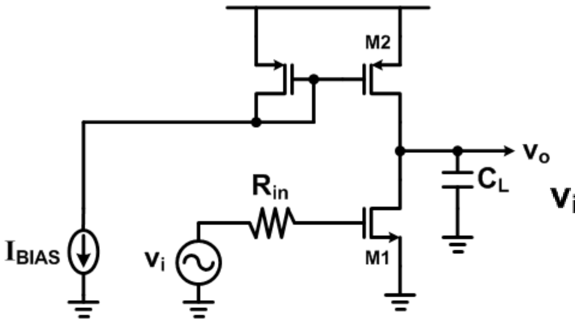
**(2)** $v_{gs1} = -i_{2o} R_{in}$

**Plugging (2) into (1) and solving for** $\frac{v_{2o}}{i_{2o}}$

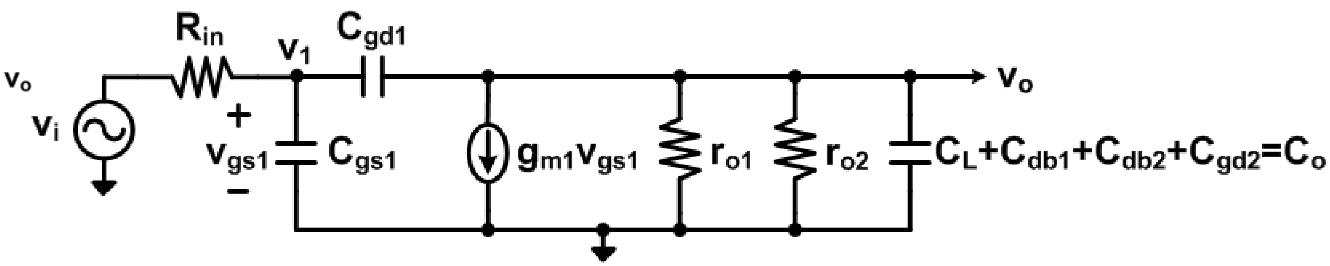
$$R_{2o} = \frac{v_{2o}}{i_{2o}} = R_{in}(1 + g_{m1} r_o) + r_o$$

$$\tau_{2o} = (R_{in}(1 + g_{m1} r_o) + r_o) C_{gd1}$$

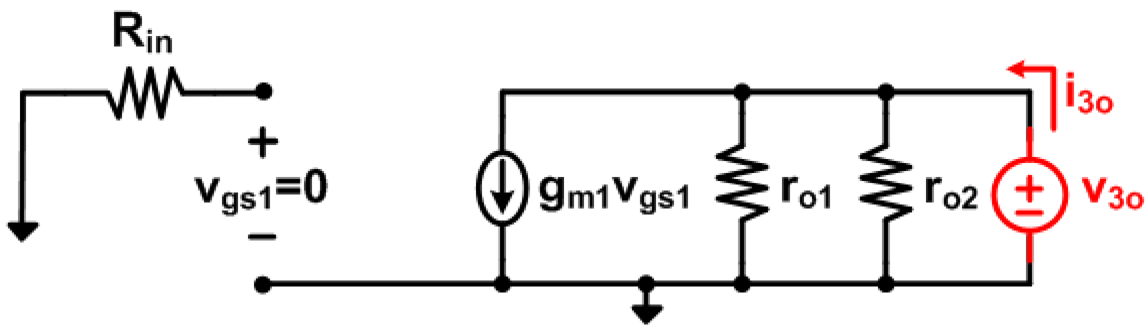
# Common-Source Amp w/ OCτ




Small-Signal Model (Assuming $V_{G2}$ is AC gnd)



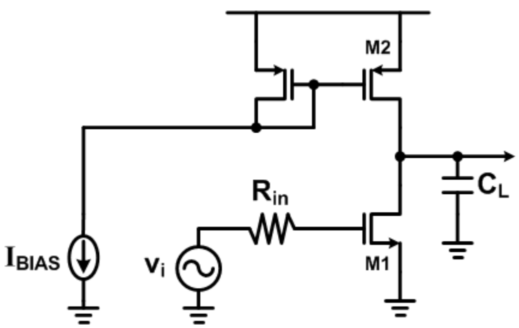

- For $C_o$



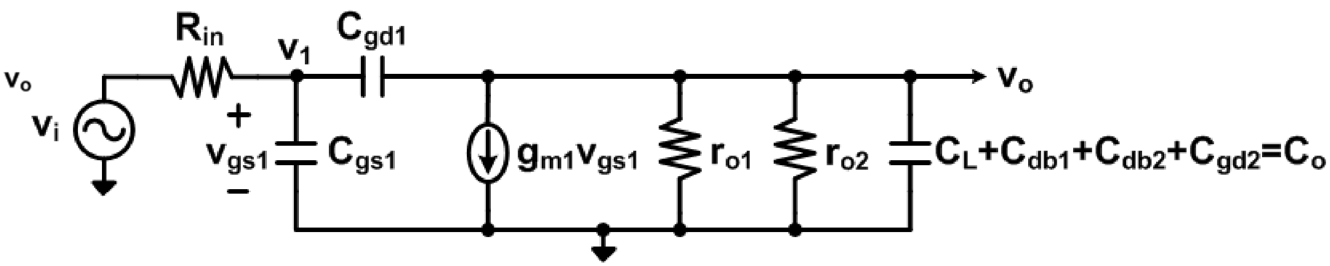

$$R_{3o} = \frac{v_{3o}}{i_{3o}} = \frac{v_{3o}}{\left(\dfrac{v_{3o}}{r_o}\right)} = r_o$$

$$\tau_{3o} = r_o C_o$$

# Common-Source Amp w/ OCτ




Small-Signal Model (Assuming $V_{G2}$ is AC gnd)




**3 Time Constants :** $\tau_{1o} = R_{in}C_{gs1}$, $\tau_{2o} = \left(R_{in}(1+g_m r_o)+r_o\right)C_{gd1}$, $\tau_{3o} = r_o C_o$

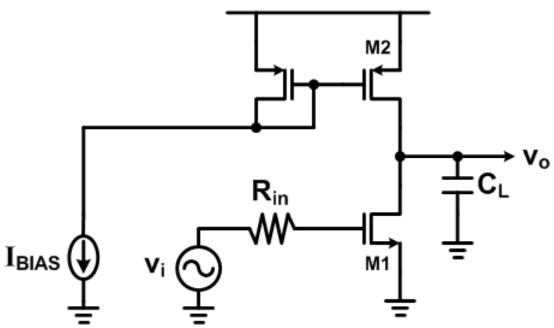
$$b_1 = \sum_{i=1}^{n} \tau_i = R_{in}C_{gs1} + \left(R_{in}(1+g_m r_o)+r_o\right)C_{gd1} + r_o C_o$$

$$\omega_{h,est} = \frac{1}{b_1} = \frac{1}{R_{in}C_{gs1} + \left(R_{in}(1+g_m r_o)+r_o\right)C_{gd1} + r_o C_o}$$

**Exactly the same as what we derived in Slide 6!**

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1}r_o}{1+s\left(R_{in}\left[C_{gs1}+C_{gd1}(1+g_{m1}r_o)\right]+r_o\left(C_{gd1}+C_o\right)\right)}$$

# Common-Source Amp w/ Large $R_{in}$




- Example: Using common-source output stage in a 2-stage OpAmp

$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+s\left(R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]+r_o\left(C_{gd1}+C_o\right)\right)}$$

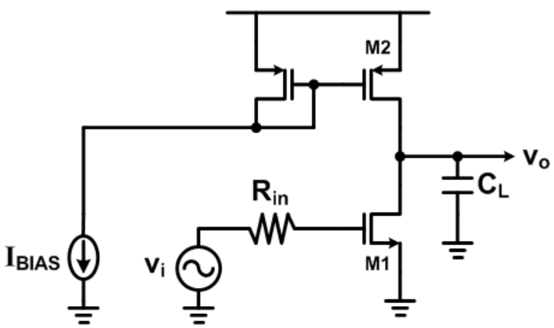
**with** $R_{in} >> r_o$

$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+sR_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]}$$

$$\omega_{p1}=-\frac{1}{R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]}$$

- Dominant pole is formed by input resistance times transistor $C_{gs}$ and Cgd which has been multiplied by $1\text{-}A_{dc}$
  - $C_{gd}(1\text{-}A_{dc})$ is called the Miller capacitance

# Common-Source Amp w/ Large $R_{in}$




- What about the second pole?

**Exact Transfer Function :** $\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$

**Denominator** $D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$

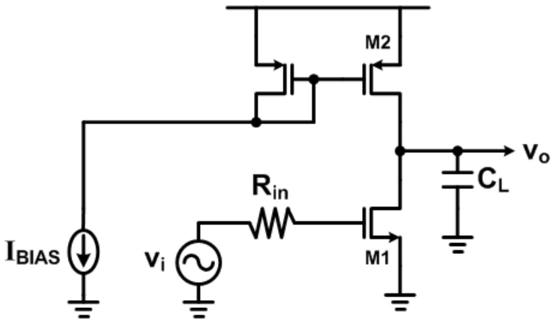
$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1} r_o\right)\right]}{R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

**Assuming that the Miller Cap,** $C_{gd1}\left(1 + g_{m1} r_o\right)$**, dominates**

$$\boxed{\omega_{p2} \cong -\frac{g_m C_{gd1}}{C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o}}$$

# Common-Source Amp w/ Small $R_{in}$




- Example: Source-follower driving the common-source amp

$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+s\left(R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]+r_o\left(C_{gd1}+C_o\right)\right)}$$

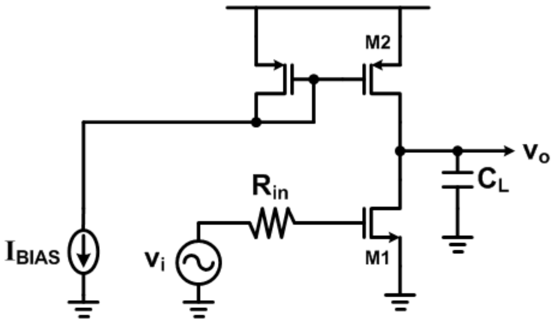
**with** $r_o >> R_{in}$

$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+sr_o\left(C_{gd1}+C_o\right)}$$

$$\omega_{p1}=-\frac{1}{r_o\left(C_{gd1}+C_o\right)}$$

- Dominant pole is formed by output resistance times output capacitance plus transistor Cgd

# Common-Source Amp w/ Small $R_{in}$




- What about the second pole?

**Exact Transfer Function :** $\dfrac{v_o}{v_i} = \dfrac{-g_m r_o\left(1 - s\dfrac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$

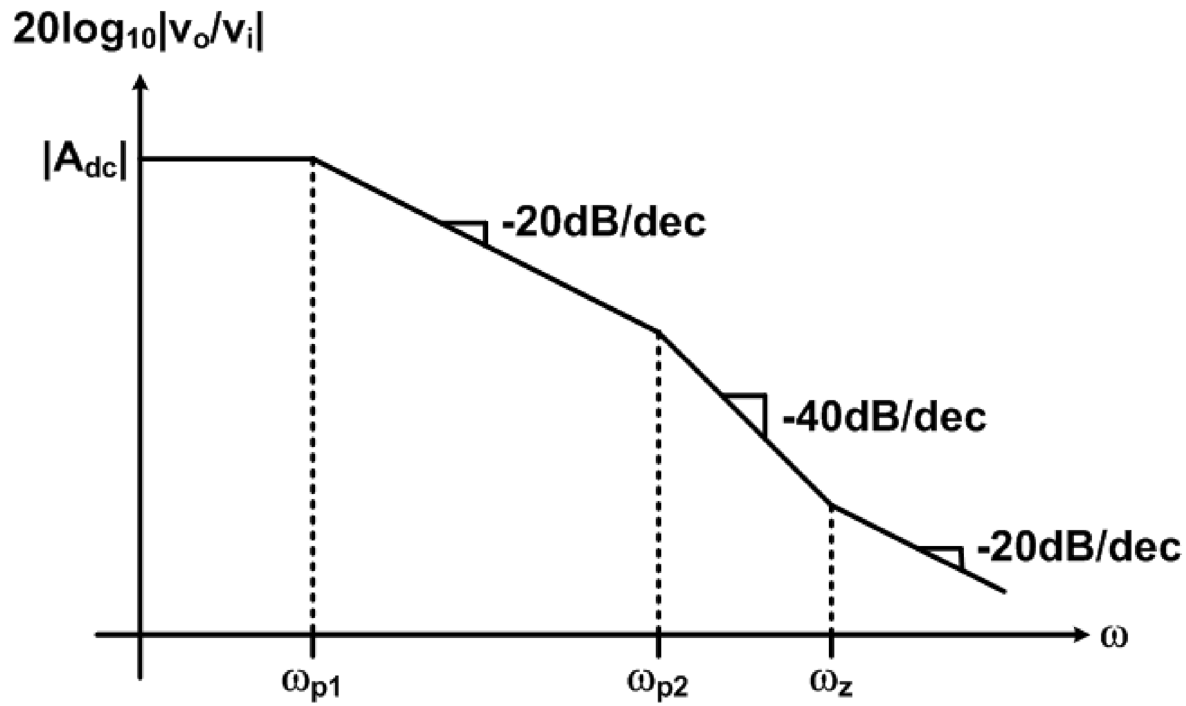
**Denominator** $D(s) = \left(1 - \dfrac{s}{\omega_{p1}}\right)\left(1 - \dfrac{s}{\omega_{p2}}\right) = 1 - s\left(\dfrac{1}{\omega_{p1}} + \dfrac{1}{\omega_{p2}}\right) + \dfrac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \dfrac{s}{\omega_{p1}} + \dfrac{s^2}{\omega_{p1}\omega_{p2}}$

$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{r_o\left(C_{gd1} + C_o\right)}{R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

$$\omega_{p2} = -\frac{C_{gd1} + C_o}{R_{in}\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} \cong -\frac{1}{R_{in}\left(C_{gs1} + C_{gd1}\right)} \quad \textbf{(with large } \mathbf{C_o}\textbf{)}$$

# Common-Source Amp Frequency Response




**Large $R_{in}$** **Small $R_{in}$**

$$A_{dc} = -g_m r_o \qquad \omega_z = \frac{g_{m1}}{C_{gd1}}$$

Large $R_{in}$:

$$\omega_{p1} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1} r_o\right)\right]}$$

$$\omega_{p2} \cong -\frac{g_{m1} C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o}$$

Small $R_{in}$:

$$\omega_{p1} = -\frac{1}{r_o\left(C_{gd1} + C_o\right)}$$

$$\omega_{p2} \cong -\frac{1}{R_{in}\left(C_{gs1} + C_{gd1}\right)}$$

# Next Time

- Single-Stage Amplifiers (cont.)
  - Common-Drain
  - Common-Gate
  - Cascode Stage
- Differential Pairs