# ECEN474: (Analog) VLSI Circuit Design
# Fall 2011

## Lecture 18: Feedback & Stability


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# Agenda

- Feedback in opamp circuits
- Stability Considerations
  - Nyquist Criteria
  - Phase & Gain Margin

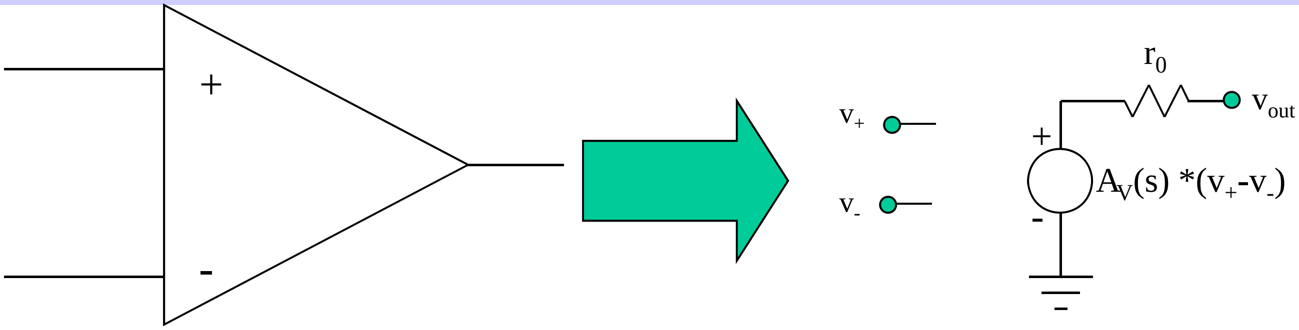
# If the OPAMP is not precise, how we can design accurate systems?

**Answer is FEEDBACK!!!**

**Examples of our daily life**

- **Can you shave yourself closing your eyes?**
- **Can you drive your car closing your eyes?**
- **Can you adjust the supply voltages without a voltage indicator?**
- **Can you measure (without any equipment) the magnetic field generated by your cellular phone?**
- **Can you control properly your daily activities without feedback?**

- **If you measure the output at the time you apply the stimuli you can better control the system!!**

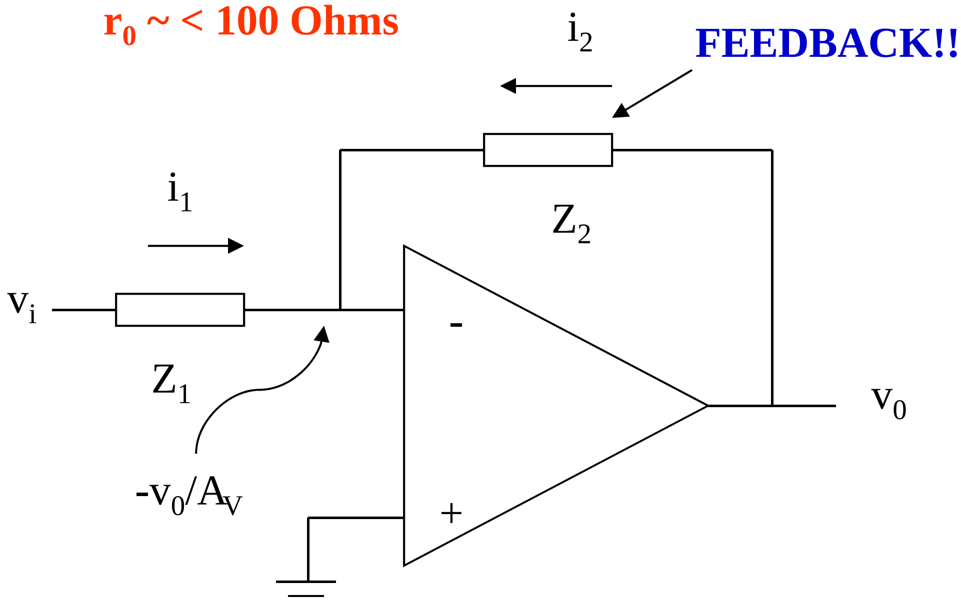



$$A_V \cong \frac{A_{VDC}}{1+\frac{s}{\omega_P}}$$

$$GBW = A_{VDC} * \omega_p \cong \frac{g_m}{C_M}$$

$$R_{out} = r_0$$

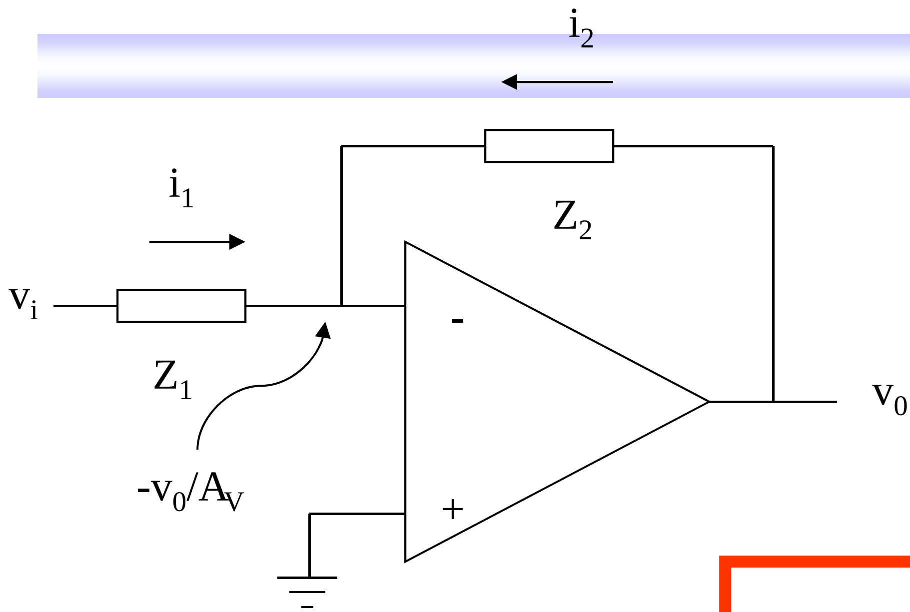
**Typical values for low-frequency opamps:**
**$A_V$~$10^5$**
**$\omega_P$~100 rad/sec**
**$r_0$ ~ < 100 Ohms**




If $A_V$~$\infty$ then
v- = 0 VIRTUAL GROUND

$$i_1 = -i_2$$

$$v_i/Z_1 = -v_0/Z_2 \qquad \frac{v_o}{v_i} = -\frac{Z_2}{Z_1}$$




**If $A_V$ is finite then v- ≠ 0**

$$i_1 = -i_2 \quad \text{if} \quad Z_{in} = \infty$$

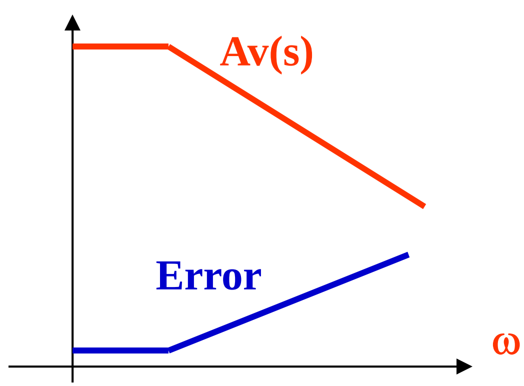
or

$$\left(v_i - (-v_0/A_V)\right)Y_1 = \left(v_0 - (-v_0/A_V)\right)Y_2$$

$$\boxed{\frac{v_0}{v_i} = -\frac{Y_1}{Y_2}\left[\frac{1}{1+\frac{1}{A_V}\left(1+\frac{Y_1}{Y_2}\right)}\right] = -\frac{Z_2}{Z_1}\left[\frac{1}{1+\frac{1}{A_V}\left(1+\frac{Z_2}{Z_1}\right)}\right]}$$

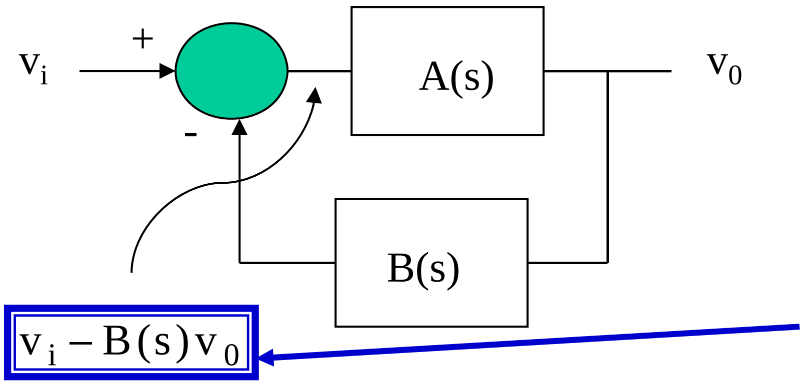
$$A_V = \frac{A_{VDC}}{1+\frac{s}{\omega_P}}$$

$$R_{out} = 0 \quad \text{and} \quad Z_{in} = \infty$$




**Using the approximation** $\frac{1}{1+x} \approx 1 - x$ **for small x**

$$\text{Error} = -\frac{1}{A_V}\left(1+\frac{Z_2}{Z_1}\right) = -\frac{1+\frac{s}{\omega_P}}{A_{DC}}\left(1+\frac{Z_2}{Z_1}\right)$$




# FEEDBACK:

•If you measure the output at the time you apply the stimuli you can better control the system!!

**Definitions:**

**A(s): Amplifier gain (very large but not very well controlled)**

**B(s): Feedback Factor (Very well controlled)**

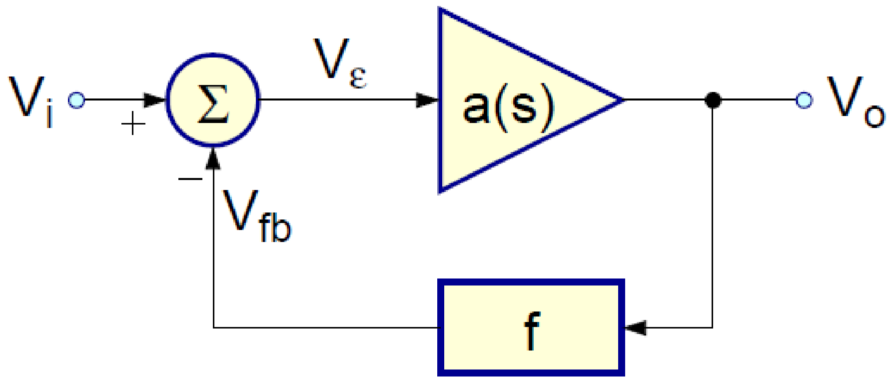
**T(s)=A(s)B(s): Loop Gain (Extremely important parameter!!)**

Applying **Mason Rule**:

$$\boxed{\frac{v_o}{v_i} = \frac{Direct\ \ trajectory}{1 - loop\ \ gain} = \frac{A(s)}{1-\left(-T(s)\right)} = \frac{A(s)}{1 + A(s)B(s)}}$$

# Feedback Configuration

[Karsilayan]



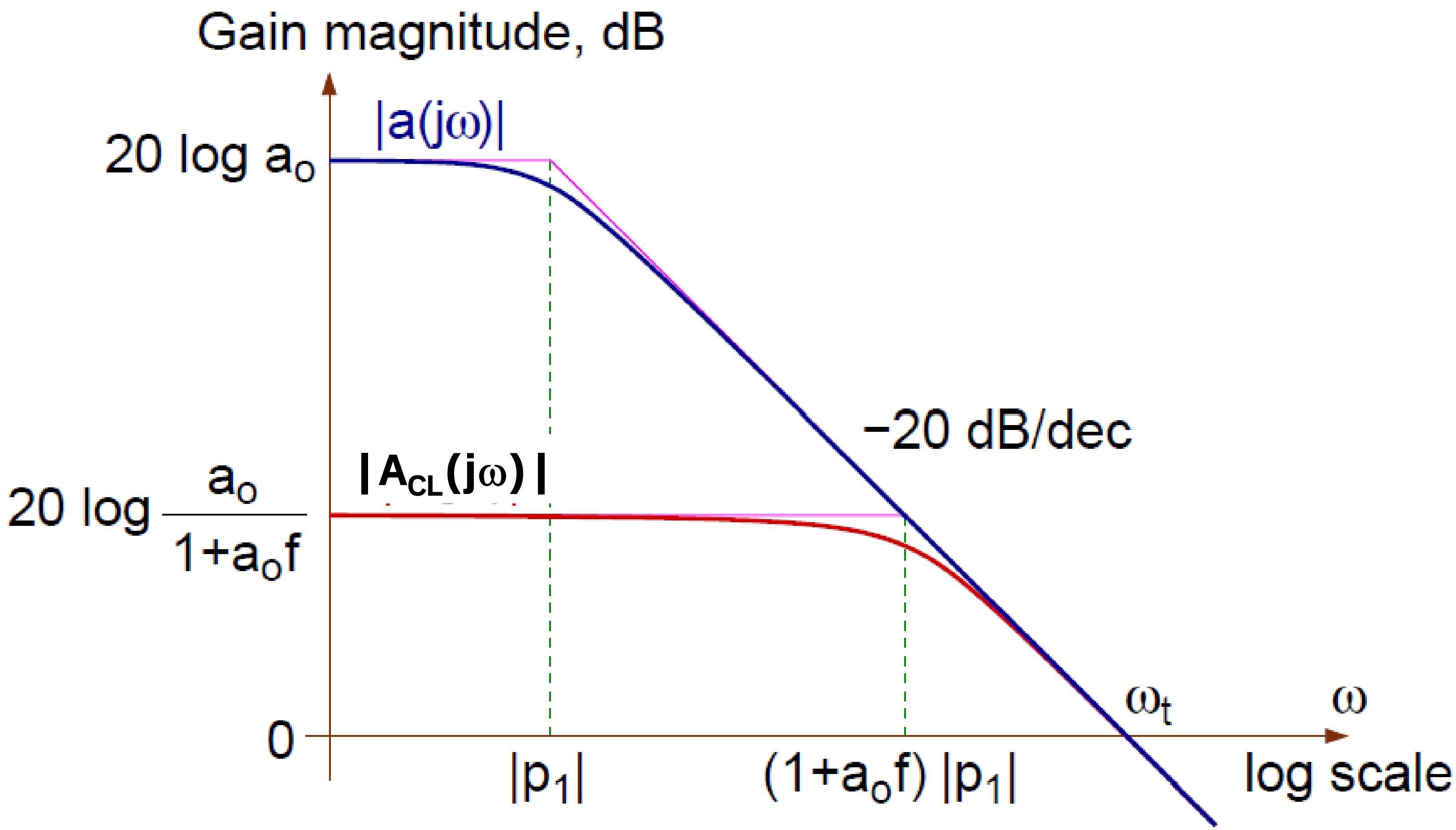

Here f = feedback factor (B(s) in previous slides)

$$a(s) = \frac{V_o}{V_\varepsilon}(s) = \frac{a_o}{1 - \dfrac{s}{p_1}}$$

$$\mathbf{A_{CL}(s)} = \frac{V_o}{V_i}(s) = \frac{a(s)}{1 + a(s)f} = \frac{\dfrac{a_o}{1 + a_o f}}{1 - \dfrac{s}{(1 + a_o f)p_1}}$$

# Gain-Bandwidth

[Karsilayan]



# Instability and the Nyquist Criterion

[Karsilayan]

Transfer function of a 3-pole amplifier:

$$\mathbf{a(s)} = \frac{\mathbf{a_o}}{\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_1}}\right)\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_2}}\right)\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_3}}\right)}$$

Nyquist criterion for stability of the amplifier:

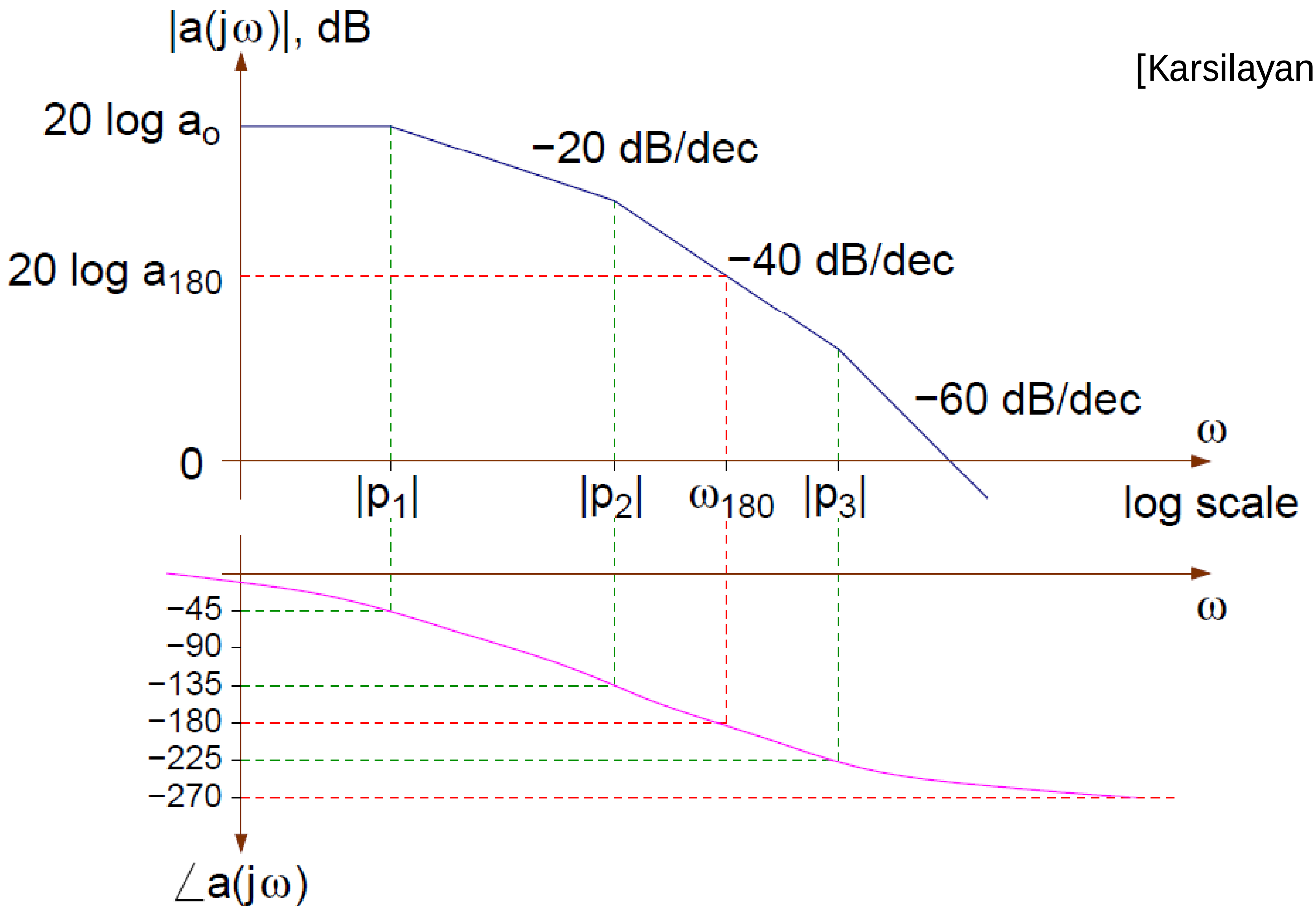
> *Consider a feedback amplifier with a stable* $\mathbf{T(s)}$. *If the Nyquist plot of* $\mathbf{T(j\omega)}$ *encircles the point (-1,0), the feedback amplifier is unstable.*

**Recall** $T(s)$ **is the loop gain**

$$T(s) = A(s)B(s) = a(s)f$$

# Magnitude & Phase

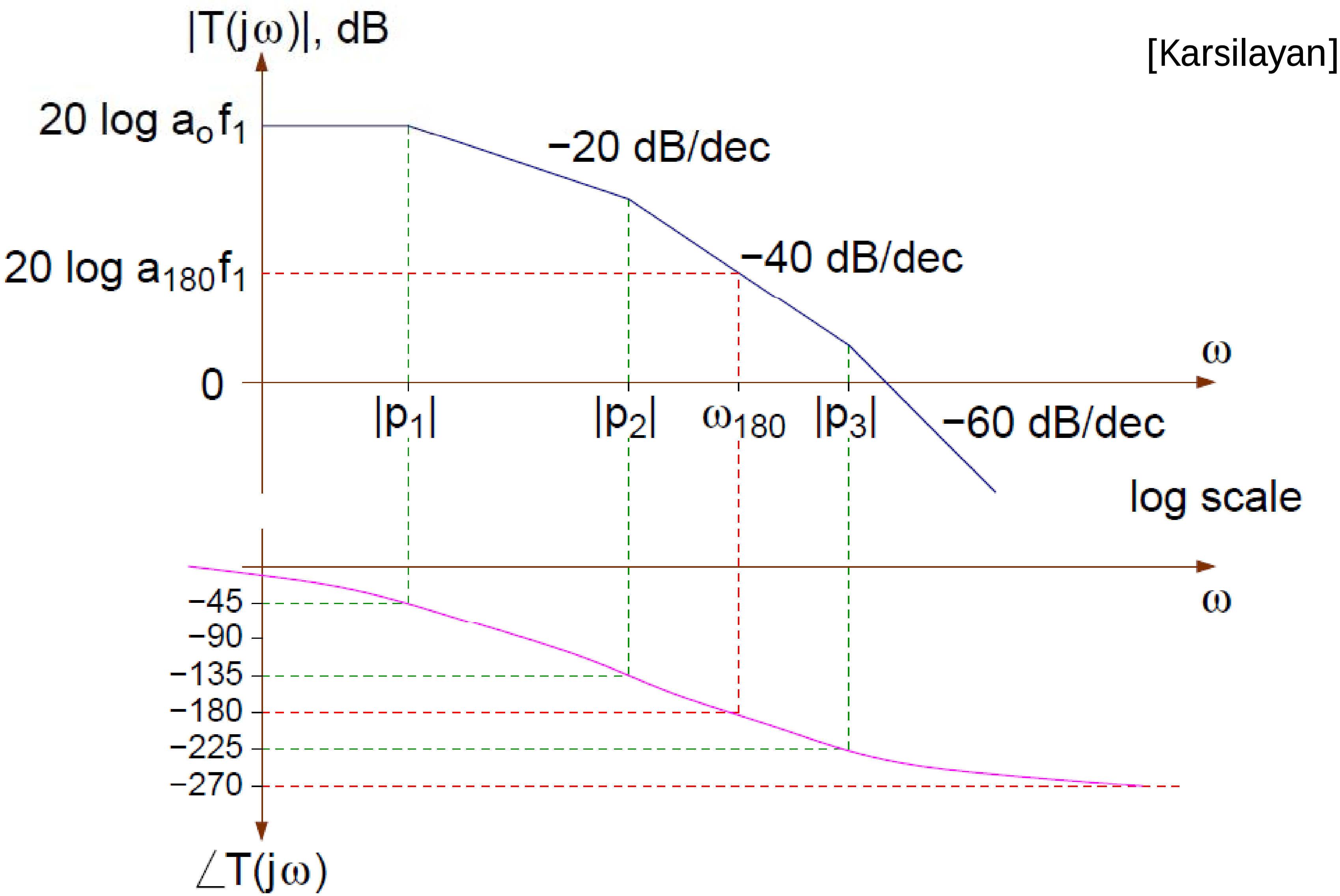
3-pole amplifier




[Karsilayan]

# Magnitude & Phase

$T(s) = a(s)f_1$

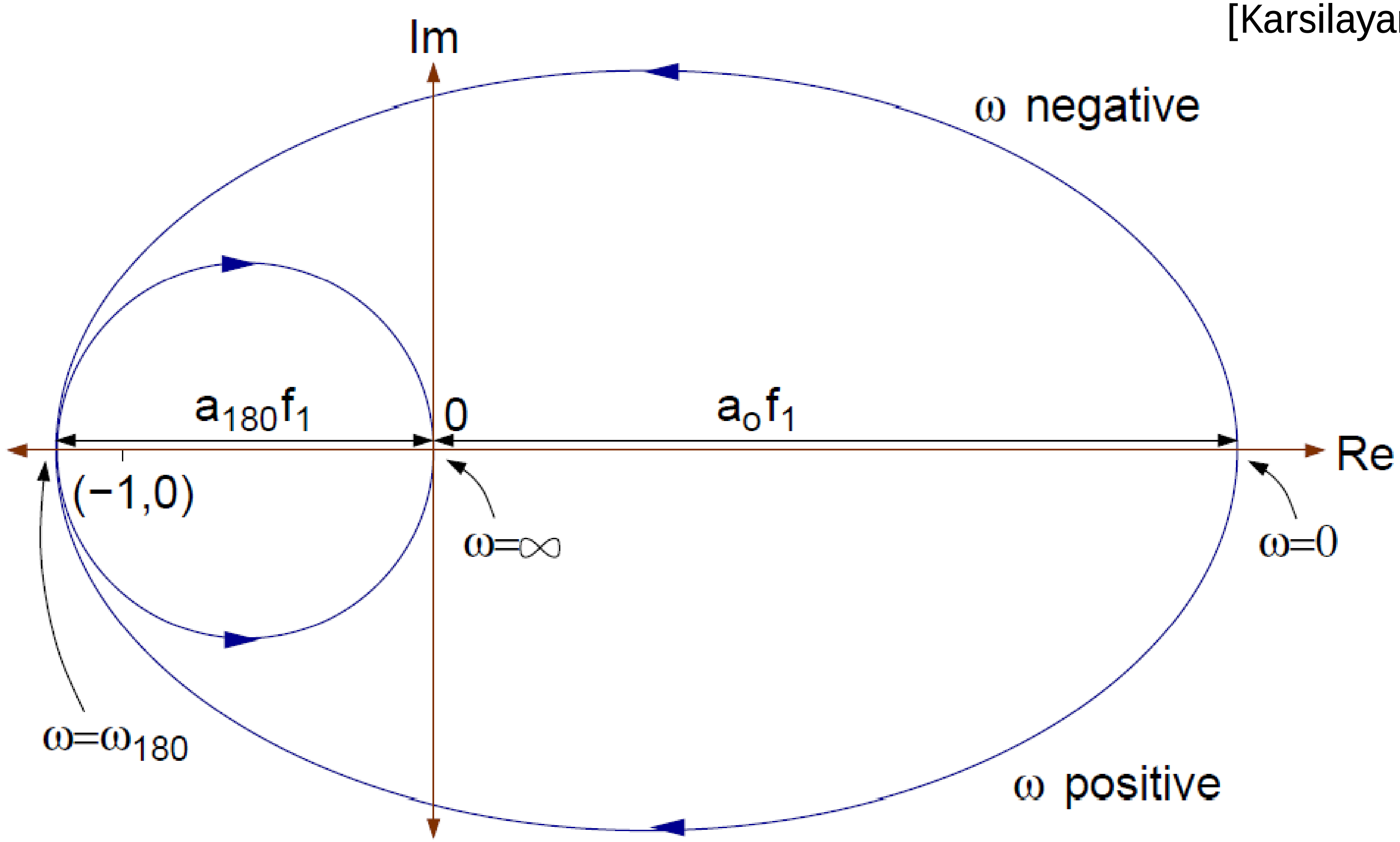


# Nyquist Plot

$T(s) = a(s)f_1$

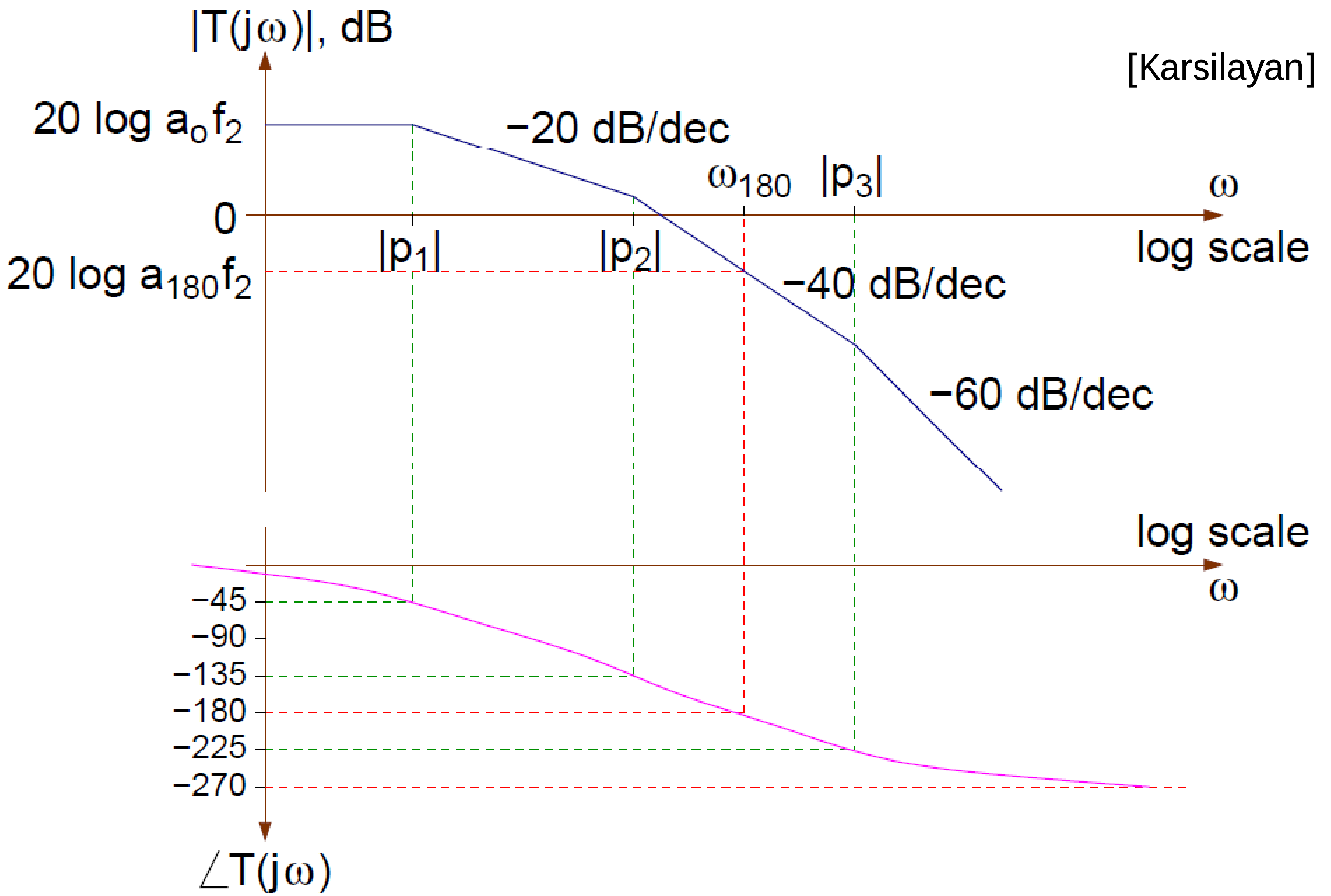
Frequency Sweep of Loop Gain, T(s)

[Karsilayan]



# Magnitude & Phase

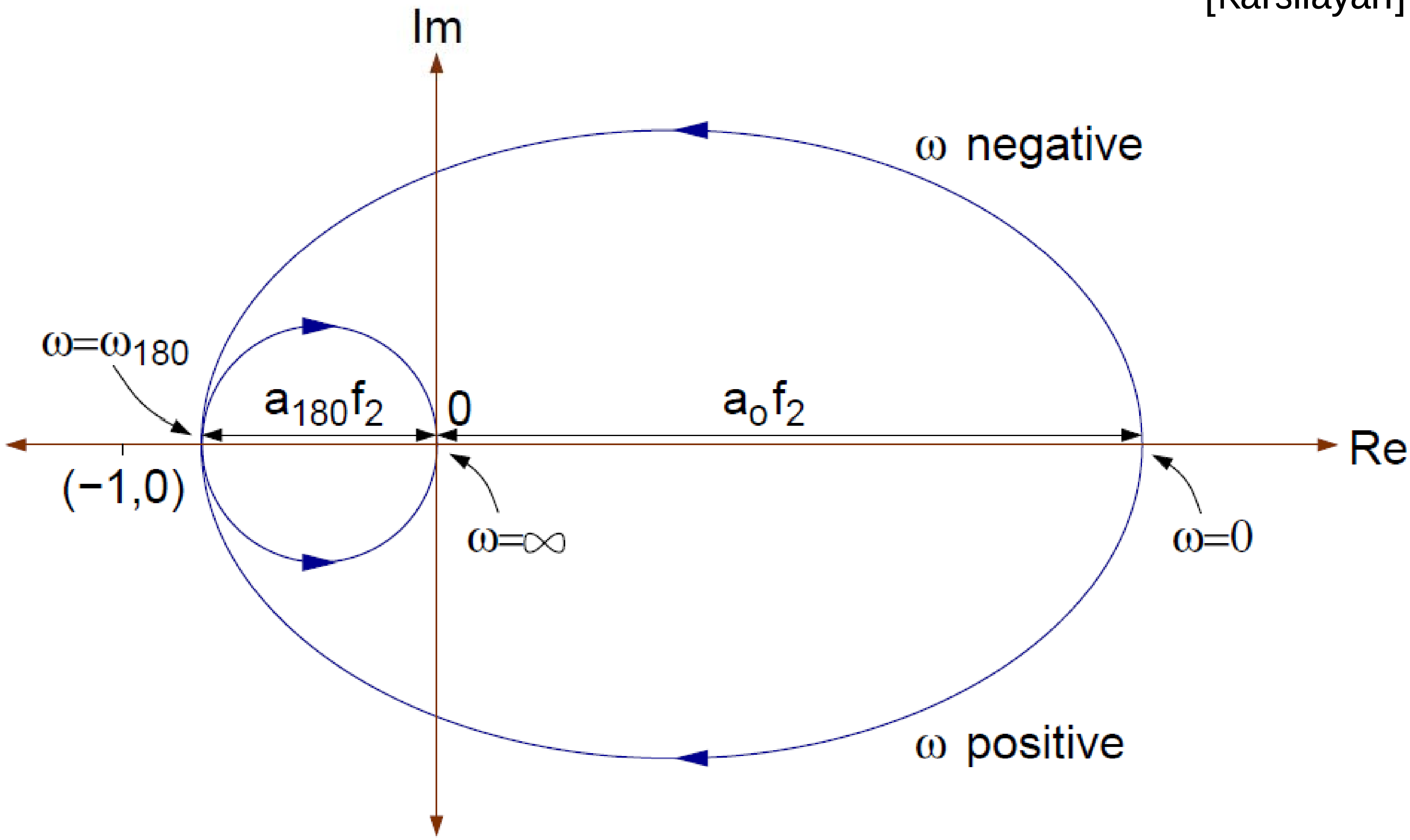
$T(s) = a(s)f_2$



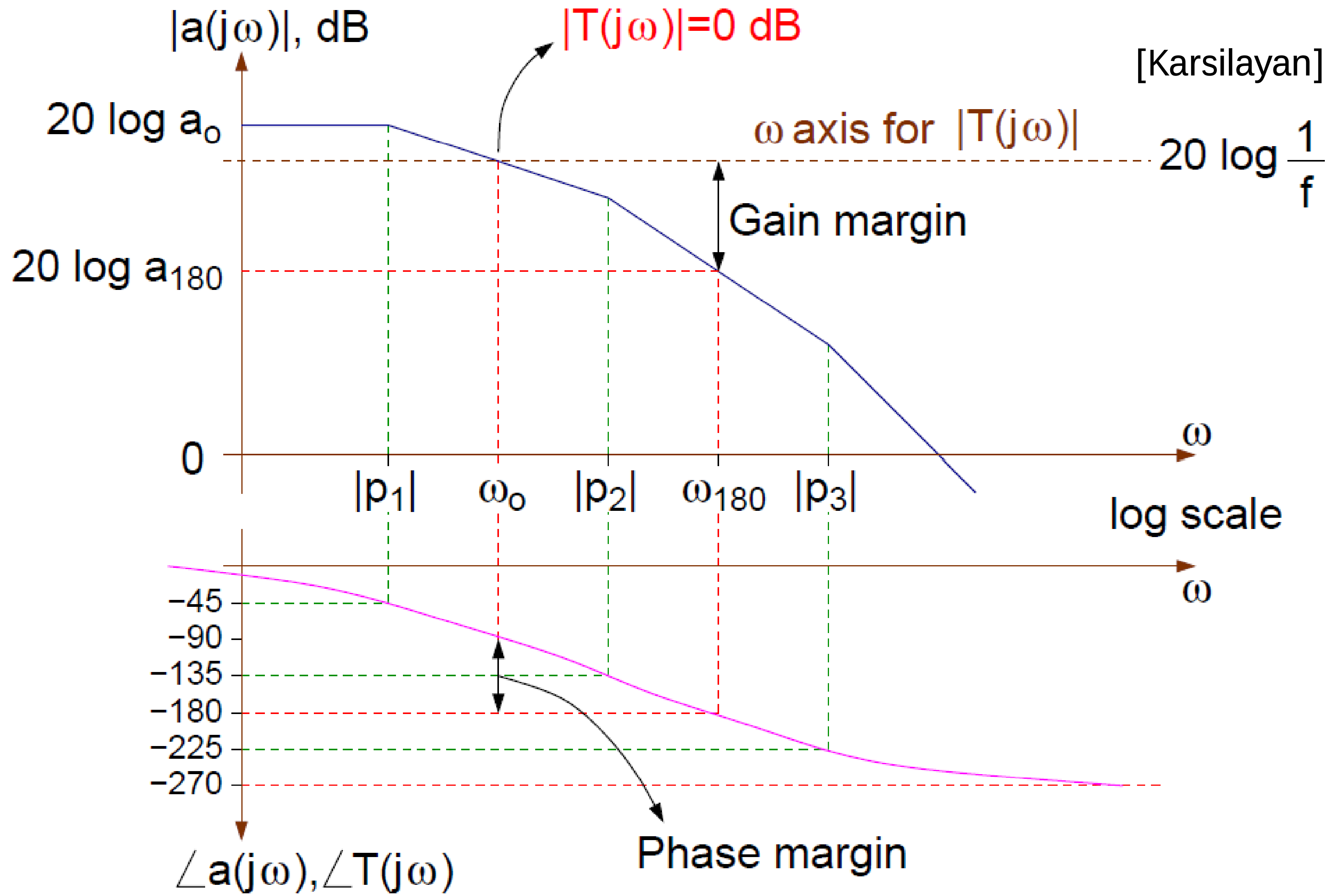
# Nyquist Plot

$T(s) = a(s)f_2$

[Karsilayan]



# Gain & Phase Margin



## Stability Criteria

Nyquist:

[Karsilayan]

$$|T(j\omega_{180})| = a_{180}f < 1 \quad \Rightarrow \quad \text{Stable}$$

Gain Margin (**GM**):

$$\mathbf{GM} = 20\log\frac{1}{|T(j\omega_{180})|} = -20\log|T(j\omega_{180})|$$

$$\mathbf{GM} > 0 \quad \Rightarrow \quad \text{Stable}$$

Phase Margin (**PM**):

$$\mathbf{PM} = 180^\circ + \angle T(j\omega_o)$$

$$\mathbf{PM} > 0 \quad \Rightarrow \quad \text{Stable}$$

# Phase Margin

[Karsilayan]

$$|T(j\omega_o)| = 1 \;\Rightarrow\; |a(j\omega_o)|f = 1 \;\Rightarrow\; |a(j\omega_o)| = \frac{1}{f}$$

$$PM = 45^\circ \;\Rightarrow\; \angle T(j\omega_o) = -135^\circ\,,\; A_{CL}(j\omega_o) = \frac{a(j\omega_o)}{1+T(j\omega_o)}$$

$$A_{CL}(j\omega_o) = \frac{a(j\omega_o)}{1+e^{-j135^\circ}} = \frac{a(j\omega_o)}{1-0.7-0.7j}$$

$$|A_{CL}(j\omega_o)| = \frac{|a(j\omega_o)|}{|0.3-0.7j|} = \frac{1}{0.76f} = \frac{1.3}{f}$$

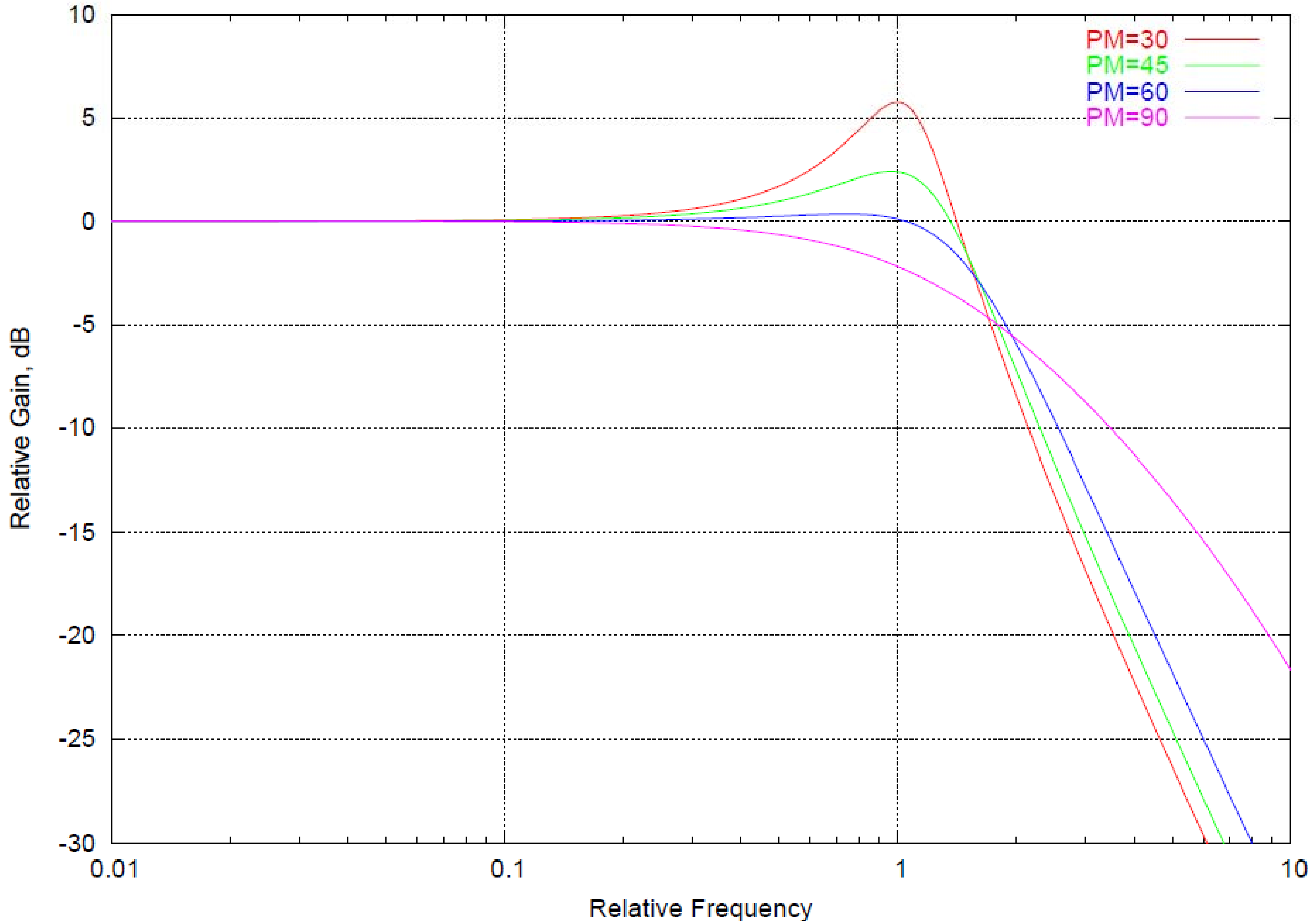
$$PM = 30^\circ \;\Rightarrow\; \angle T(j\omega_o) = -150^\circ\,,\; |A_{CL}(j\omega_o)| = 1.92/f$$

$$PM = 60^\circ \;\Rightarrow\; \angle T(j\omega_o) = -120^\circ\,,\; |A_{CL}(j\omega_o)| = 1/f$$

$$PM = 90^\circ \;\Rightarrow\; \angle T(j\omega_o) = -90^\circ\,,\; |A_{CL}(j\omega_o)| = 0.7/f$$

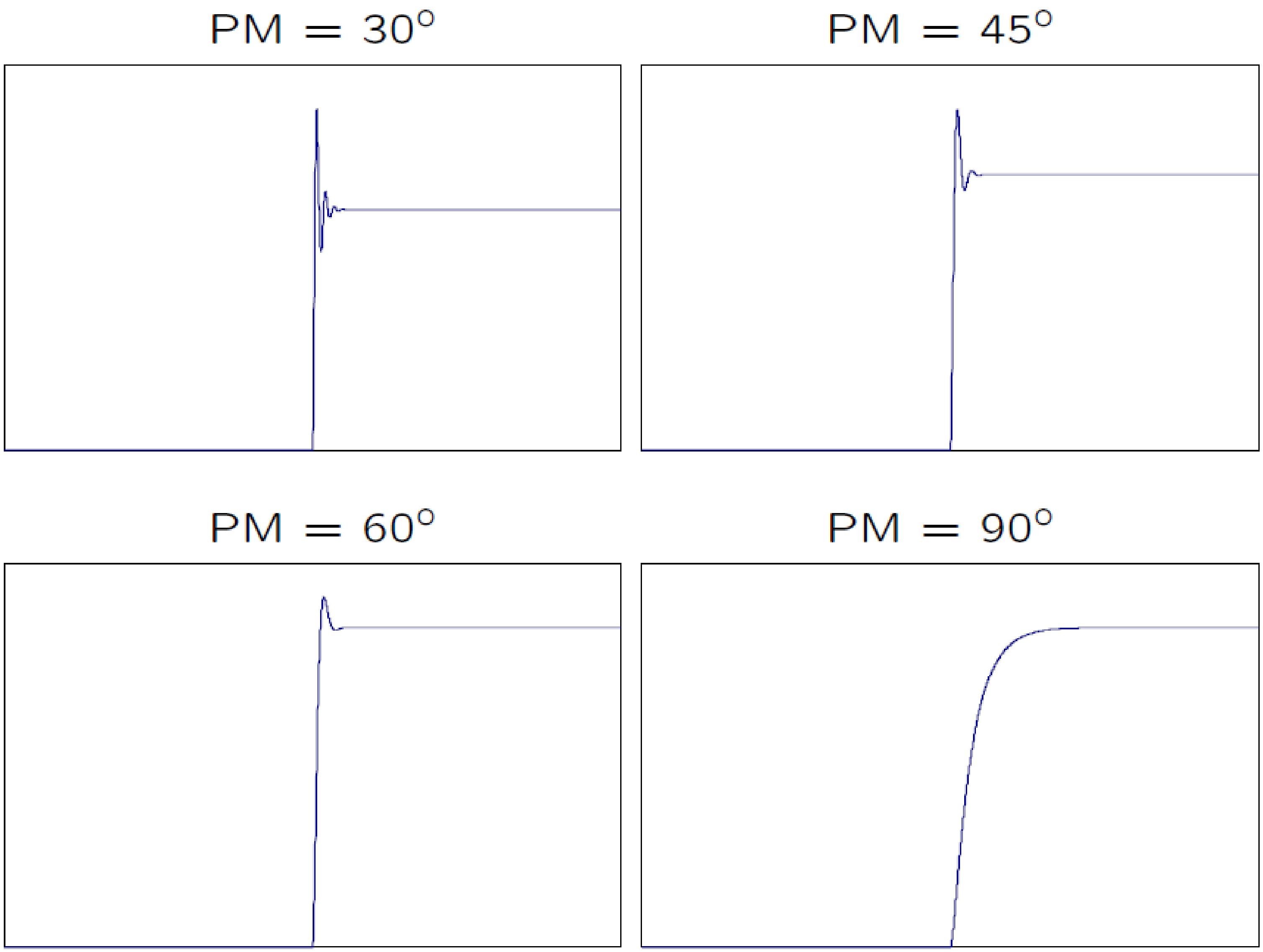
# Closed-Loop Frequency Response

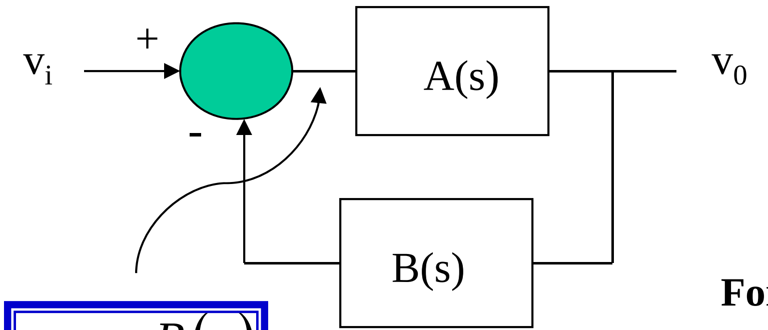
[Karsilayan]



# Closed-Loop Step Response

[Karsilayan]






$$\boxed{v_i - v_o B(s)}$$

$$\frac{v_o}{v_i} = \frac{A(s)}{1-(-T(s))}$$

**If** $T(s) >> 1$, **then** $\frac{v_o}{v_i} \cong \frac{A(s)}{T(s)} = \frac{1}{B(s)}$

**For Error, can write :** $\frac{v_o}{v_i} = \frac{1}{B(s)}\left[\frac{T(s)}{1+T(s)}\right] = \frac{1}{B(s)}\left[\frac{1}{1+\frac{1}{T(s)}}\right]$

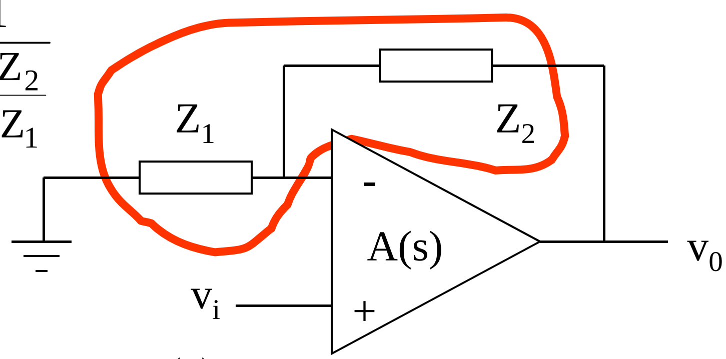
**Error** $\propto -\frac{1}{T(s)}$

**Key points:**

**If you want to amplify your signal: B(s) must be an attenuator (voltage divider!!)**

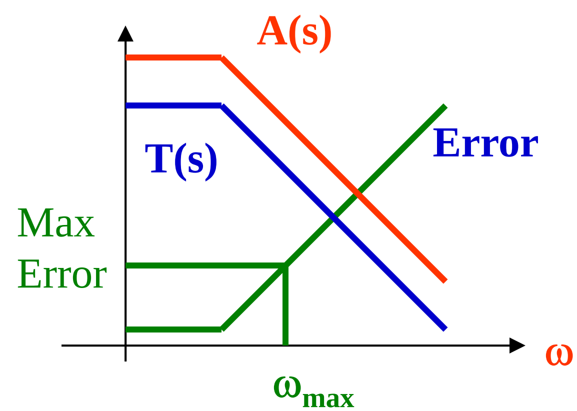
**The error is determined by the overall loop gain: T(s)=A(s)B(s)**

$$B(s) = \frac{Z_1}{Z_1+Z_2} = \frac{1}{1+\frac{Z_2}{Z_1}}$$

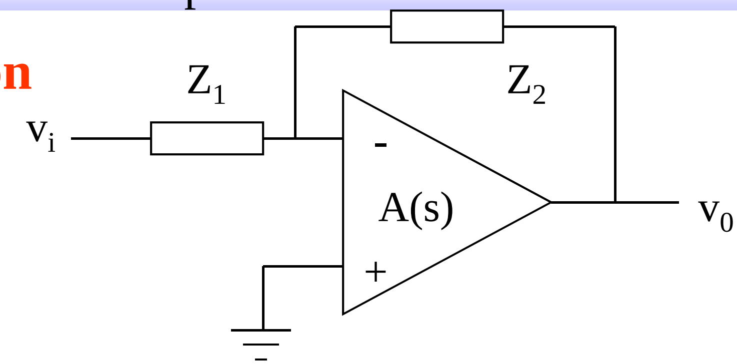



$$\boxed{\frac{v_o(s)}{v_i(s)} \cong 1+\frac{Z_2}{Z_1}}$$

$A(s)$ **is amplifier response only**

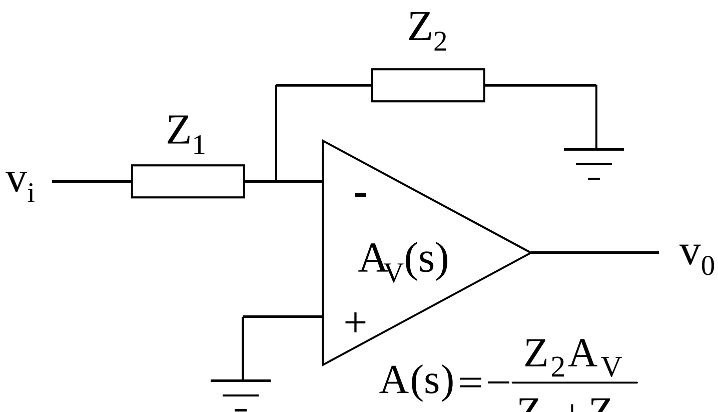


## Inverting Amplifier: Apply superposition

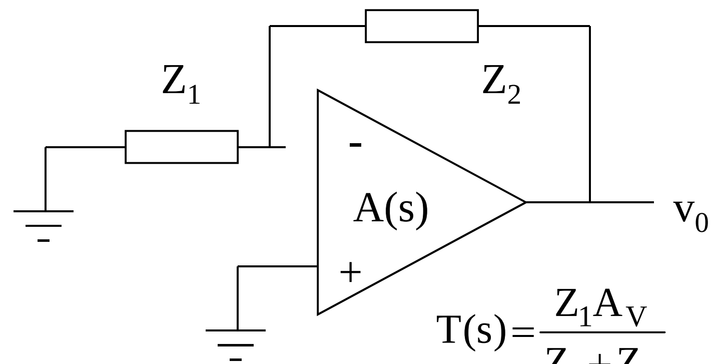



**A(s) and B(s) are sharing some elements!!**

**A(s)= ? B(s) = ? T(s) = ?**




$$A(s) = -\frac{Z_2 A_V}{Z_1 + Z_2}$$




$$T(s) = \frac{Z_1 A_V}{Z_1 + Z_2}$$

**From** $T(s) = A(s)B(s)$

$$B(s) = \frac{T(s)}{A(s)} = -\frac{Z_1}{Z_2}$$

$$\boxed{\frac{v_o(s)}{v_i(s)} \cong \frac{1}{B(s)} = -\frac{Z_2}{Z_1}}$$

$$\frac{v_0}{v_i} = \frac{A(s)}{1+T(s)} = -\frac{\frac{Z_2 A_V}{Z_1 + Z_2}}{1 + \frac{Z_1 A_V}{Z_1 + Z_2}} = -\frac{Z_2}{Z_1}\left( \frac{1}{1 + \frac{1 + \frac{Z_2}{Z_1}}{A_V}} \right)$$

# Next Time

- Common-Mode Feedback Techniques