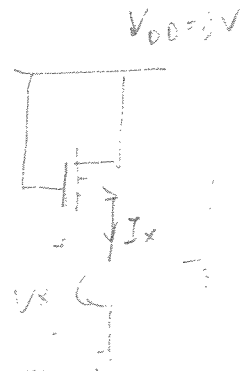


1a) $\lambda = 0.1$, $\gamma = 0.45$, $\phi_F = 0.7$, $V_{TH0} = 0.7$

$V_{GS} = 3 - V_x$, $V_{DS} = 3 - V_x$, $V_{SB} = V_x$

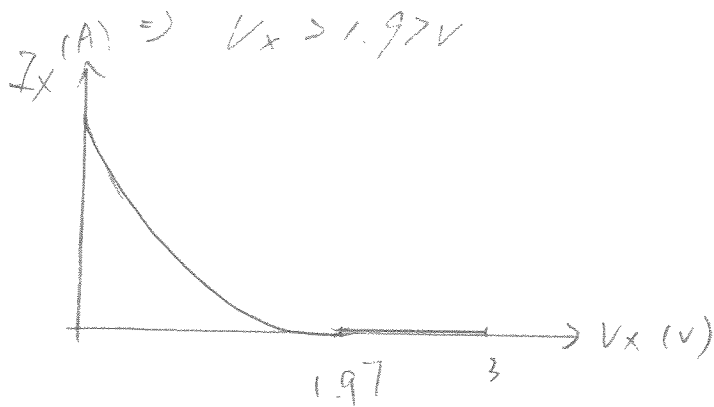
$V_{TH} = V_{TH0} + \gamma (\sqrt{\phi_F + V_{SB}} - \sqrt{\phi_F})$

So, $I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - V_x - 0.7 - 0.45 (\sqrt{0.9 + V_x} - \sqrt{0.9}))^2 \times (1 + \lambda (3 - V_x))$

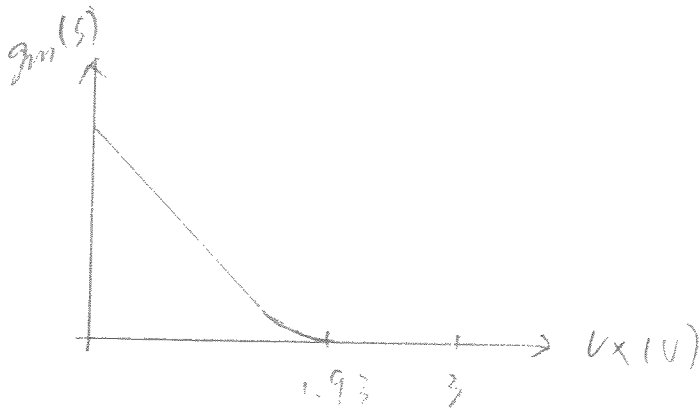


MOS is off when

$3 - V_x - 0.7 - 0.45 (\sqrt{0.9 + V_x} - \sqrt{0.9}) > 0$



$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_x}$



(b) $\lambda = \gamma = 0$, $V_{TH} = 0.7$.

(1) $V_x < 1V \Rightarrow$ S. D Terminals exchange.

$$V_{GS} = 1.9 - V_x, \quad V_{DS} = 1 - V_x, \quad V_{DD} = 1.2 - V_x$$

$$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(1.2 - V_x) \times 0.2 \times (1 - V_x) - (1 - V_x)^2 \right]$$

$$\Rightarrow I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (1 - V_x)(1.2 - V_x)$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} (1 - V_x)$$

(2) $V_x > 1V$

$$V_{GS} = 1.9 - 1 = 0.9 \quad V_{DS} = V_x - 1, \quad V_{DD} = 0.9 - 0.7 = 0.2$$

if $V_x < 1.2V \Rightarrow$ Triode

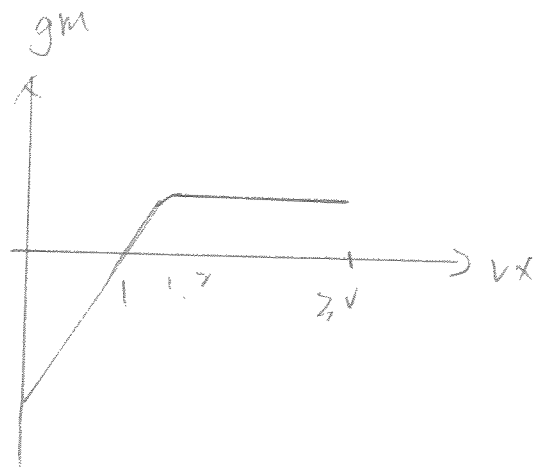
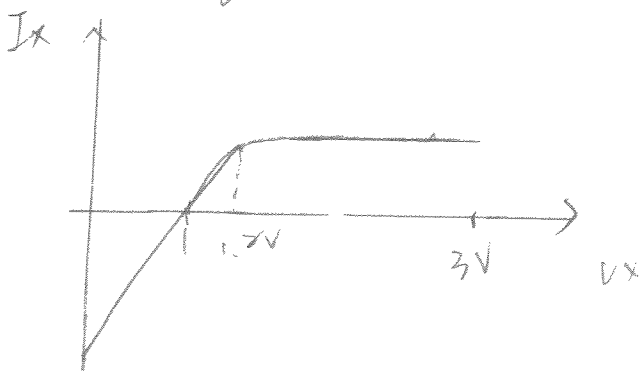
$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[0.2 \times 0.2 \times (V_x - 1) - (V_x - 1)^2 \right]$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1)$$

if $V_x > 1.2V \Rightarrow$ Saturation

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$$



(1) $\lambda = \gamma = 0$. $V_{TH} = 0.7$, S. D terminals exchange

$$V_{GS} = 1 - V_x \quad V_{DS} = 1.9 - V_x \quad V_{GD} = V_{GS} - V_{TH} = 0.3 - V_x$$

I_x {

$V_x < 0.3V \Rightarrow$ saturation

$$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.3 - V_x)^2$$

$V_x > 0.3V \Rightarrow$ off

$$I_x = 0$$

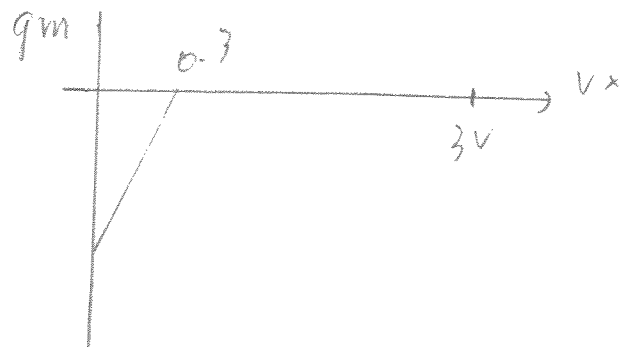
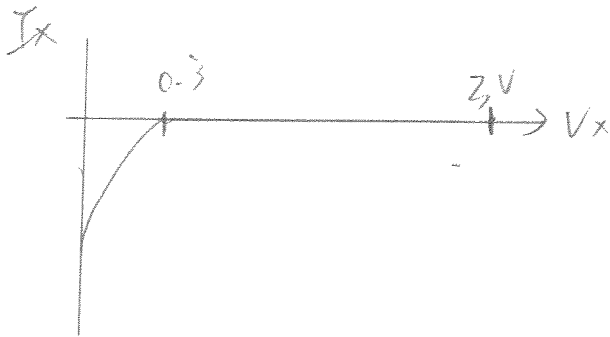
g_m {

$V_x < 0.3V \Rightarrow$ sat.

$$g_m = -\mu_n C_{ox} \frac{W}{L} (0.3 - V_x)$$

$V_x > 0.3$

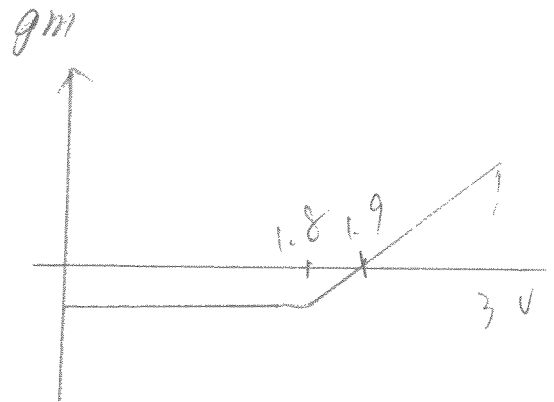
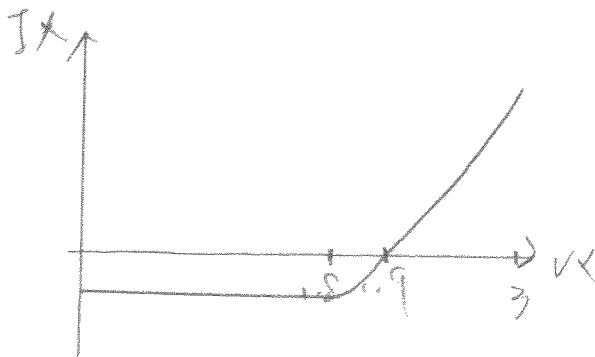
$$g_m = 0$$



(d) $V_{TH} = -0.8$ $r = 0$

$$0 < V_x < 1.8 \Rightarrow \begin{cases} I_x = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L} (0.1)^2 \\ g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)^2 \end{cases}$$

$$1.8 < V_x < 3 \Rightarrow \begin{cases} I_x = \mu_p C_{ox} \frac{W}{L} \left[\frac{1}{2} (V_x - 1.9)(V_x - 1.7) \right] \\ g_m = \mu_p C_{ox} \frac{W}{L} (V_x - 1.9) \end{cases}$$



(e)

$$V_{TH} = 0.7, \quad \gamma = 0.45, \quad \gamma_F = 0.9, \quad \lambda = 0.$$

$$V_{SB} = 1 - V_X$$

$$V_{TH} = 0.7 + 0.45 (\sqrt{0.9 + 1 \cdot V_X} - \sqrt{0.9})$$

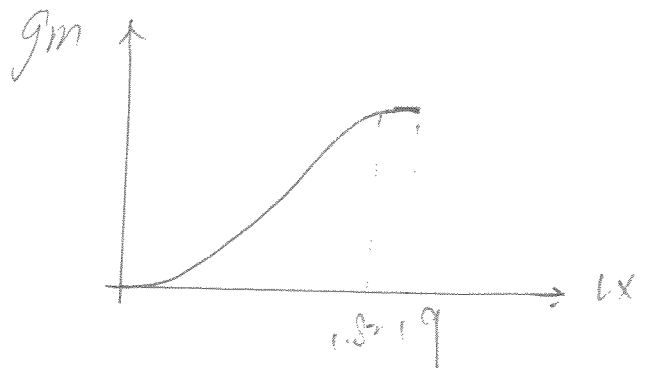
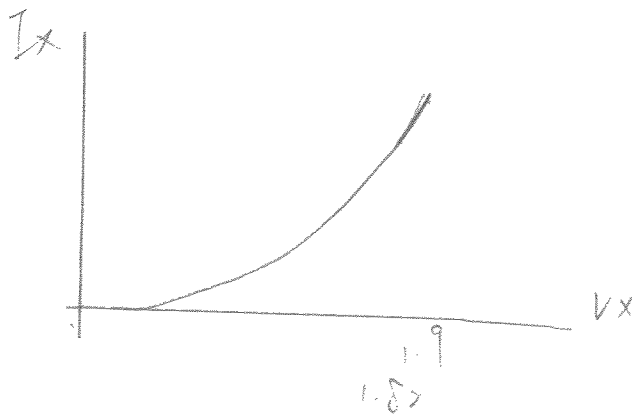
$$V_{GS} = 0.7 \quad V_{DS} = 0.5, \quad V_{GS} - V_{TH} = 0 \Rightarrow V_X = 1.82V$$

(1) $V_X < 1.82V \Rightarrow \text{Sat.}$

$$\begin{cases} I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2 - 0.45 (\sqrt{1.9 - V_X} - \sqrt{0.9}))^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} (0.2 - 0.45 (\sqrt{1.9 - V_X} - \sqrt{0.9})) \end{cases}$$

(2) $V_X > 1.82V$

$$\begin{cases} I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 \times 0.5 \times (0.2 - 0.45 (\sqrt{1.9 - V_X} - \sqrt{0.9})) - 0.5]^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} (0.5) \end{cases}$$

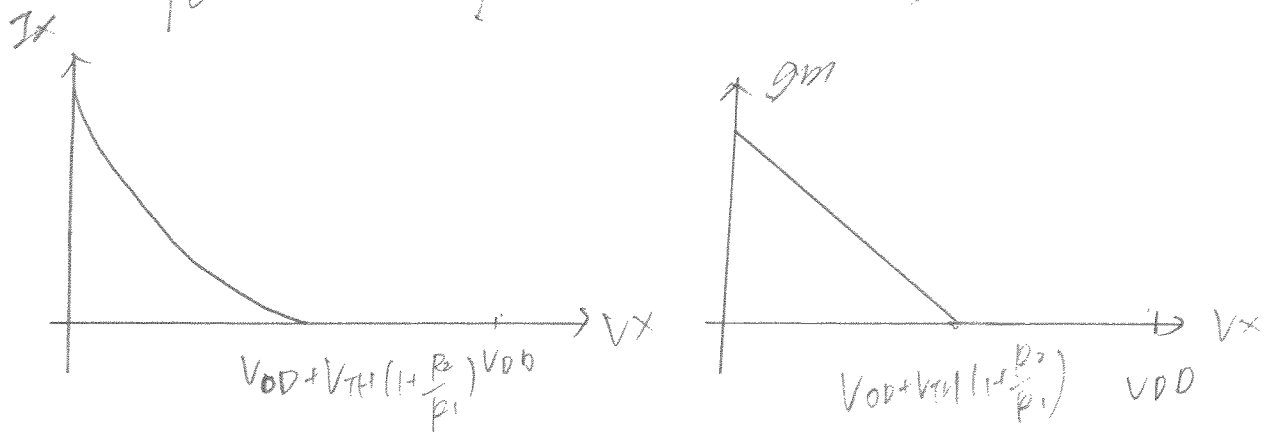


2. b (a) $V_{SG} > |V_{TH}|$

$$\Rightarrow (V_{DD} - V_X) \frac{R_1}{R_1 + R_2} > -V_{TH}$$

$$\Rightarrow V_X < V_{DD} + V_{TH} \left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow \begin{cases} I_X = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} \left[(V_{DD} - V_X) \frac{R_1}{R_1 + R_2} + V_{TH} \right]^2 \\ g_m = \mu_p C_{ox} \frac{W}{L} \left[(V_{DD} - V_X) \frac{R_1}{R_1 + R_2} + V_{TH} \right] \end{cases}$$

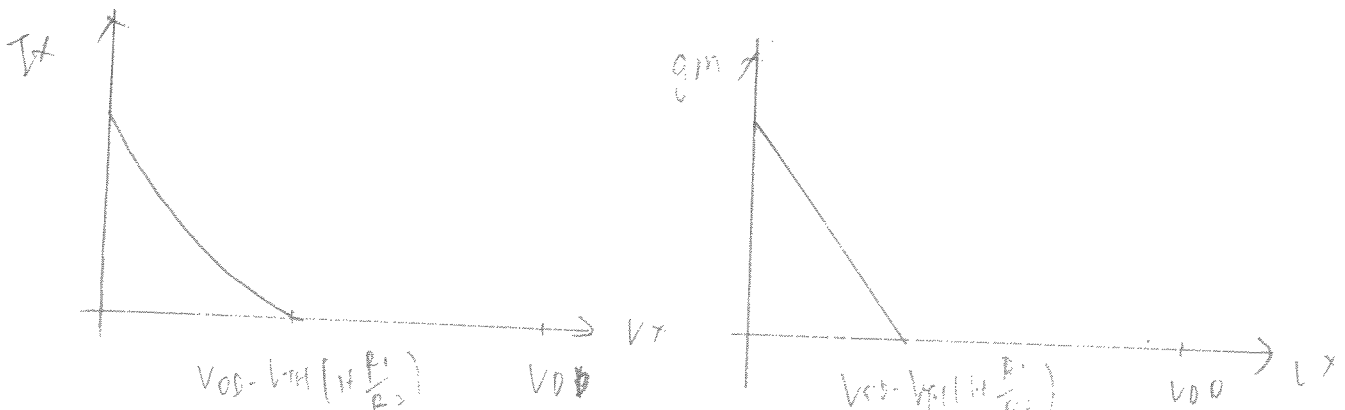


if $V_{DD} + V_{TH} \left(1 + \frac{R_2}{R_1}\right) < 0 \Rightarrow$ off.

(b) $V_{DS} > V_{TH0} \Rightarrow V_X < V_{DD} - V_{TH0} \left(1 + \frac{R_2}{R_1}\right)$

$$\Rightarrow \begin{cases} I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_X) \frac{R_2}{R_1 + R_2} - V_{TH} \right]^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_X) \frac{R_2}{R_1 + R_2} - V_{TH} \right] \end{cases}$$

if $V_{DD} - V_{TH} \left(1 + \frac{R_2}{R_1}\right) < 0 \Rightarrow$ off



1c)

① $0 < V_x < 2 - V_{TH} \Rightarrow$ Triode

$$V_{GS} = 2 - V_x + R_1(I_1 - I_x), \quad V_{DS} = R_1(I_1 - I_x)$$

$$\Rightarrow I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [R_1(I_1 - I_x) + 2(2 - V_{TH} - V_x)] (R_1(I_1 - I_x))$$

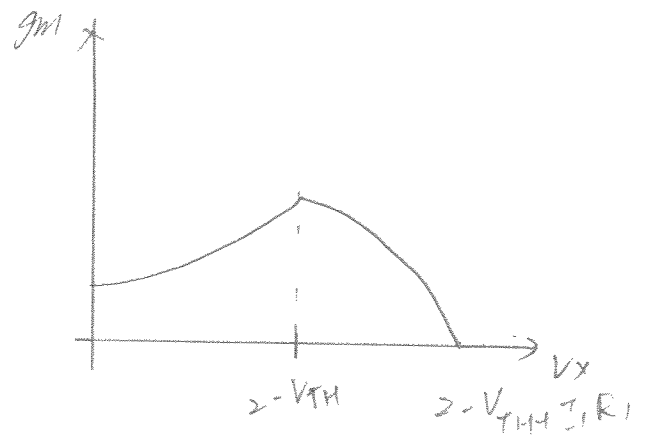
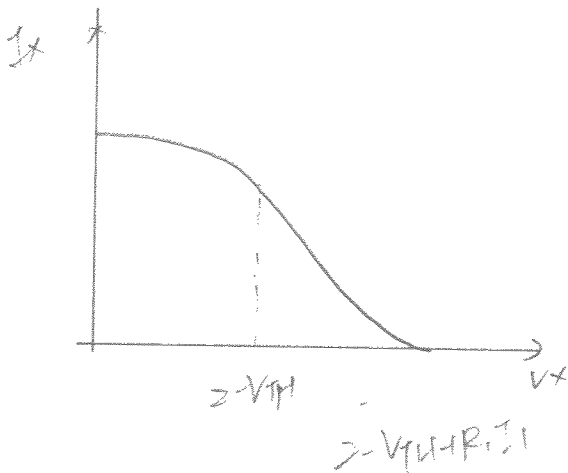
$$g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$$

② $V_{GS} = 2 - V_x + R_1(I_1 - I_x) \Rightarrow$ Sat

$$\Rightarrow I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 - V_x - R_1(I_1 - I_x) - V_{TH}]^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} [2 - V_x + R_1(I_1 - I_x) - V_{TH}]$$

③ $V_x = 2 - V_{TH} + R_1 I_1 \Rightarrow$ Off

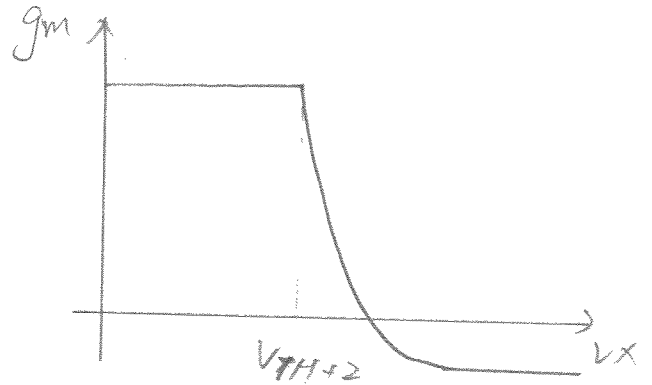
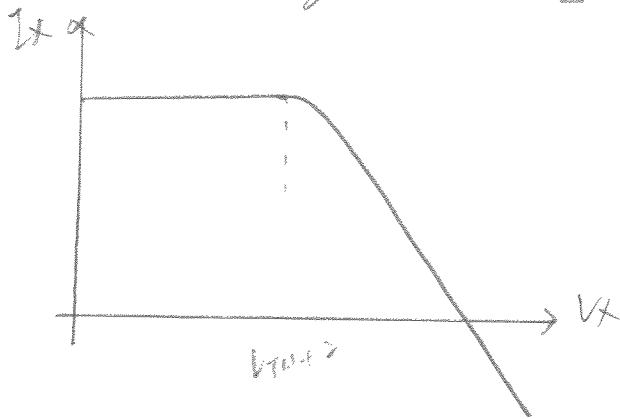


(d) $0 < v_x < v + v_{TH} \Rightarrow \text{Sat}$

$$\begin{cases} I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) - V_{TH}]^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) - V_{TH}] \end{cases}$$

$v_x > v + v_{TH} \Rightarrow \text{Triode}$

$$\begin{cases} I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) - V_{TH}]^2 - (v_x - v - V_{TH})^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) + v - v_x] \end{cases}$$

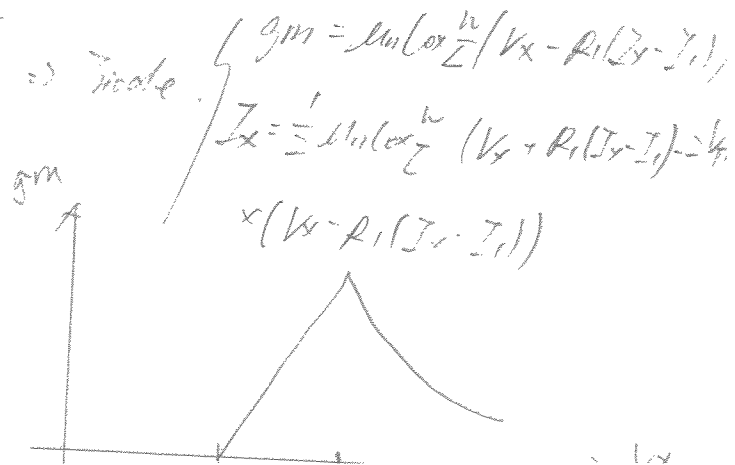
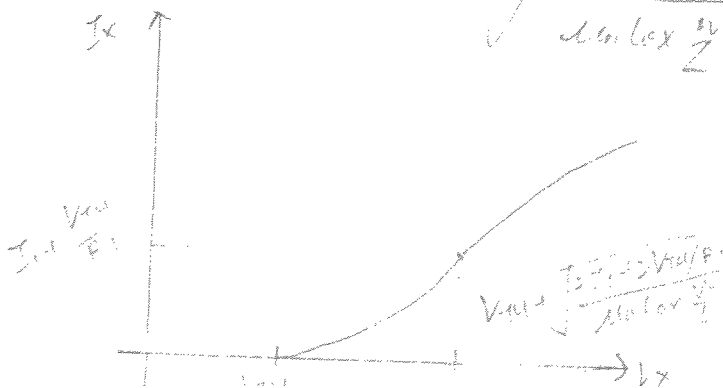


(e) 1) $0 < v_x < v_{TH} \Rightarrow \text{OFF}$

2) $v_{TH} < v_x < v_{TH} + \sqrt{\frac{2I_1 - 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}}$

$$\Rightarrow \begin{cases} I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_x - v_{TH})^2 \\ g_m = \mu_n C_{ox} \frac{W}{L} (v_x - v_{TH}) \end{cases}$$

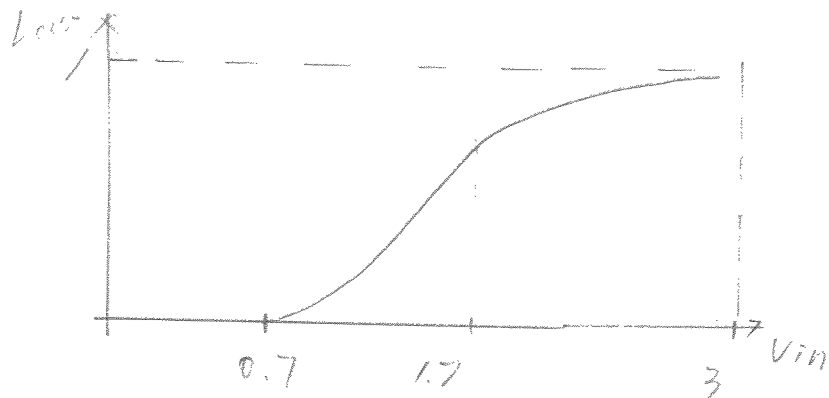
3) $v_x > v_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow \text{Triode}$



2.7 (a) $0 < v_{in} < 0.7 \Rightarrow \text{OFF} \Rightarrow v_{out} = 0$

$0.7 < v_{in} < 1.7 \Rightarrow \text{Sat}$

$1.7 < v_{in} < 3 \Rightarrow \text{Triode} \Rightarrow v_{OS} = 1 - v_{out}$

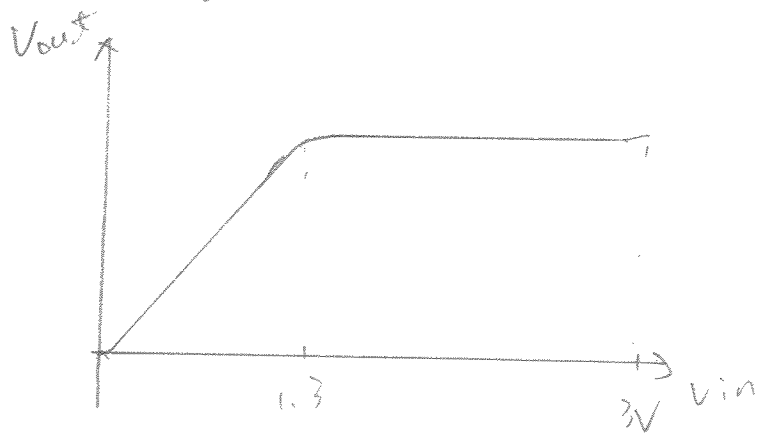


(b) $0 < v_{in} < 1.3 \Rightarrow \text{Triode}$

$$v_{OS} = v_{in} - v_{out}$$

$1.3 < v_{in} < 3 \Rightarrow \text{Sat}$

$v_{out} \text{ constant}$

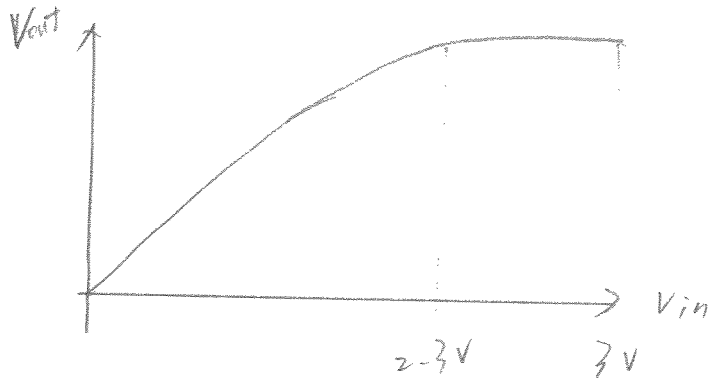


(C) $0 < V_{in} < 2.3 \Rightarrow$ Triode

$$V_{os} = V_{in} - V_{out}$$

$2.3 < V_{in} < 3 \Rightarrow$ Sat

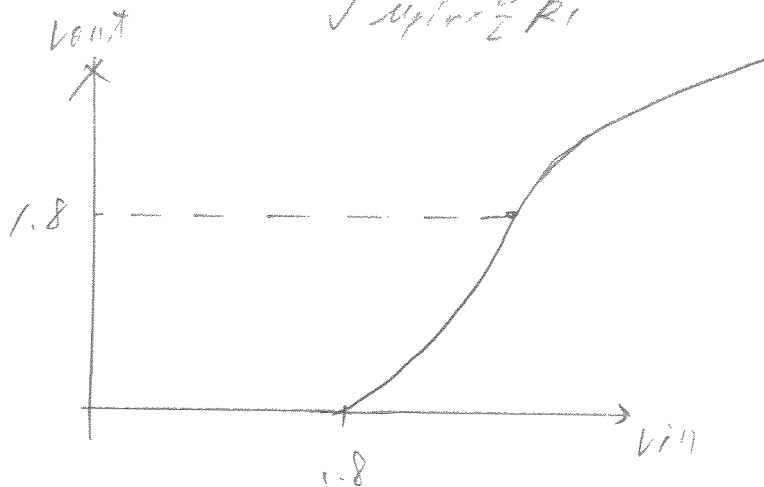
$$V_{out} = \text{constant}$$



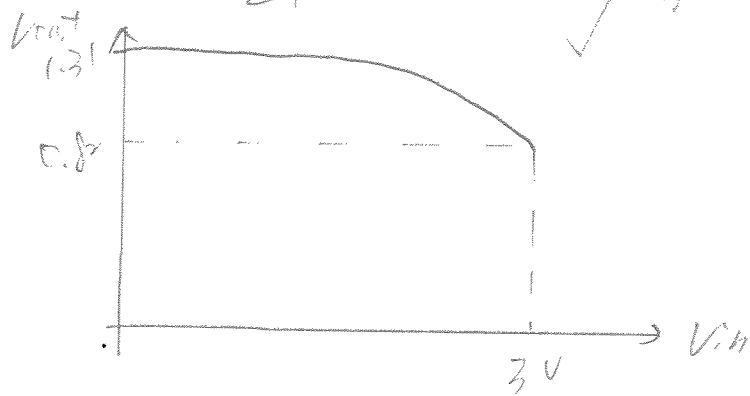
(D) $0 < V_{in} < 1.8 \Rightarrow$ OFF, $V_{out} = 0$

$$1.8 < V_{in} < 1.8 + \sqrt{\frac{2 \times 1.8}{\mu_p \left(\frac{W}{L} \right) R_1}} \Rightarrow \text{Sat}$$

$$V_{in} > 1.8 + \sqrt{\frac{2 \times 1.8}{\mu_p \left(\frac{W}{L} \right) R_1}} \Rightarrow \text{Triode}$$

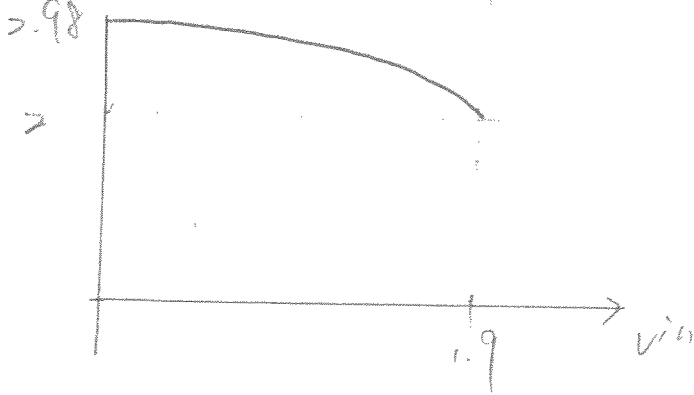


(a)
$$I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{out} - V_{TH0} - V)^2 \sqrt{I_D - I_{D0} - I_{D1} - I_{D2} - \sqrt{I_D}}$$



$\frac{W}{L} = \frac{50}{0.5}$ $I_D = 1 \text{ mA}$

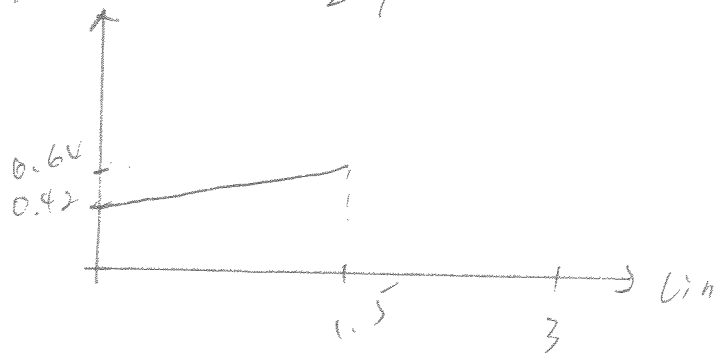
(b)
$$V_{out} = 3 - R_1 \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.3 - 0.45 (\sqrt{1.9 - V_{in}} - \sqrt{0.9}))^2$$



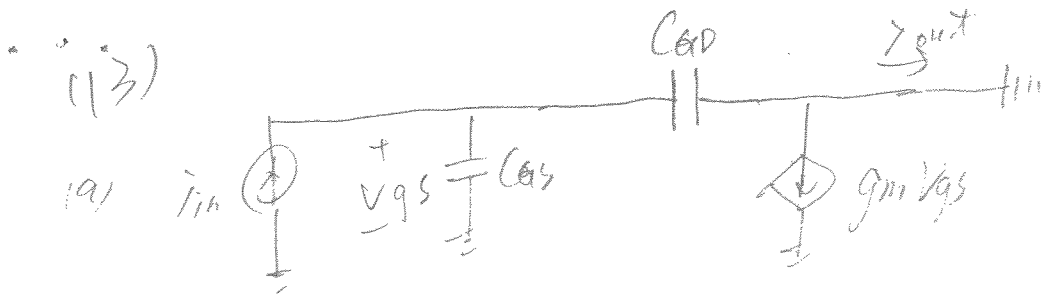
$\frac{W}{L} = \frac{50}{0.5}$

$R_1 = 0.2 \text{ k}$

(c)
$$\frac{V_{out}}{R_1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_{out} - 0.7 - 0.45 (\sqrt{0.9 - V_{out} - V_{in}} - \sqrt{0.9}))^2$$



$\frac{W}{L} = \frac{50}{0.5}$ $R_1 = 100 \text{ k}$



$$i_{in} = (C_{GS} + C_{GD}) s V_{GS}$$

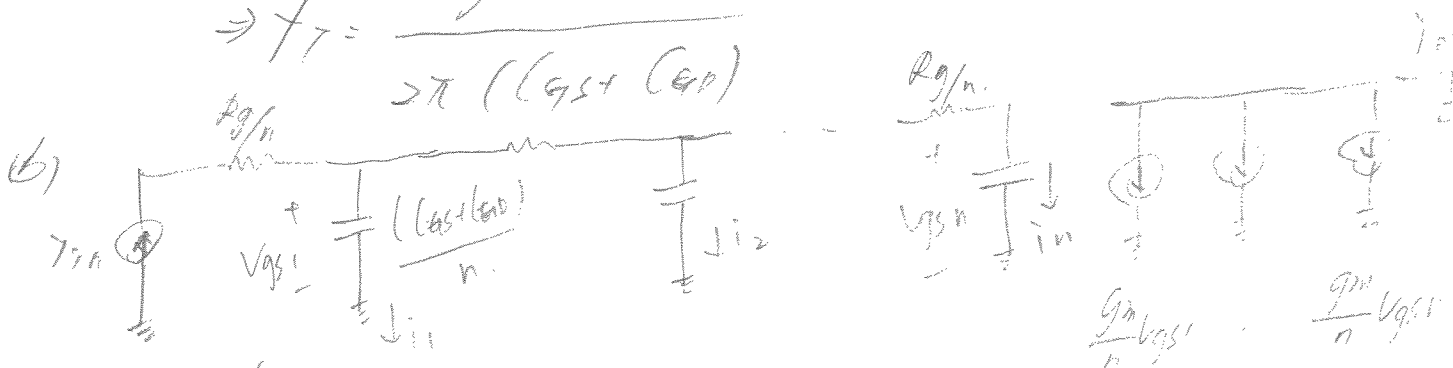
$$i_{out} = g_m V_{GS}$$

$$\Rightarrow \beta = \frac{i_{out}}{i_{in}} = \frac{g_m}{(C_{GS} + C_{GD}) s}$$

By definition $|\beta| = 1 \Rightarrow \frac{g_m}{\omega_T (C_{GS} + C_{GD})} = 1$

$$\Rightarrow \omega_T = \frac{g_m}{C_{GS} + C_{GD}}$$

$$\Rightarrow f_T = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$



$$i_{K1} = \frac{1}{n} (C_{GS} + C_{GD}) s V_{GS1} \quad L=1$$

$$i_{in} = i_{K1} + i_{K2} + \dots + i_{in} = \frac{1}{n} (C_{GS} + C_{GD}) s (V_{GS1} + V_{GS2} + \dots + V_{GSn})$$

$$i_{out} = \frac{g_m}{n} V_{GS1} + \dots + \frac{g_m}{n} V_{GSn} = \frac{g_m}{n} (V_{GS1} + \dots + V_{GSn})$$

$$\Rightarrow \beta = \frac{i_{out}}{i_{in}} = \frac{g_m}{(C_{GS} + C_{GD}) s} \Rightarrow \omega_T = \frac{g_m}{C_{GS} + C_{GD}}$$

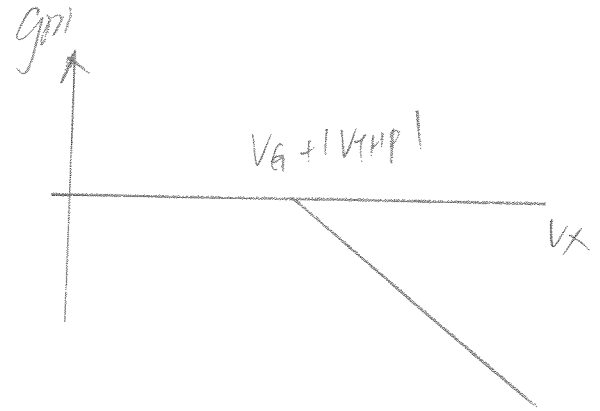
$$\dots (c) f_T = \frac{g_{m1}}{2\pi (C_{GS} + C_{GD})}$$

$$g_{m1} = \mu_1 \left(\text{cox} \frac{W}{L} (V_{GS} - V_{TH}) \right)$$

$$C_{GS} + C_{GD} \approx \text{cox} \frac{W}{L}$$

$$\Rightarrow f_T = \frac{\mu_1 \left(\text{cox} \frac{W}{L} (V_{GS} - V_{TH}) \right)}{2\pi \text{cox} W \cdot L} \approx \frac{\mu_1}{2\pi} \frac{(V_{GS} - V_{TH})}{L^2}$$

24 (a) (Case I : $V_G < V_{THN}$)

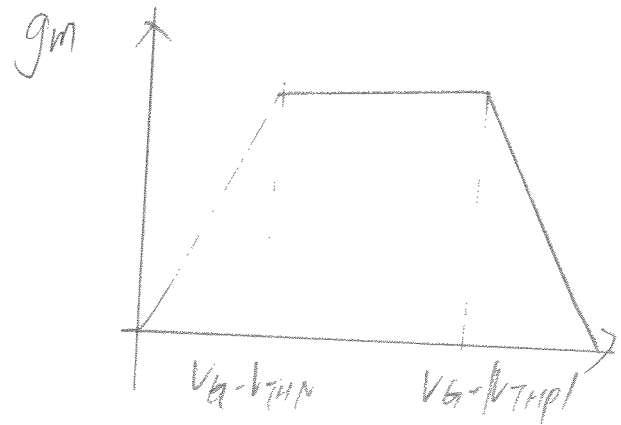
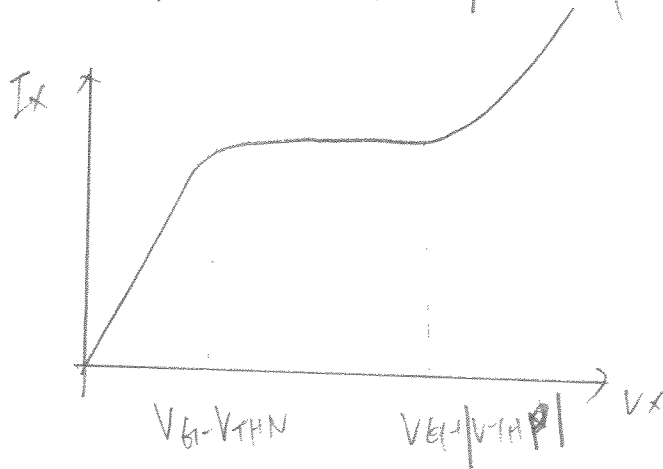


Case II : $V_G > V_{THN}$

$0 < V_X < V_G - V_{THN} \Rightarrow (M_2 \text{ off, } M_1 \text{ triode})$

$V_G - V_{THN} < V_X < V_G + |V_{THP}| \Rightarrow (M_2 \text{ off, } M_1 \text{ sat})$

$V_X > V_G + |V_{THP}| \Rightarrow (M_1, M_2 \text{ sat})$



1b). Case I $V_G < V_{THN} \Rightarrow \mu_n \text{ off, } \begin{cases} I_x = 0 \\ g_m = 0 \end{cases}$

Case II $V_G > V_{THN}$

$0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0. (\mu_n \text{ off! } g_m = 0)$

Case III

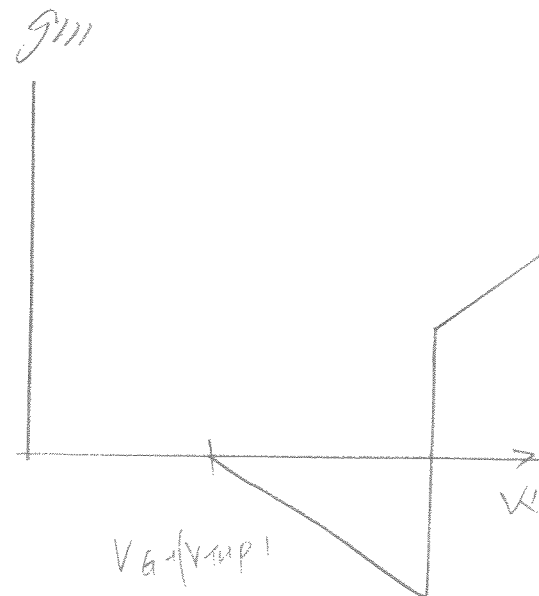
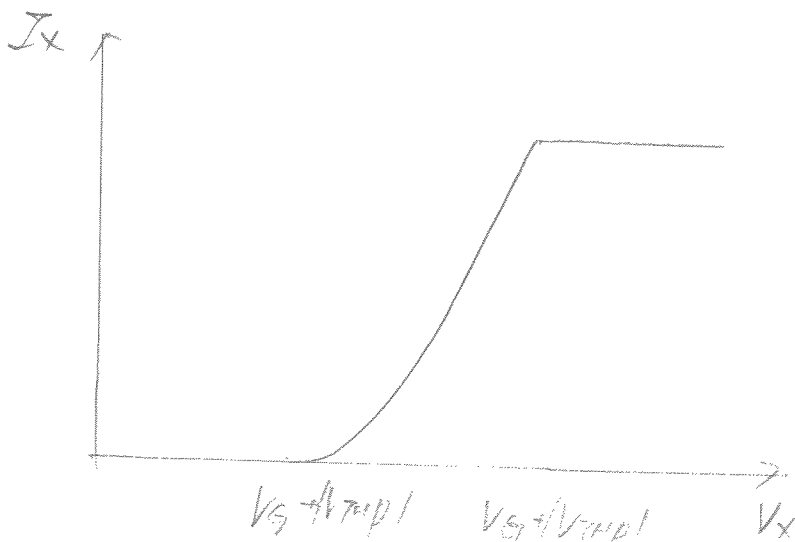
$$V_G + |V_{THP}| < V_X < V_G + |V_{THP}| + \sqrt{\frac{\mu_n (W/L)_n}{\mu_p (W/L)_p} (V_G - V_{THN})}$$

(μ_n sat. μ_p triode)

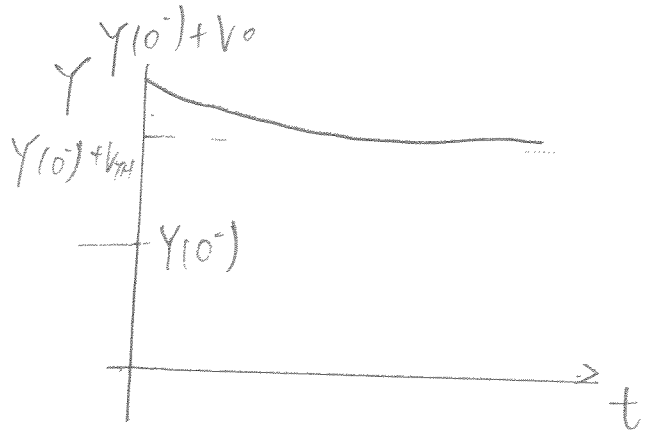
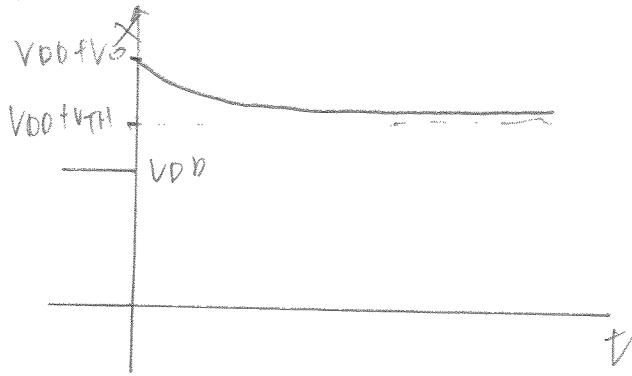
Case IV

$$V_X > V_G + |V_{THP}| + \sqrt{\frac{\mu_n (W/L)_n}{\mu_p (W/L)_p} (V_G - V_{THN})}$$

(μ_n sat. μ_p sat)



(2) 6) 1a)



1b)

