ECEN474: (Analog) VLSI Circuit Design
Fall 2011

Lecture 13: Noise

1876

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Agenda

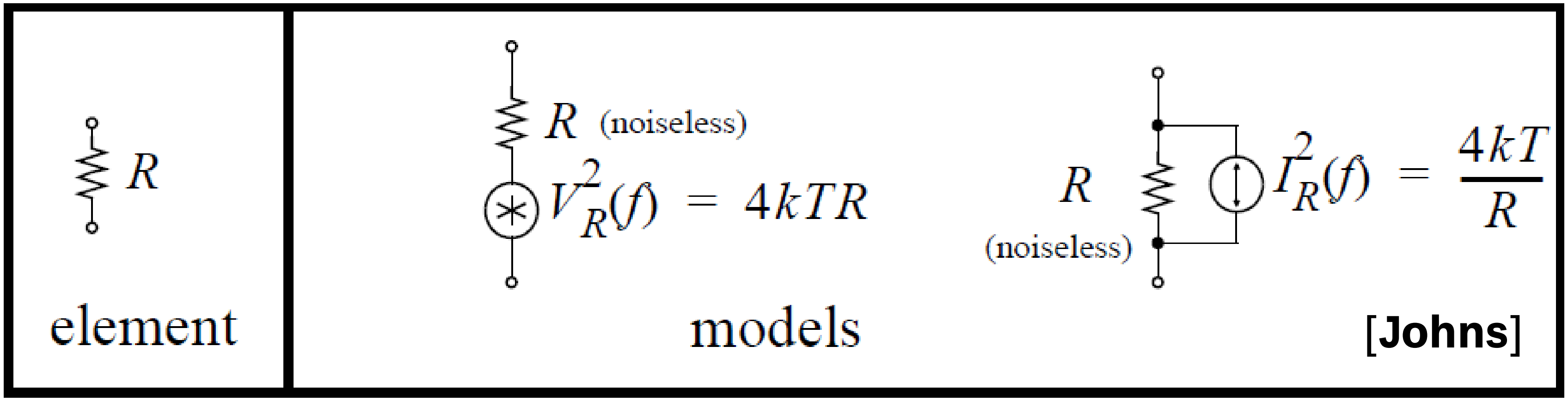
- MOSFET Noise
- Filtered Noise
- OTA Noise Example

Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

$$\boxed{V_{Rn}^2 = P_n R = 4kTR\Delta f}$$

$$\boxed{I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R}\Delta f}$$

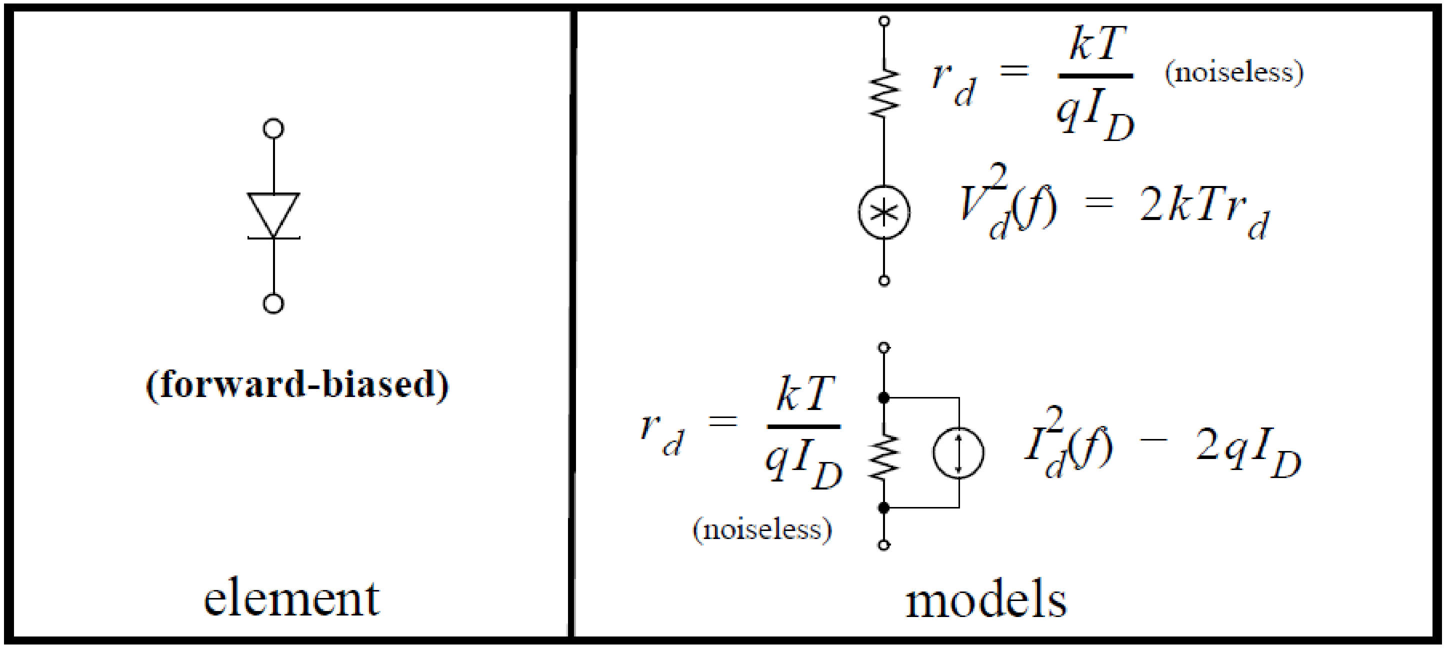


[Johns]

- Recall the PSD is white (uniform w/ frequency)

Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions
- White spectral density

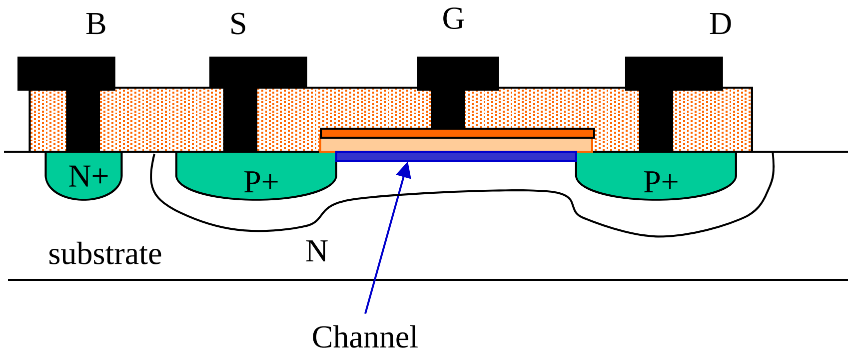


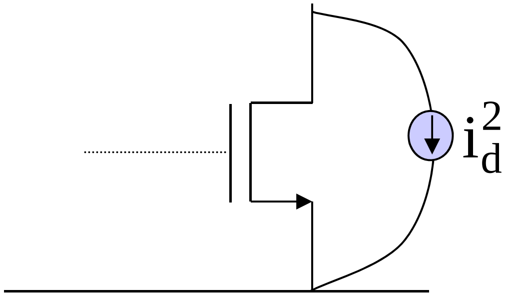
[Johns]

- Where $q = 1.6\text{x}10^{-19}$C and I_D is the diode DC current

Thermal Noise

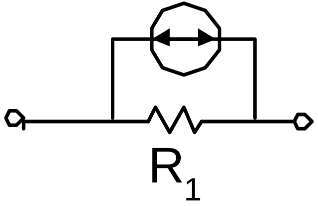
=> Spectral Density of the thermal noise drain current (CMOS transistor biased @ linear region)





Resistor

$$i_{n1}^2 = 4kT/R1$$

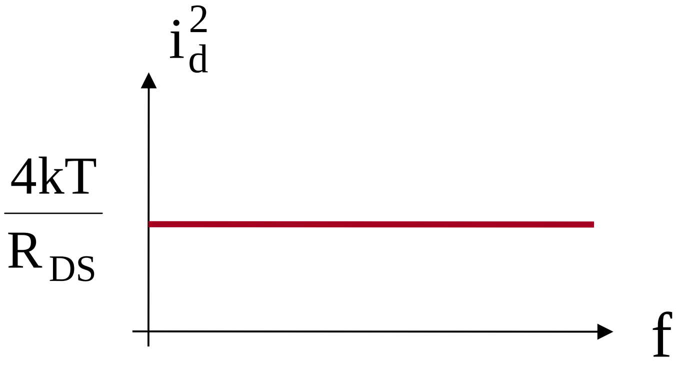


Transistor

$$i_d^2 = \frac{4kT}{R_{DS}}$$

$$R_{DS} \cong \frac{1}{\mu C_{ox} \frac{W}{L} \left(V_{GS} - V_T - V_{DS} \right)}$$

White Noise



@ Triode region

$$i_d^2 = \left[4kT\mu C_{ox}\right]\left[\frac{W}{L}\right]\left[V_{GS} - V_T - V_{DS}\right]$$

Low current noise => W/L ↓ => g_m or g_o ↓

@ Saturation

$$g_o = \frac{1}{R_{DS}} \rightarrow \frac{2}{3} g_m$$

$$\boxed{i_d^2 = \frac{8}{3} kTg_m}$$

$$\boxed{\Rightarrow \quad i_d^2 = \left(\frac{8kT}{3}\right)\left(\mu C_{ox}\right)\left(\frac{W}{L}\right)\left(V_{GS} - V_T\right)}$$

MOSFET 1/f (Flicker) Noise

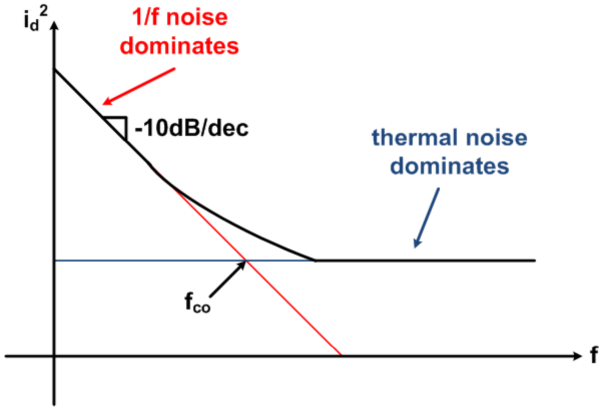
- Caused by traps near Si/SiO$_2$ interface that randomly capture and release carriers

$$\boxed{\overline{i_d^2}(f) = \frac{K_F g_m^2}{W L C_{ox} f}}$$

- K$_F$ is strongly dependent on the technology

1/f Noise Corner Frequency

- This is the frequency at which the flicker noise density equals the thermal noise density



$$\frac{K_F g_m^2}{WLC_{ox} f_{co}} = 4kT\gamma g_m$$

$$f_{co} = \frac{K_F}{4kT\gamma C_{ox}} \frac{g_m}{WL} = \frac{K_F}{4kT\gamma C_{ox}} \frac{1}{L} \left(\frac{g_m}{I_D}\right)\left(\frac{I_D}{W}\right)$$

- For a given g_m/I_D (which sets I_D/W), the only way to reduce f_{co} is to use longer channel devices

Output and input referred noise

Current noise is the real one

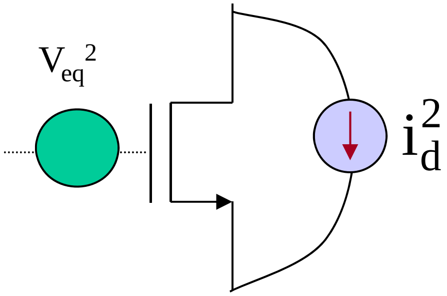
Thermal Noise

$$i_d = g_m V_{gs}$$

$$i_d^2 = g_m^2 V_{gs}^2$$

$$\Rightarrow \quad V_{gseq}^2 = \frac{8}{3}\frac{kT}{g_m}$$

Voltage noise representation is an artifact to facilitate system analysis



Flicker Noise

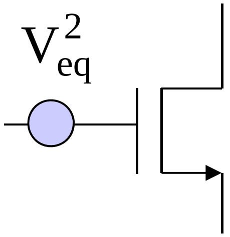
$$i_d^2 = \frac{K_F g_m^2}{WLC_{ox} f}$$

Referred to the input

$$v_{eq}^2 = \frac{K_F g_m^2}{WLC_{ox} f}\frac{1}{g_m^2}$$

$$v_{eq}^2 = \frac{K_F}{C_{ox}}\left(\frac{1}{WL}\right)\left(\frac{1}{f}\right)$$

Equivalent input referred voltage noise



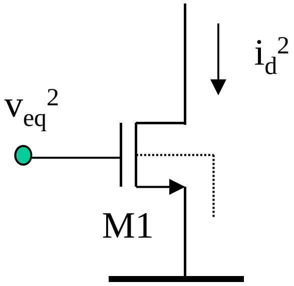
$$V_{eq}^2 = \frac{i_{dth}^2 + i_{df}^2}{g_m^2}$$

Equivalent input referred noise voltage means that all **current noise sources are accounted as drain current and represented by an "equivalent" noise voltage at transistor gate**

$$V_{eq}^2 = \frac{8}{3}\frac{kT}{g_m} + \frac{K_F}{C_{ox}}\frac{1}{WL}\frac{1}{f}$$

$$V_{eq_{total}}\left(\mathrm{RMS}\right) = \sqrt{\int_{BW} v_{eq}^2(f)\,df}$$

NOISE COMPONENTS (values provided are for a 0.8 μm technology)



Noise density (V^2/Hz)

Spice model

$$v_{eq}^2 = v_{th}^2 + v_{1/f}^2$$

$$v_{eq}^2 = \frac{8}{3}\frac{kT}{g_m}df + \frac{K_F}{WLC_{OX}f}df$$

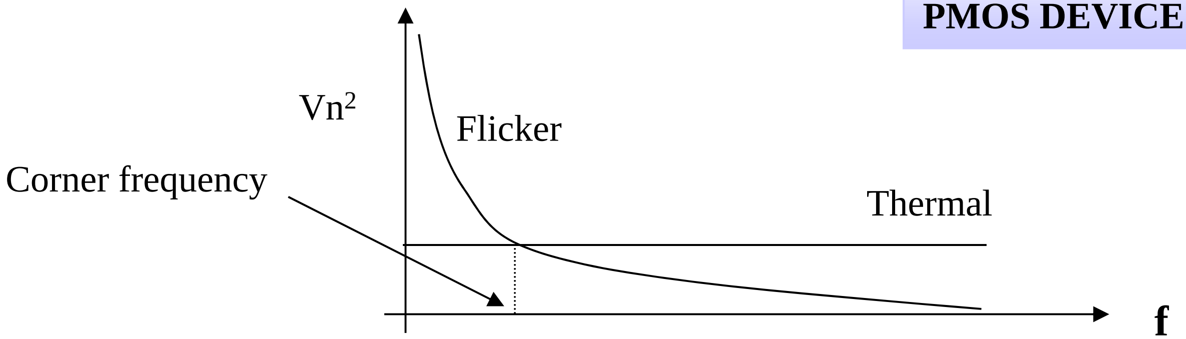
$$\frac{K_F}{C_{OX}} = 9.8x10^{-9}V^2/\mu m - Hz(NMOS)$$

$$= 0.5x10^{-9}V^2/\mu m - Hz(PMOS)$$

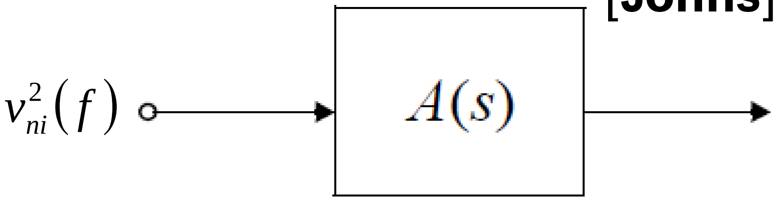
FOR LOW-FREQUENCY APPLICATIONS,

WHEREIN 1/F NOISE IS DOMINANT,

PMOS DEVICES MUST BE USED.



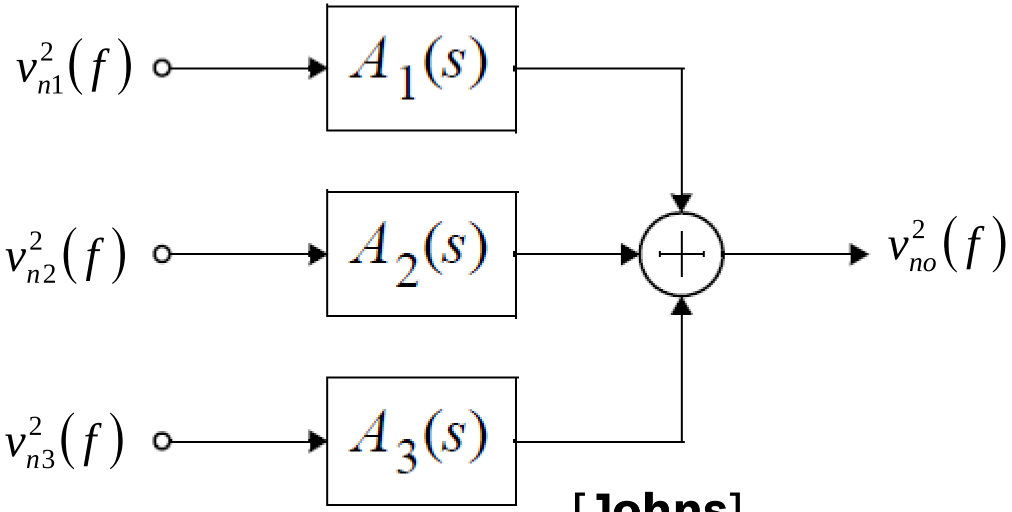
Filtered Noise



[Johns]

$$v_{no}^2(f) = |A(j2\pi f)|^2 v_{ni}^2(f)$$
$$v_{no}(f) = |A(j2\pi f)| v_{ni}(f)$$

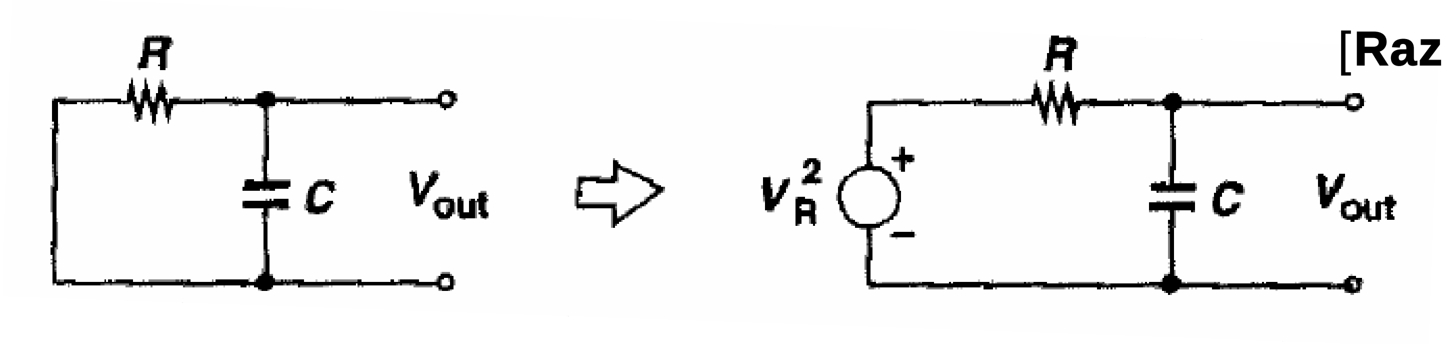
- Noise output spectral density is a function only of the magnitude of the transfer function, and not its phase
- With multiple uncorrelated noise sources, combined output is also uncorrelated



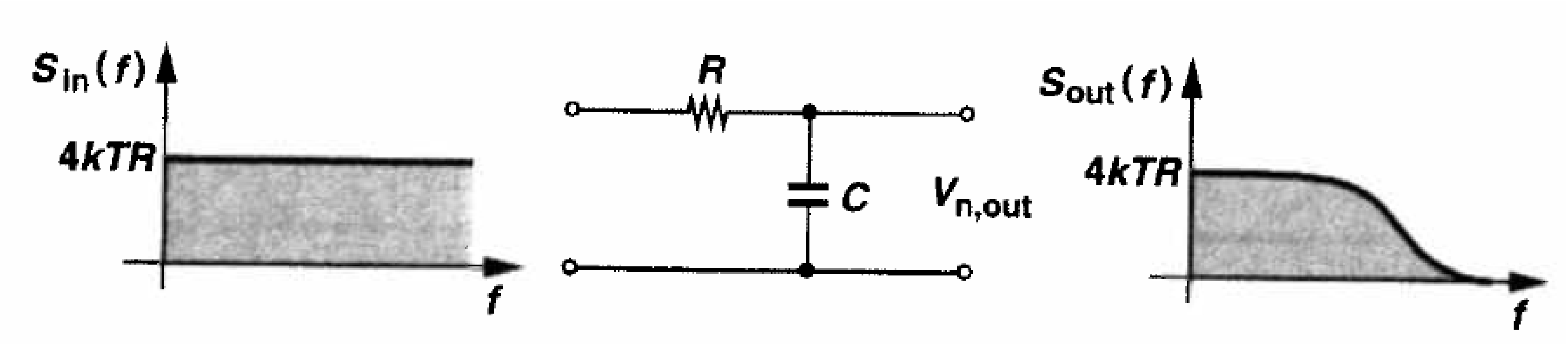
$$v_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 v_{ni}^2(f)$$

[Johns]

First-Order RC Circuit Example

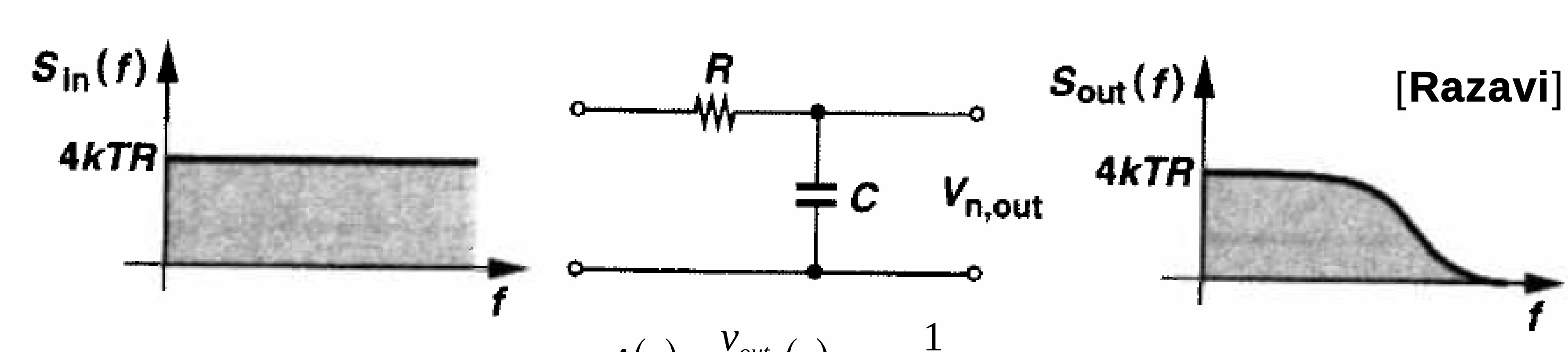


[Razavi]



What is the total output noise power?

First-Order RC Circuit Example



[Razavi]

$$A(s)=\frac{v_{out}}{v_R}(s)=\frac{1}{1+sRC}$$

$$v_{out}^2(f)=\left|A(j2\pi f)\right|^2 v_R^2(f)=\frac{1}{1+4\pi^2 f^2 R^2 C^2}4kTR$$

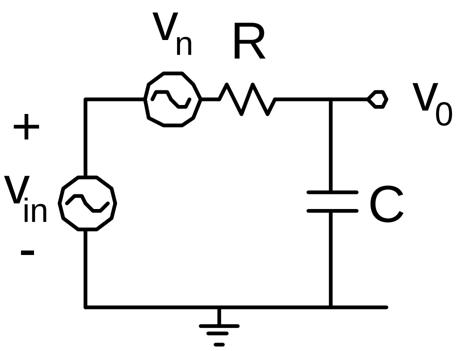
To calculate Total Noise Power integrate over all frequencies

$$v_{out}^2=\int_0^\infty \frac{4kTR}{1+4\pi^2 f^2 R^2 C^2}$$

Using $\int \frac{dx}{x^2+1}=\tan^{-1}x$

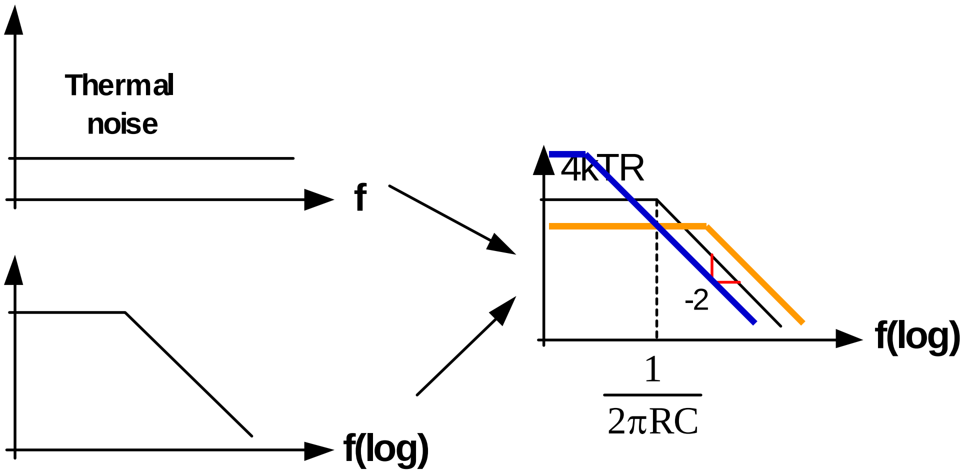
$$v_{out}^2=\frac{2kT}{\pi C}\tan^{-1}(2\pi fRC)\Big|_{f=0}^{f=\infty}=\frac{2kT}{\pi C}\left[\frac{\pi}{2}-0\right]=\frac{kT}{C}$$

Noise is generated by R but integrated noise is function of C (??)



$$v_{total}^{\;2} = \int_0^\infty \left(\left(\frac{1}{1+(\omega RC)^2}\right)(4kTR)\right)df = \frac{kT}{C}$$

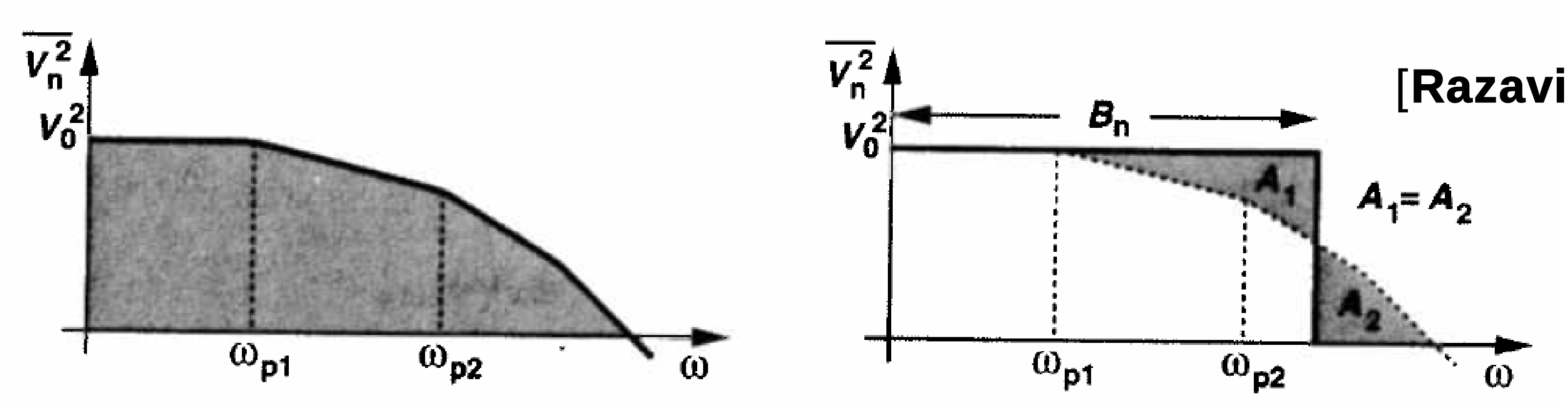
To get more insight, lets have a closer look on the operations!



Notice that:

When R increases thermal noise increases too but the corner frequency decreases, leading to a constant area under the curves!

Noise Bandwidth



[Razavi]

- The noise bandwidth is equal to the frequency span of a brickwall filter having the same output noise rms value

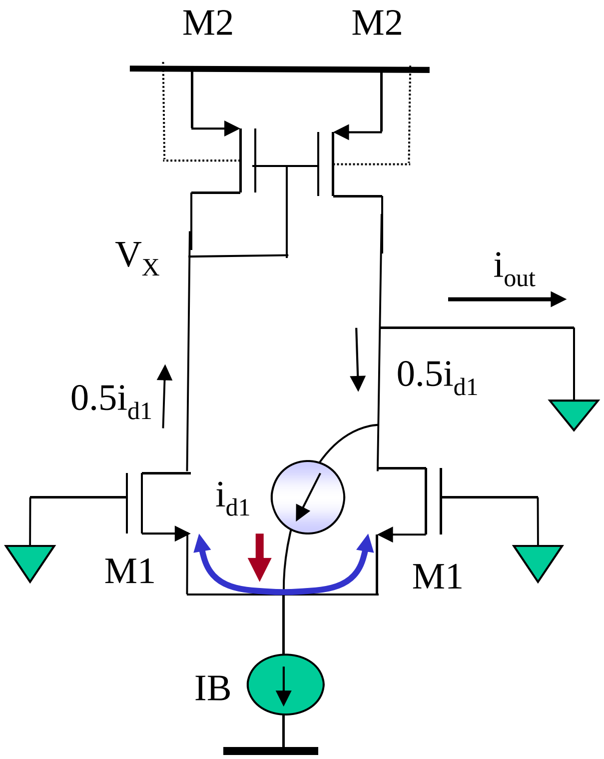
$$v_0^2 B_n = \int_0^\infty v_{no}^2 \, df$$

$$\textbf{For a first-order filter } B_n = \frac{\pi}{2}\omega_p$$

Validating with previous slides derivation :

$$\textbf{Total Noise Output} = v_0^2 B_n = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right) = \frac{kT}{C}$$

Output referred noise: Take advantage of SYMMETRIES!



Noise injected into the common-source node equally splits into the two branches

Output referred current noise density

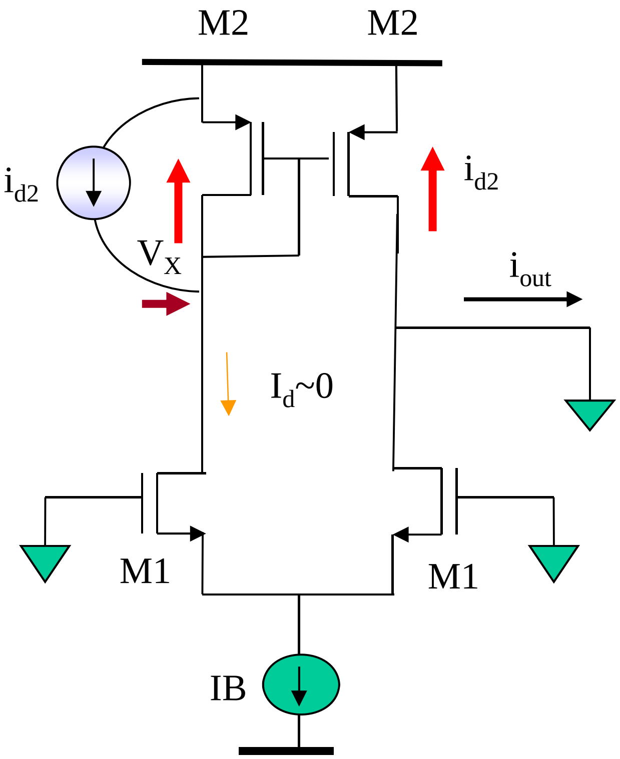
Superposition: Every transistor contributes; consider one at the time.

Analysis: You can use standard circuit analysis techniques but at the end of the day you have to consider POWER.

Output noise density: Each noise component represent the RMS value of random uncorrelated noise! Then add the power noise components

$$i_{out1}{}^2 = \frac{8}{3}kTg_{m1}$$

Output referred noise: Take advantage of SYMMETRIES!



Noise injected into the common-source node equally splits into the two branches

Noise due to the current source is mainly common-mode noise

Output referred current noise density due to the P-type devices:

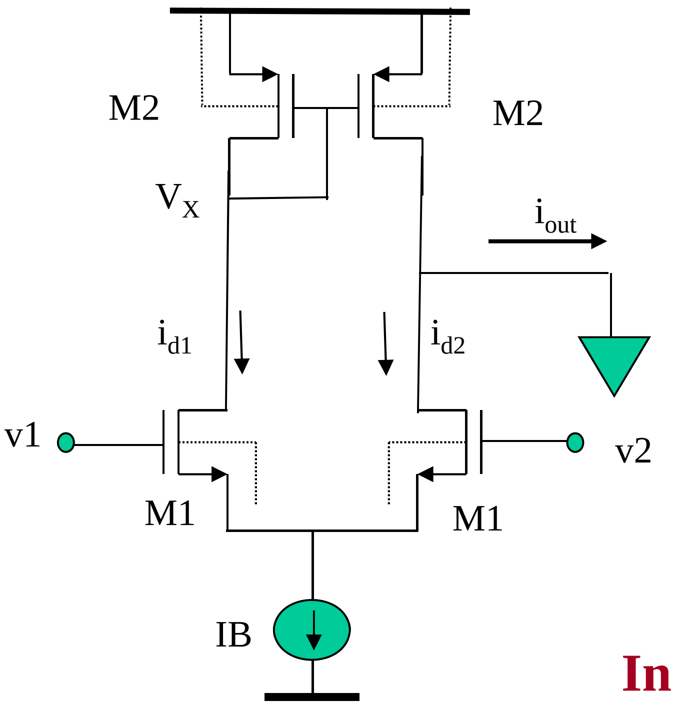
Left hand side transistor:

$$\overline{i_{out2}^{\,2}} \cong \overline{i_{d2}^{\,2}} = \frac{8}{3}kTg_{m2}$$

Right hand side transistor

$$\overline{i_{out2}^{\,2}} = \frac{8}{3}kTg_{m2}$$

Output and input referred noise



Output referred current noise density

$$i_{out}^2 = 2\left(\frac{8}{3}kTg_{m1}\right) + 2\left(\frac{8}{3}kTg_{m2}\right)$$

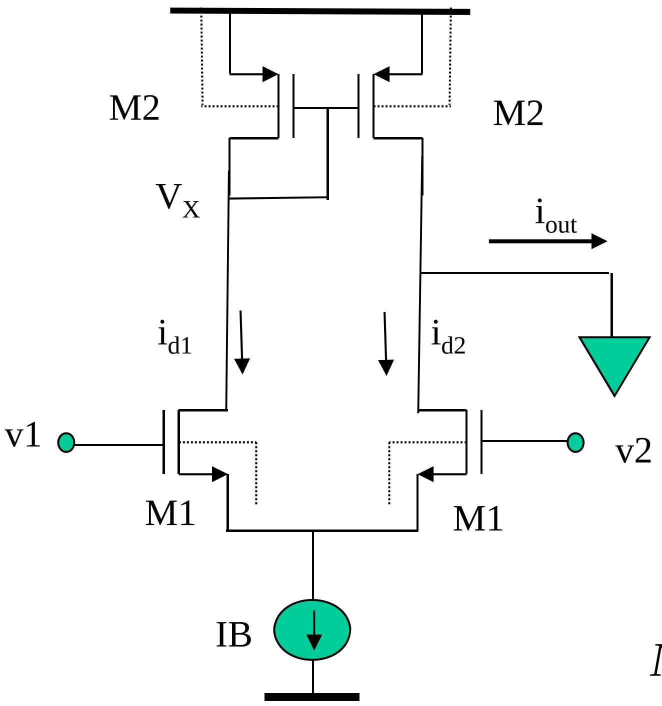
Input referred noise density (V^2/H_Z)

$$v_{in,eq}^2 = 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\right) + 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\frac{g_{m2}}{g_{m1}}\right)$$

In this case, noise due to the current source is mainly common-mode noise

Be careful because this is not always the case!

Integrated Input referred noise



Input referred thermal noise density (V^2/Hz)

$$v_{in,eq}^{\;2} = 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\right) + 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\frac{g_{m2}}{g_{m1}}\right)$$

Input referred noise level (volts)

$$\text{Noise}(V_{RMS}) = \sqrt{\int_{BW} V_{in,eq}^{\;2}\,df}$$

Example: for thermal noise, the noise level becomes (assuming a single-pole system)

$$Noise(V_{RMS}) = \sqrt{\frac{16kT}{3}}\sqrt{\frac{1}{g_{m1}}}\sqrt{1+\frac{g_{m2}}{g_{m1}}}\left(\sqrt{\frac{\pi}{2}BW}\right)$$

I should advise you to use:

$$\text{Noise}(V_{RMS}) \cong \sqrt{\frac{8kT}{g_{m1}}}\sqrt{1+\frac{g_{m2}}{g_{m1}}}\left(\sqrt{BW}\right)$$

$4kT \approx 16\text{x}10^{-21}$ coul.V

Next Time

- OTAs