ECEN474: (Analog) VLSI Circuit Design
Fall 2011

Lecture 12: Noise

Sebastian Hoyos
Analog & Mixed-Signal Center
Texas A&M University

Announcements

- Reading
  - Razavis’ CMOS Book Chapter 7

Agenda

- Noise Types
- Noise Properties
- Resistor Noise Model
- Diode Noise Model

Noise Significance

- Why is noise important?
  - Sets minimum signal level for a given performance parameter
  - Directly trades with power dissipation and bandwidth
- Reduced supply voltages in modern technologies degrades noise performance

$$\textbf{Signal Power} \propto (\alpha Vdd)^2 \Rightarrow \textbf{SNR} = P_{sig} / P_{noise} \propto \left(\frac{\alpha Vdd}{V_{noise}}\right)^2$$

- Noise is often proportional to kT/C
  - Increasing capacitance to improve noise performance has a cost in increase power consumption for a given bandwidth

Interference Noise

- Interference "Man-Made" Noise
  - Deterministic signal, i.e. not truly "random"
    - Could potentially be modeled and predicted, but practically this may be hard to do
  - Examples
    - Power supply noise
    - Electromagnetic interference (EMI)
    - Substrate coupling
  - Solutions
    - Fully differential circuits
    - Layout techniques

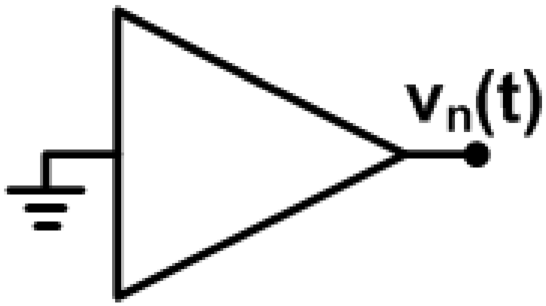
- Not the focus of this lecture
  - Unless the deterministic noise is approximated as a random process

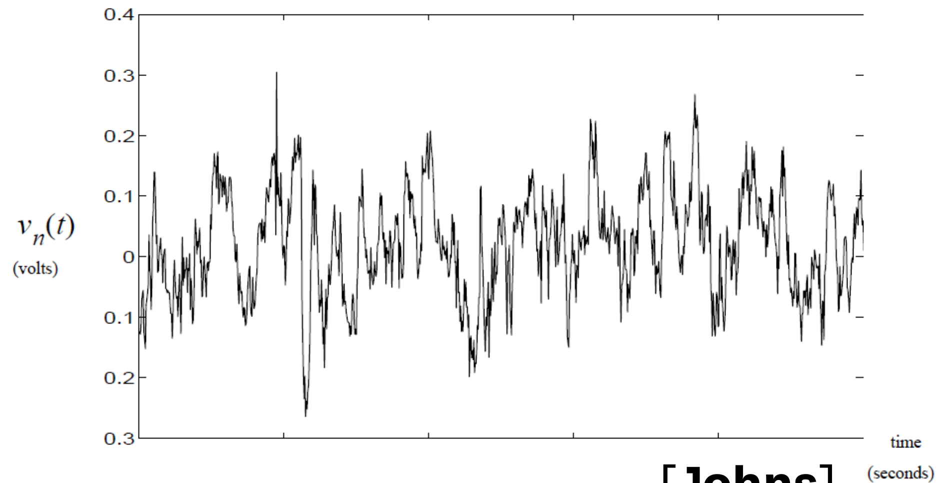
Inherent Noise

- “Electronic” or “Device” Noise
  - Random signal
  - Fundamental property of the circuits
  - Examples
    - Thermal noise caused by thermally-excited random motion of carriers
    - Flicker (1/f) noise caused by material defects
    - Shot noise caused by pulses of current from individual carriers in semiconductor junctions
  - Solutions
    - Proper circuit topology
    - More power!!!

- Is the focus of this lecture

Noise Properties





[**Johns**]

- Noise is **random**
  - Instantaneous noise value is unpredictable and the noise must be treated statistically
  - Can only predict the average noise power
  - Model with a Gaussian amplitude distribution
  - Important properties: mean (average), variance, power spectral density (noise frequency spectrum)

RMS Value

- If we assume that the noise has zero mean (generally valid)
- RMS or "sigma" value is the square-root of the noise variance over a suitable averaging time interval, T

$$\boxed{V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)dt\right]^{1/2}}$$

- Indicates the **normalized noise power**, i.e. if $v_n(t)$ is applied to a 1Ω resistor the average power would be

$$P_n = \frac{V_{n(rms)}^2}{1\Omega} = V_{n(rms)}^2$$

Signal-to-Noise Ratio (SNR)

$$\boxed{SNR \equiv 10\log\left[\frac{\textbf{signal power}}{\textbf{noise power}}\right]}$$

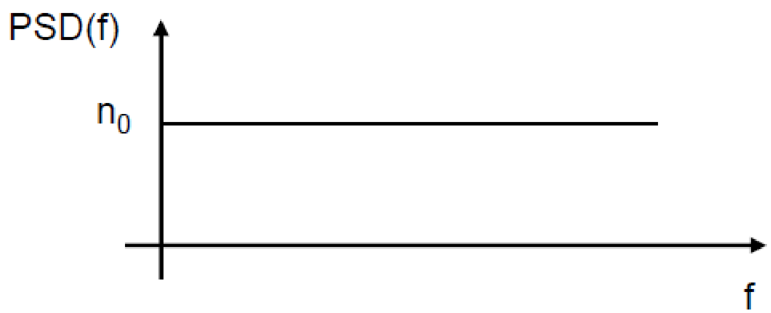
For a signal with normalized power of $\mathbf{V^2_{x(rms)}}$

$$SNR \equiv 10\log\left[\frac{V^2_{x(rms)}}{V^2_{n(rms)}}\right] = 20\log\left[\frac{V_{x(rms)}}{V_{n(rms)}}\right]$$

- Quantified in units of dB

Thermal Noise Spectrum

- The power spectral density (PSD) quantifies how much power a signal carries at a given frequency
- Thermal noise has a uniform or "white" PSD



- The total average noise power P_n in a particular frequency band is found by integrating the PSD

$$\boxed{P_n = \int_{f_1}^{f_2} PSD(f)\,df}$$

For white noise spectrum : $P_n = n_0\left(f_2 - f_1\right) = n_0 \Delta f$

Thermal Noise of a Resistor

- The noise PSD of a resistor is

$$PSD(f) = n_0 = 4kT$$

where k is the Boltzmann constant and T is the absolute temperature (K)

- The total average power of a resistor in a given frequency band is

$$P_n = \int_{f_1}^{f_2} 4kTdf = 4kT(f_2 - f_1) = 4kT\Delta f$$

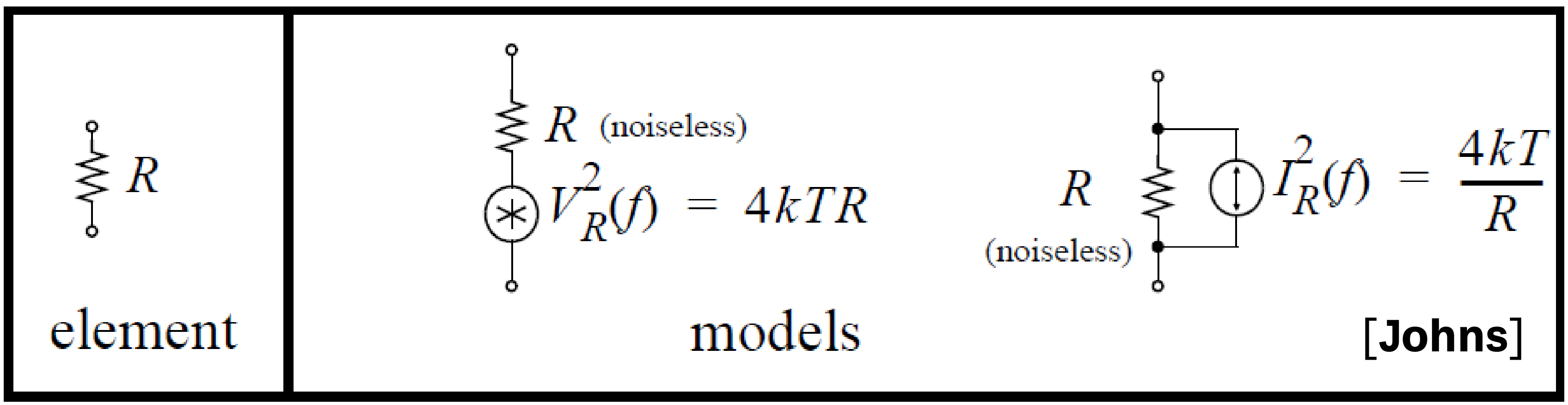
- Example: $\Delta f = 1Hz \rightarrow P_n = 4x10^{-21}W = -174dBm$

Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

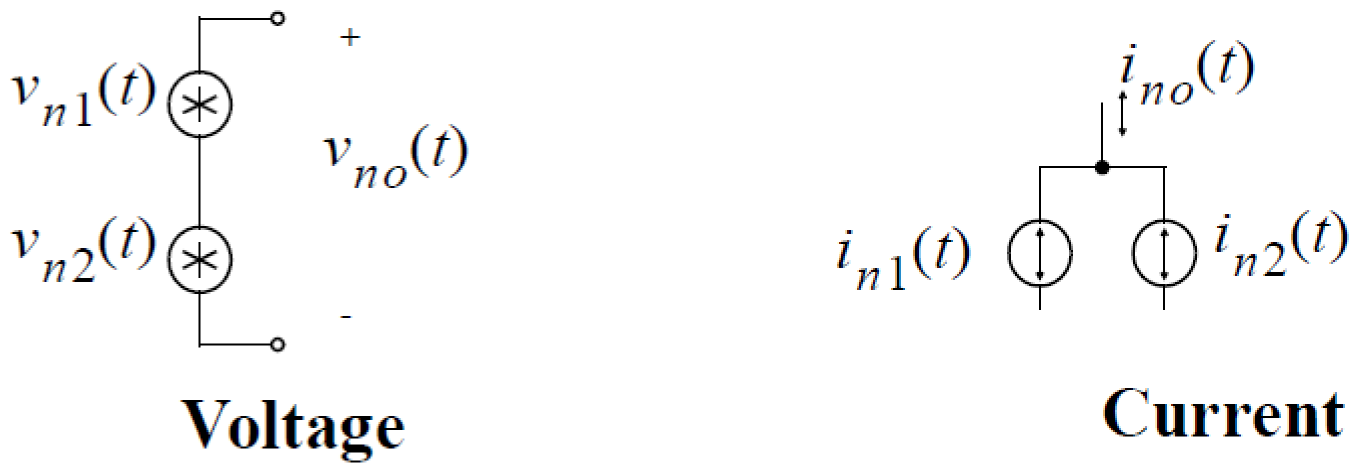
$$\boxed{V_{Rn}^2 = P_n R = 4kTR\Delta f}$$

$$\boxed{I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R}\Delta f}$$



- Recall the PSD is white (uniform w/ frequency)

Noise Summation



$$v_{no}(t) = v_{n1}(t) + v_{n2}(t)$$

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2 dt$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t) v_{n2}(t) dt$$

- Same procedure applies to noise current summing at a node

Correlation

- Last term describes the correlation between the two signals, defined by the correlation coefficient, C

$$C = \frac{\frac{1}{T}\int_0^T v_{n1}(t) v_{n2}(t)\,dt}{V_{n1(rms)} V_{n2(rms)}}$$

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2C V_{n1(rms)} V_{n2(rms)}$$

- Correlation always satisfies -1≤C≤1
  - C=+1, fully-correlated in-phase (0°)
  - C=-1, fully-correlated out-of-phase (180°)
  - C=0, uncorrelated (90°)

Uncorrelated Signals

- For two uncorrelated signals, the mean-squared sum is given by

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2$$

Add as though they were vectors at right angles

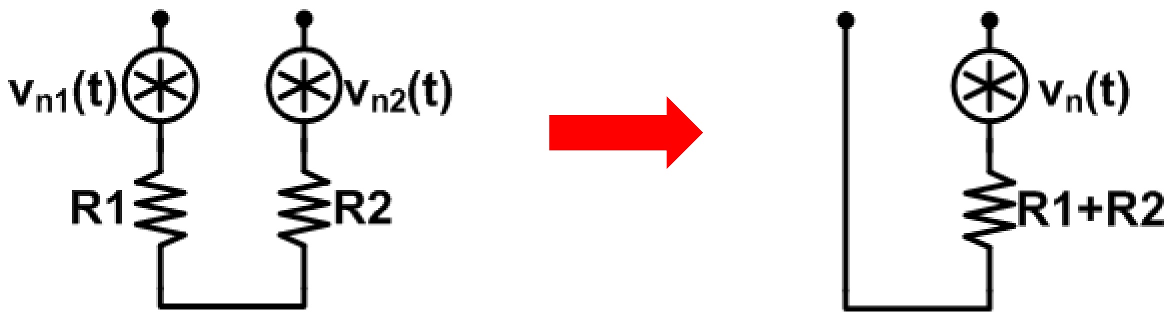
- For two fully correlated signals, the mean-squared sum is given by

$$V_{no(rms)}^2 = \left(V_{n1(rms)} \pm V_{n2(rms)}\right)^2$$

Sign is determined by phase relationship

RMS values add linearly (aligned vectors)

Noise Example #1: Two Series Resistors



$$v_{n(rms)}^2 = v_{n1(rms)}^2 + v_{n2(rms)}^2 + 2Cv_{n1(rms)}v_{n2(rms)}$$

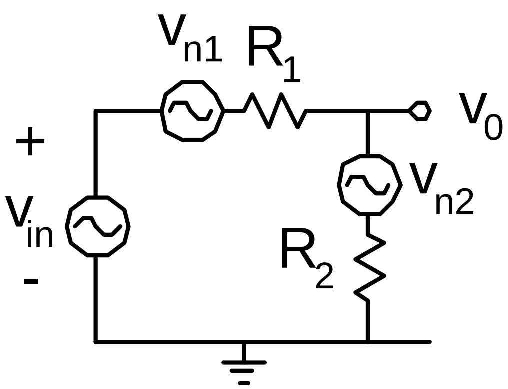
- The noise of the two resistors is uncorrelated or statistically independent, so C=0

$$v_{n(rms)}^2 = v_{n1(rms)}^2 + v_{n2(rms)}^2 = 4kT\left(R_1 + R_2\right)\Delta f$$

- Always add independent noise sources using mean squared values
  - Never add RMS values of independent sources

Noise Example #2: Voltage Divider

- Lets compute the output voltage: **Apply superposition (noise sources are small signals, you can use small signal models)!**



$$v_0 = \left(\frac{R2}{R1+R2}\right)v_{in} + \left(\frac{R2}{R1+R2}\right)v_{n1} + \left(\frac{R1}{R1+R2}\right)v_{n2}$$

Above is what you do for deterministic signals, but we cannot do this for the resistor noise

But noise is a random variable, power noise density has to be used rather than voltage; then the output referred noise density (noise in a bandwidth of 1 Hz) becomes

$$v_{0n}^{\ 2} = \left(\frac{R2}{R1+R2}\right)^2 v_{n1}^{\ 2} + \left(\frac{R1}{R1+R2}\right)^2 v_{n2}^{\ 2}$$

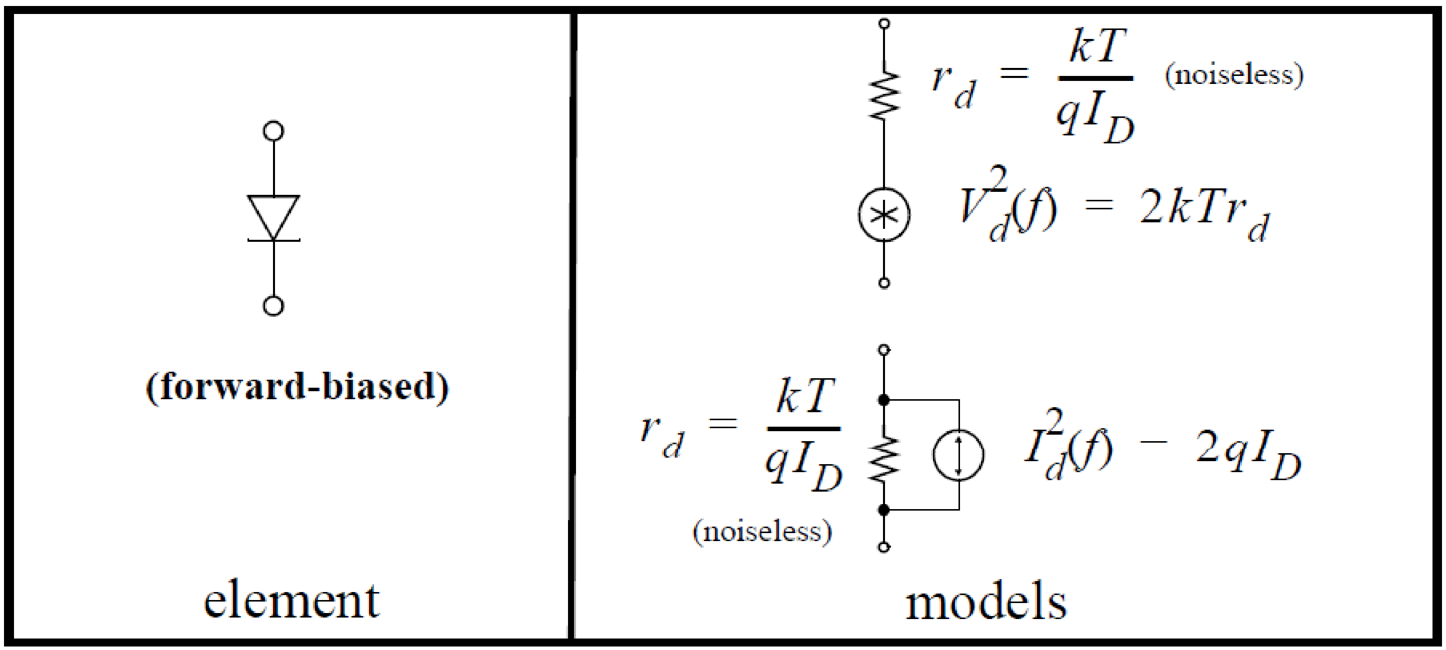
$$v_{0n}^{\ 2} = \left(\frac{R2}{R1+R2}\right)^2 4kTR1 + \left(\frac{R1}{R1+R2}\right)^2 4kTR2$$

General Case :

$$v_{on,T}^2(f) = \sum_x \left|H_x(s)\right|^2_{s=j2\pi f} v_x^2(f)$$

Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions
- White spectral density



[Johns]

- Where $q=1.6\times10^{-19}$C and I_D is the diode DC current

Next Time

- Noise in MOSFETs
- Noise Analysis