

CSCE 222
Discrete Structures for Computing

Turing Machines

Hyunyoung Lee

Chomsky Hierarchy

Type 0 – Phrase-structure Grammars

Type 1 – Context-Sensitive

$a^n b^n c^n$ (Turing Machines)

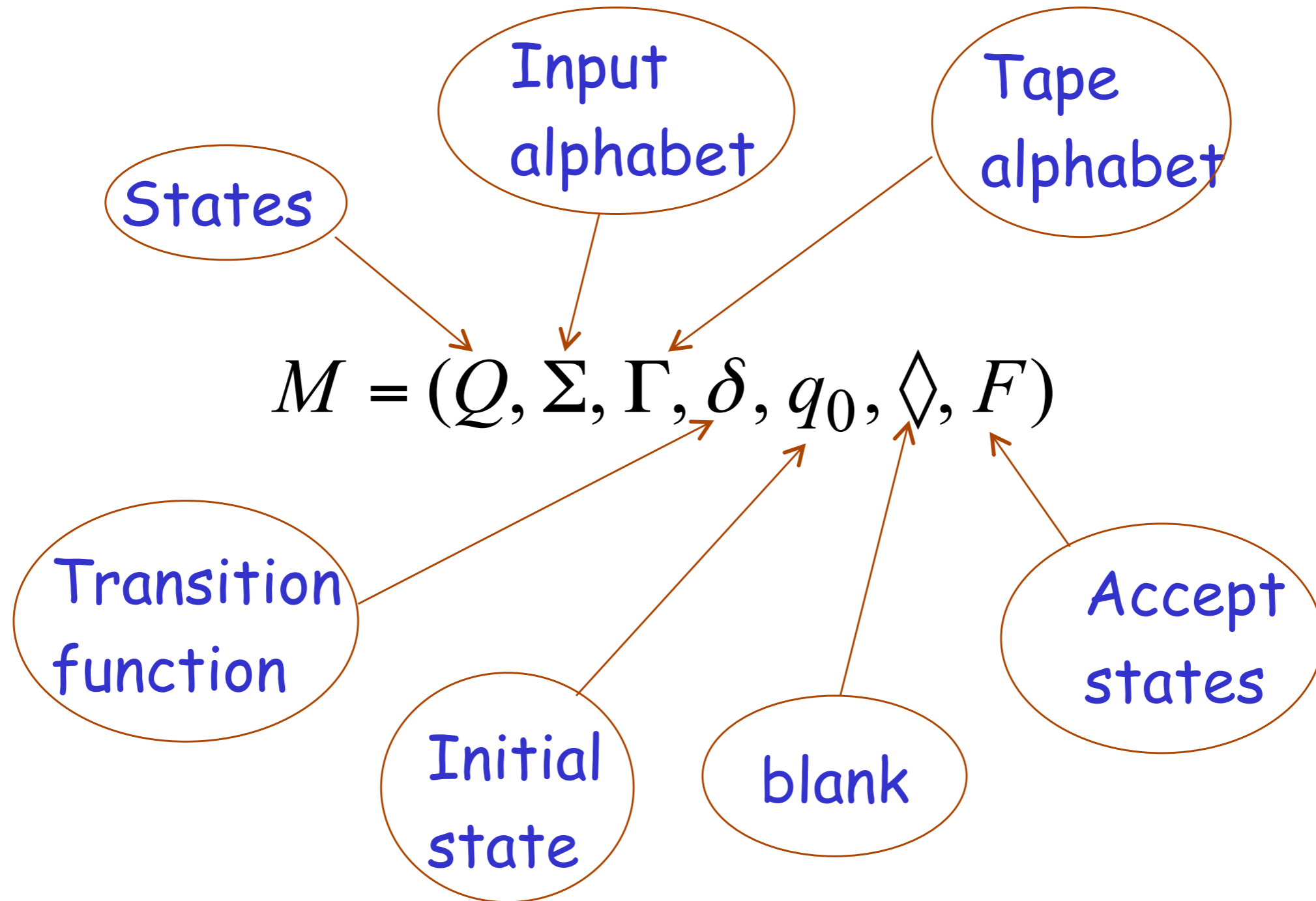
Type 2 – Context-Free

$a^n b^n$ (Pushdown automata)

Type 3 – Regular

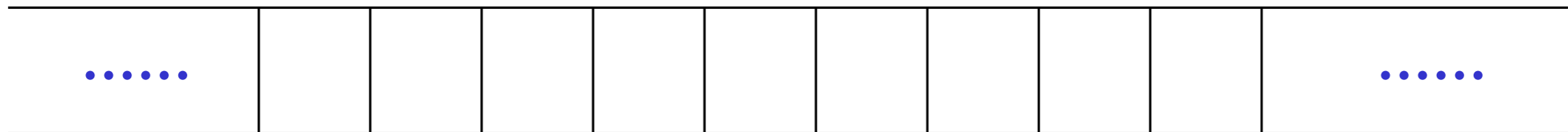
$a^* b^*$ (FSA)

A Turing Machine



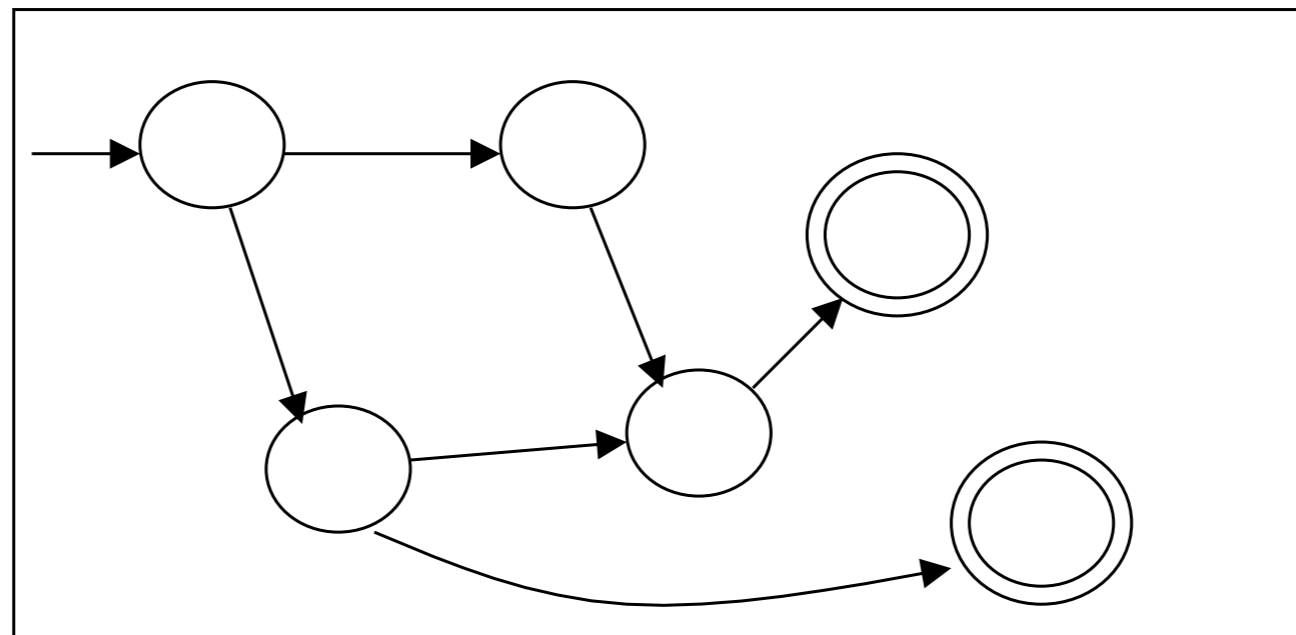
A Turing Machine

Tape — Infinite length

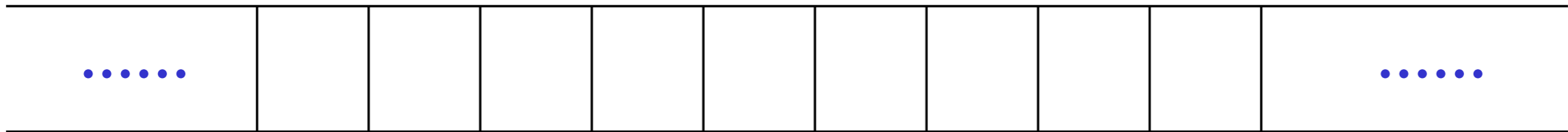


Read-Write head The head moves Left or Right

Control Unit



At Each Time Step

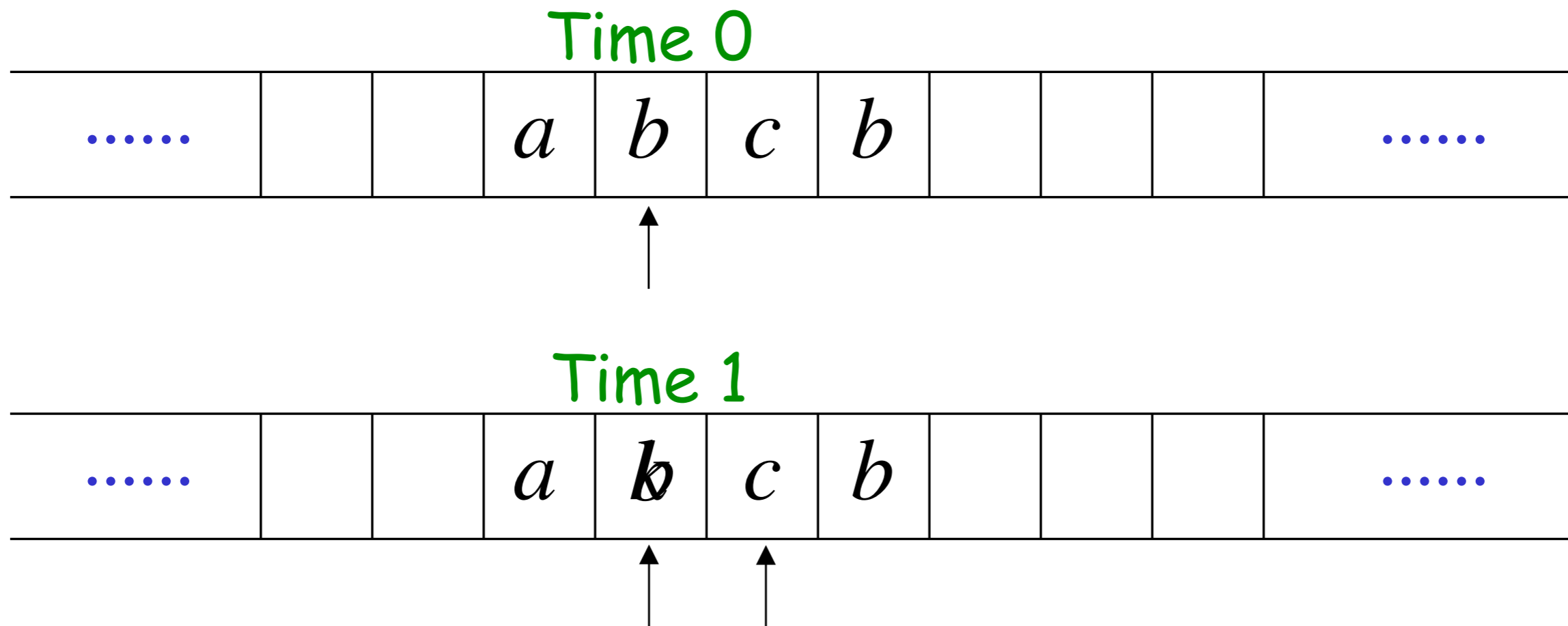


Read-Write head

The head at each transition (time step):

1. Reads a symbol
2. Writes a symbol
3. Moves Left or Right

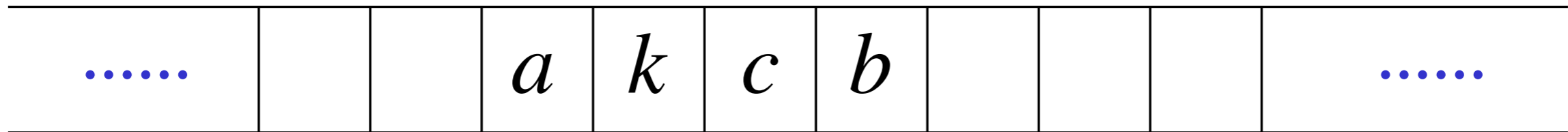
Example 1



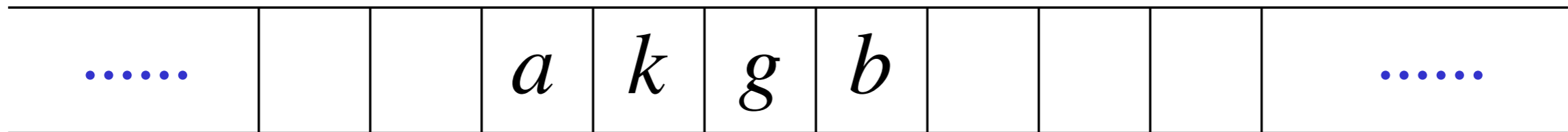
1. Reads *b*
2. Writes *k*
3. Moves Right

Example 1 (Cont.)

Time 1

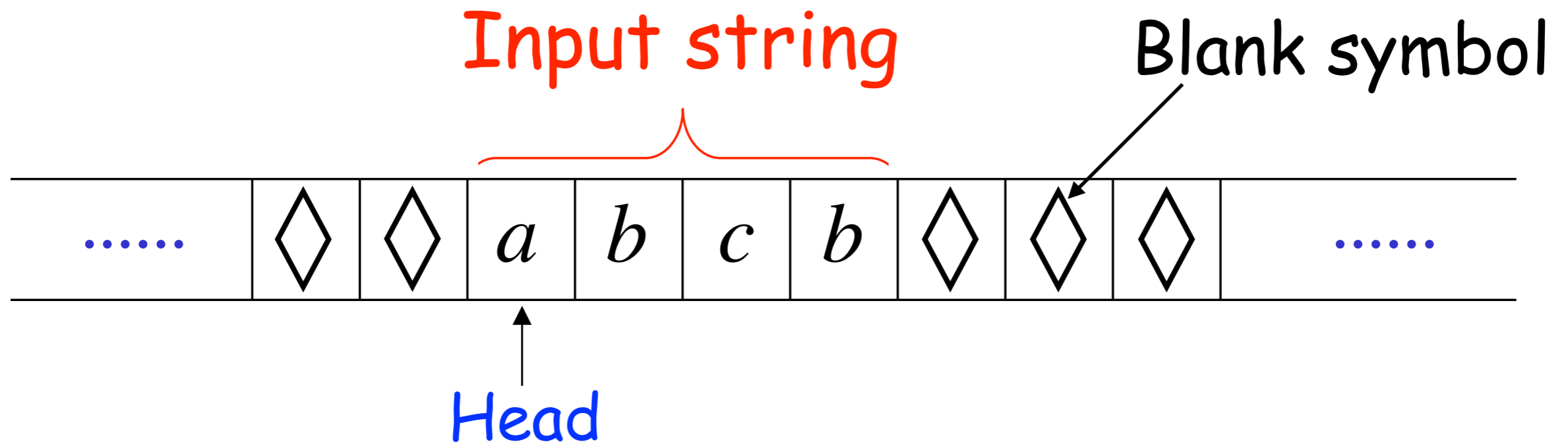


Time 2



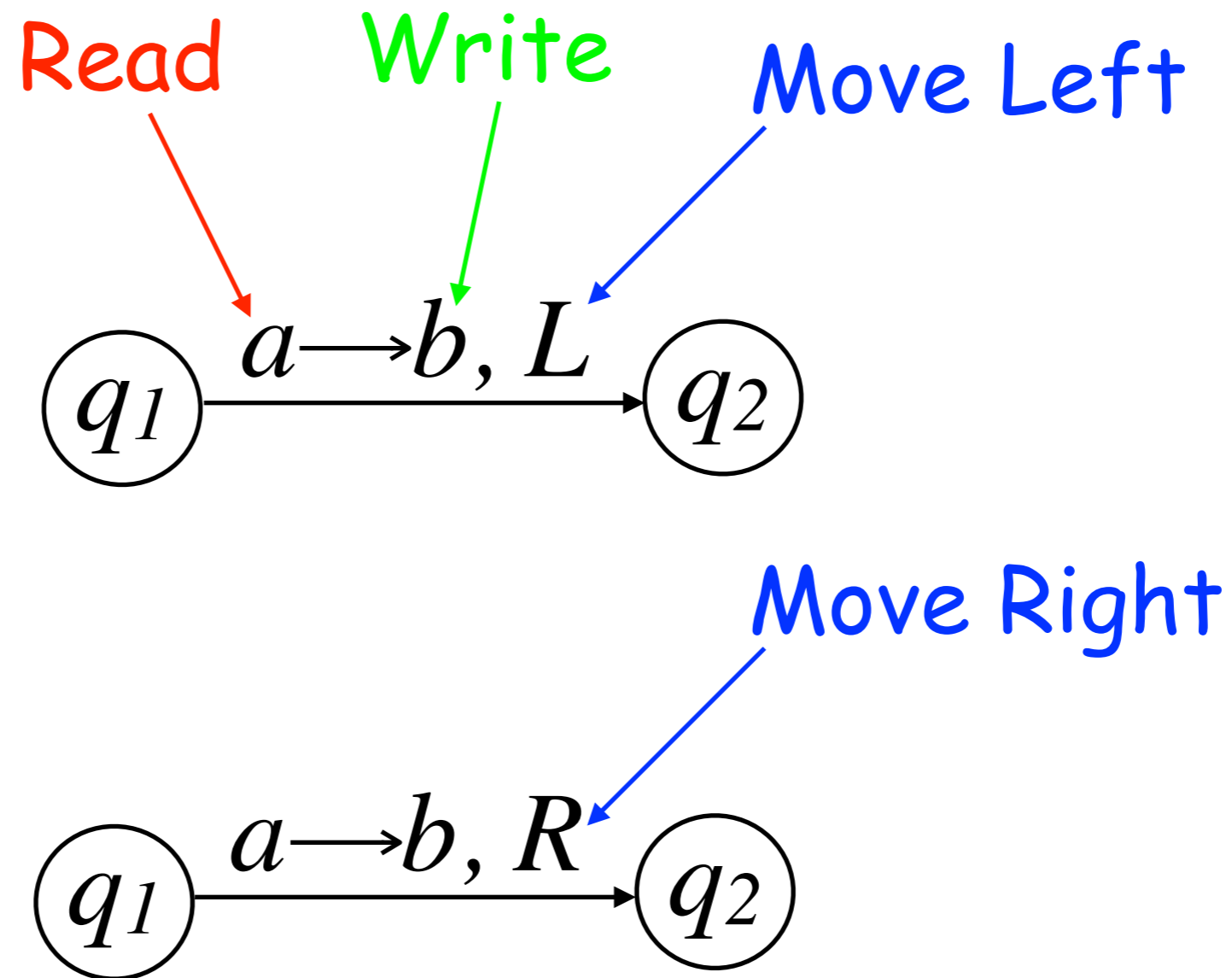
1. Reads c
2. Writes g
3. Moves Left

The Input String



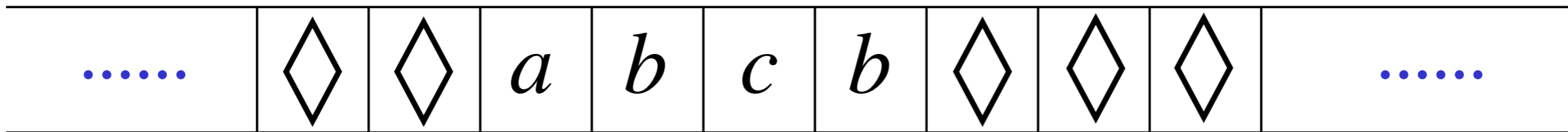
Head starts at the leftmost position of the input string

States and Transitions



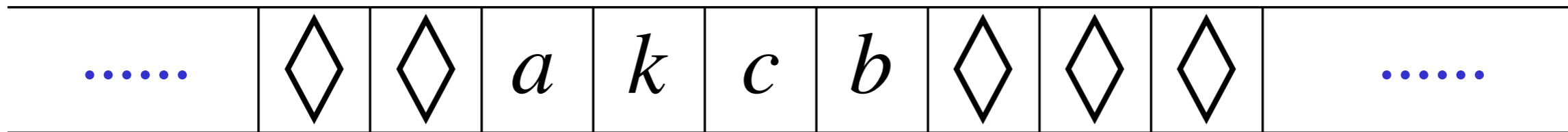
Example 2

Time 1

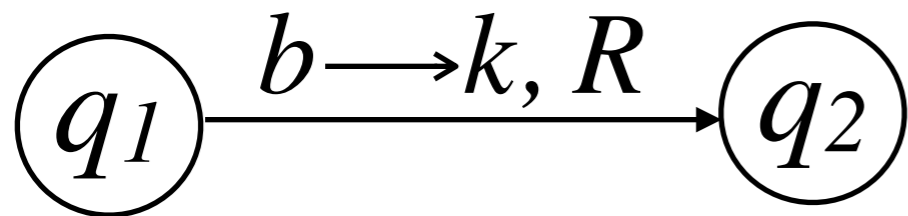


q_1
current state

Time 2

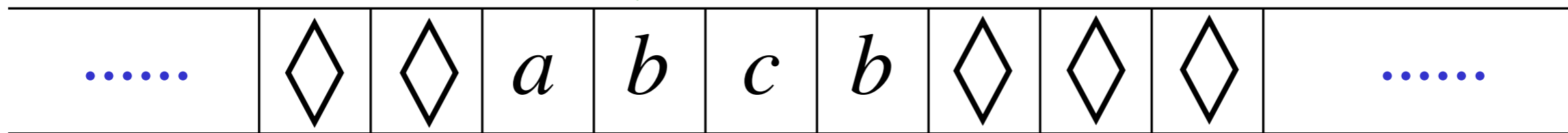


q_2



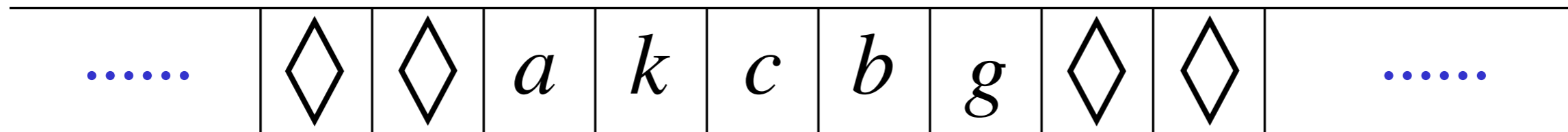
Example 3

Time 1

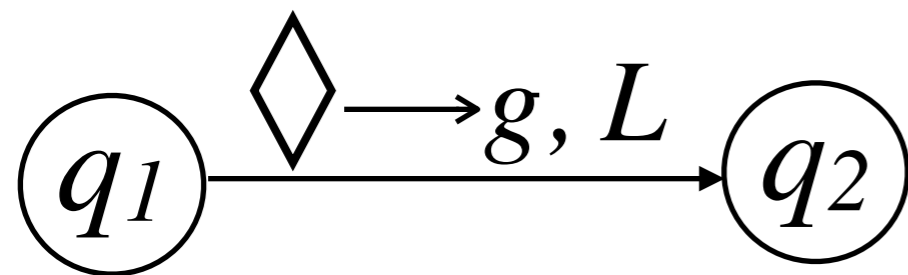


q_1

Time 2

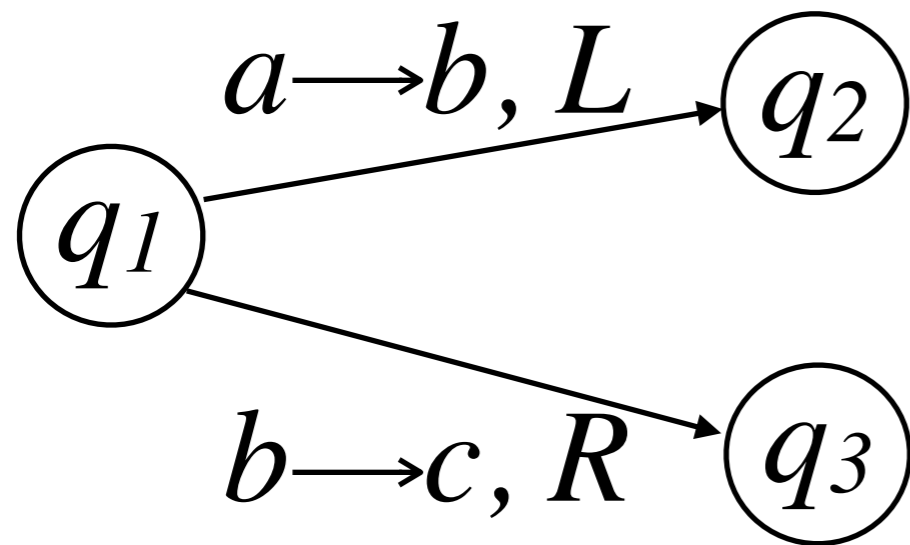


q_2

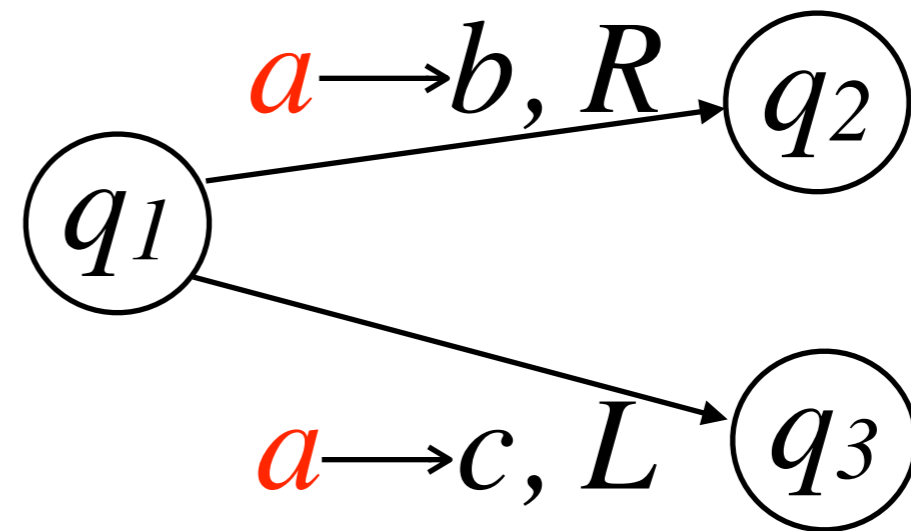


Turing Machines Are Deterministic

Allowed



Not Allowed

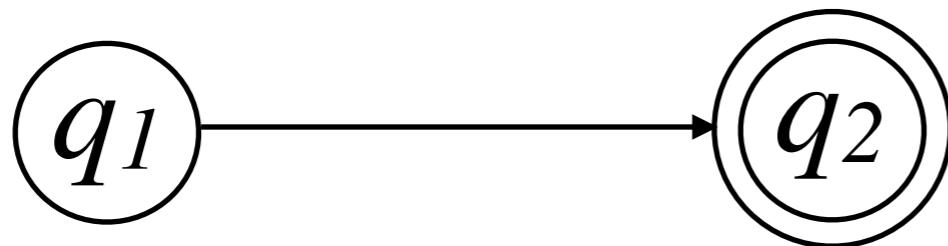


Halting

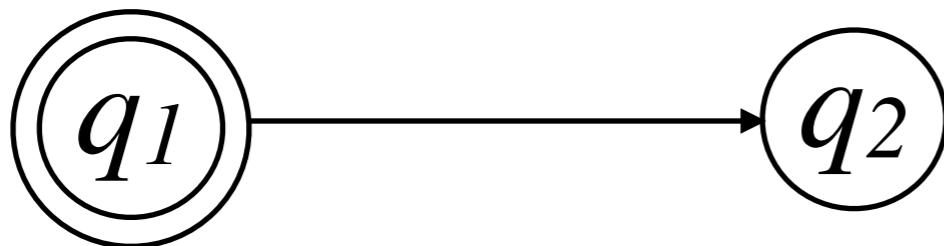
The machine halts in a state if there is no transition to follow:

- No possible transition from the current state with the current symbol.
- The machine reaches an accepting state.

Accepting States



Allowed



Not Allowed

- Accepting states have no outgoing transitions.
- The machine halts and accepts.

Acceptance

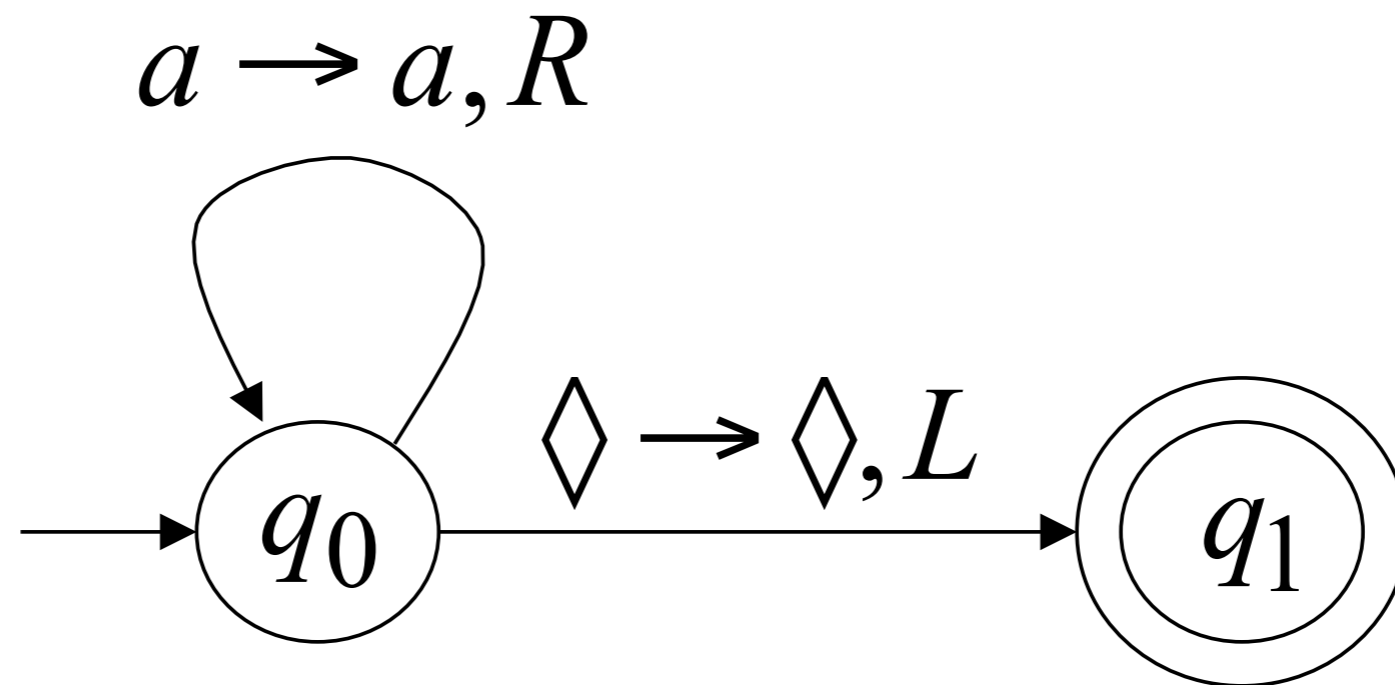
If machine halts in an **accepting** state, then it **accepts** the input string.

If machine halts in a **non-accepting** state or machine enters an **infinite loop**, then it **rejects** the input string.

Turing Machine Example 1

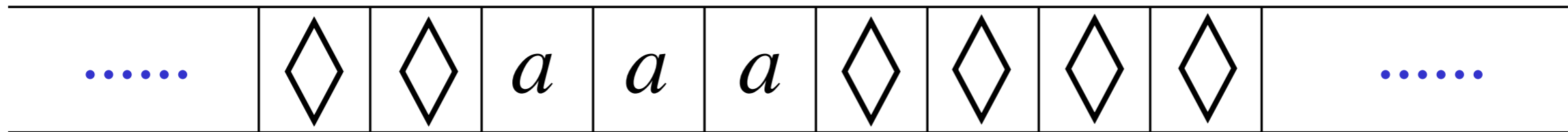
Input alphabet $\Sigma = \{a, b\}$

Accepts the language: a^*

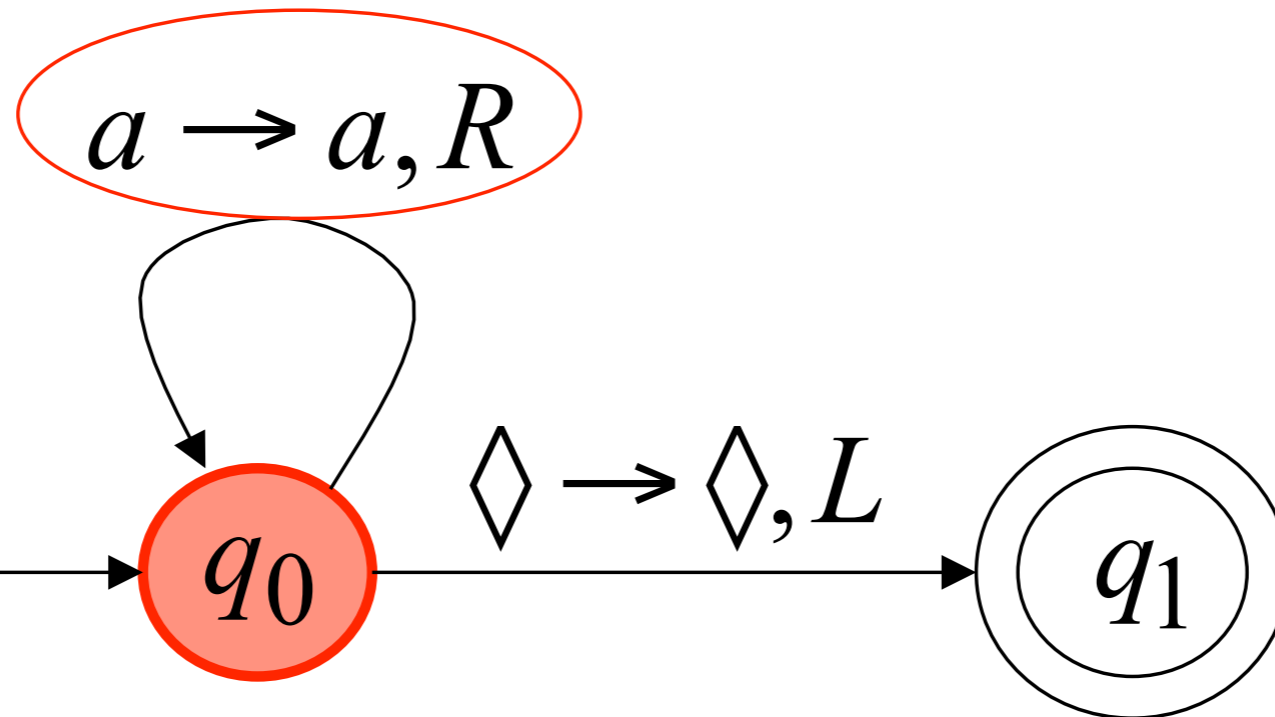


TM that accepts a^*

Time 0

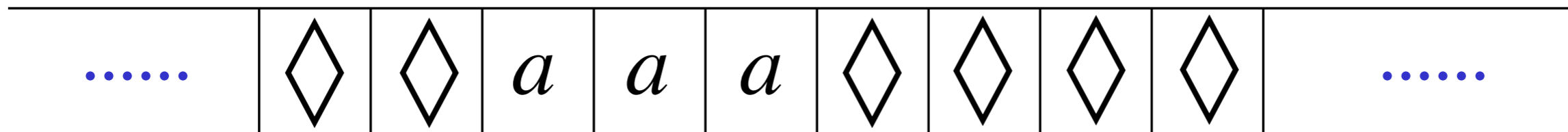


q_0



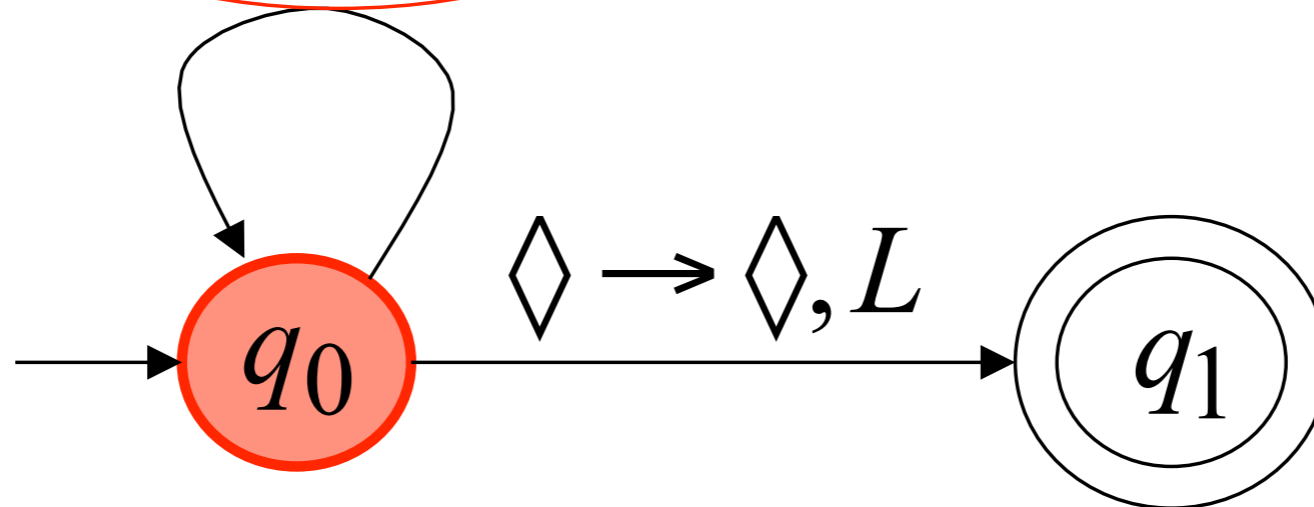
TM that accepts a^* (Cont.)

Time 1



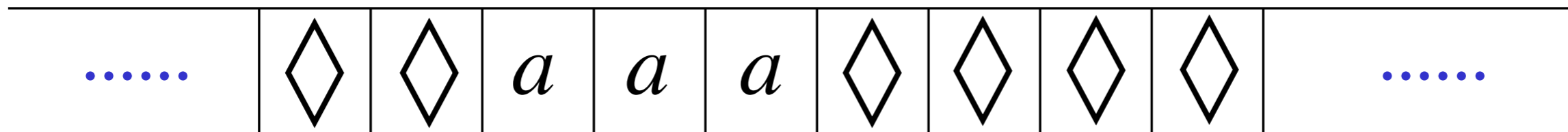
q_0

$a \rightarrow a, R$



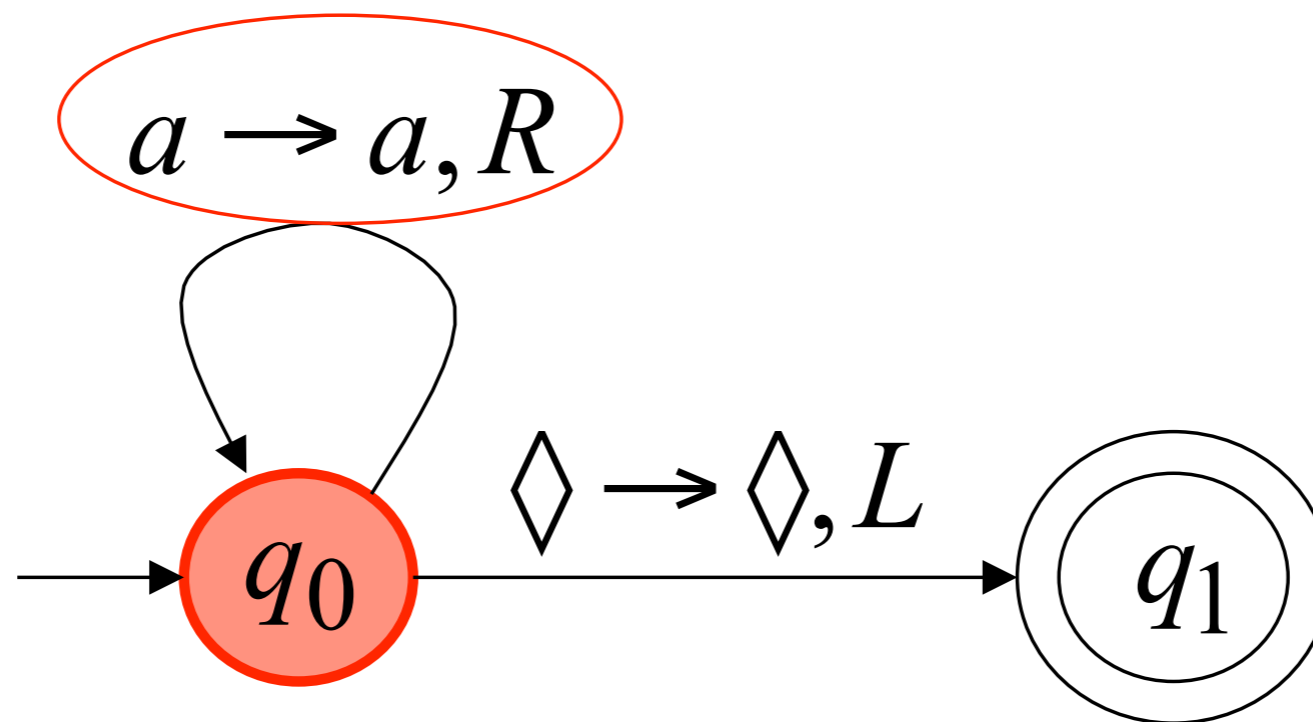
TM that accepts a^* (Cont.)

Time 2



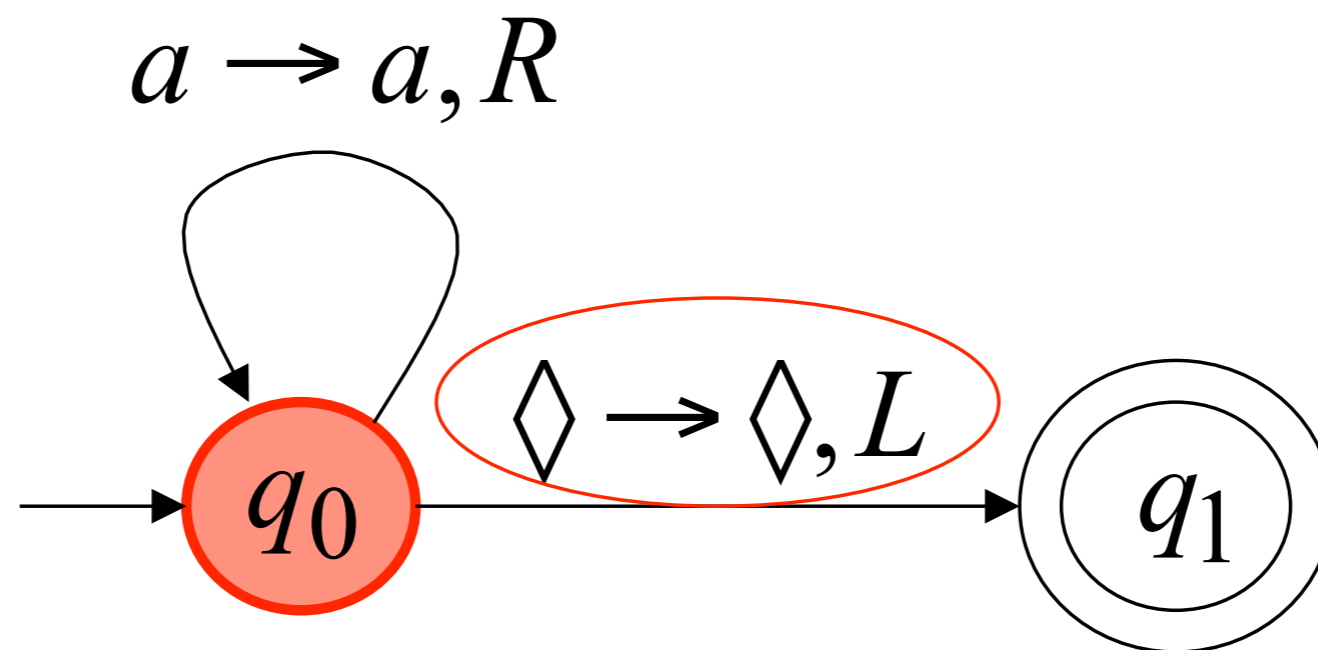
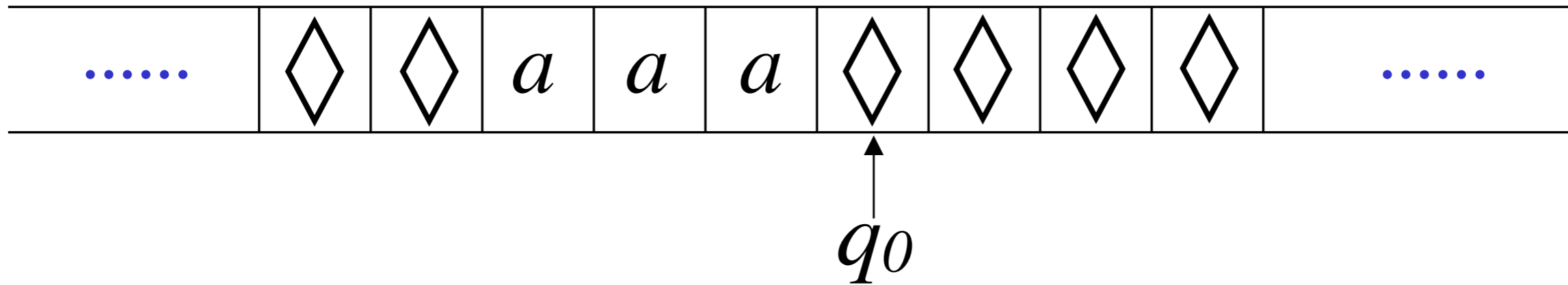
q_0

An upward-pointing arrow from the state label q_0 to the third 'a' on the tape.



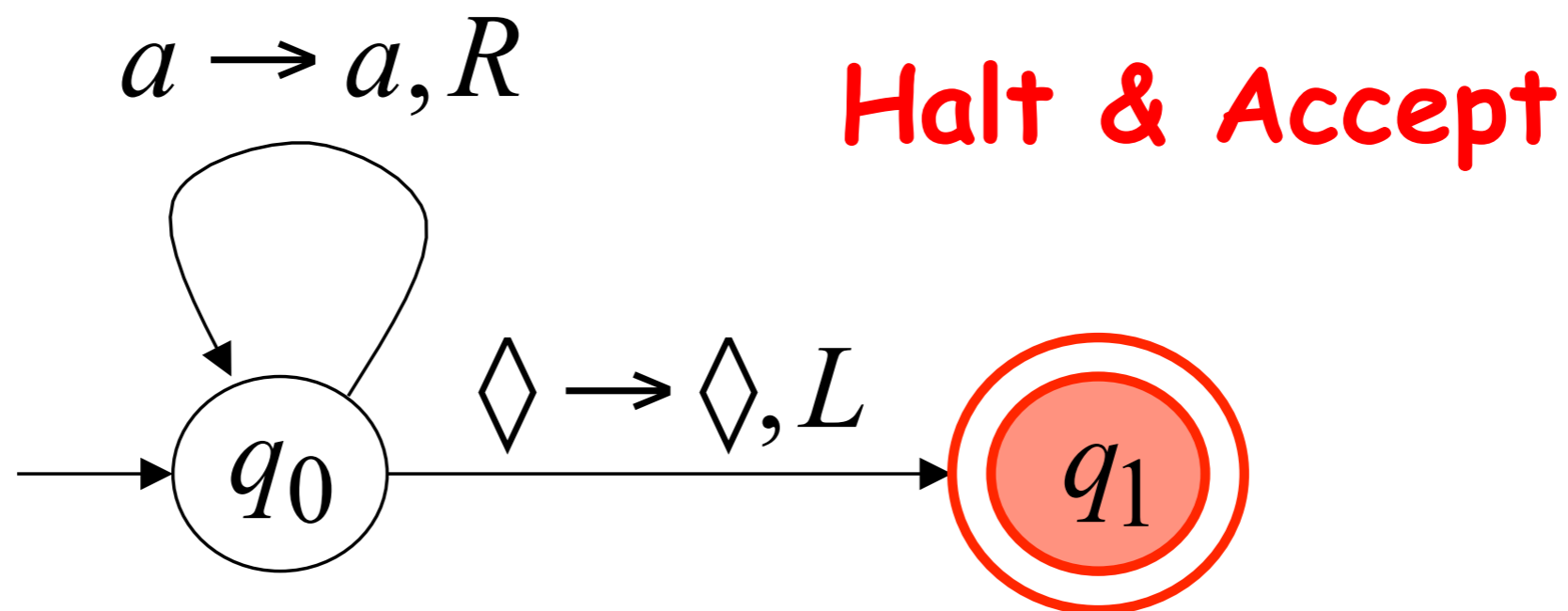
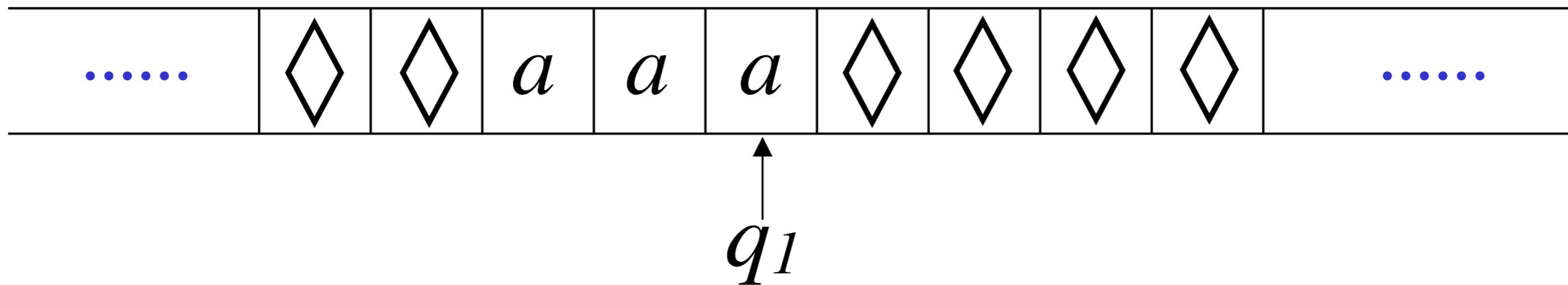
TM that accepts a^* (Cont.)

Time 3



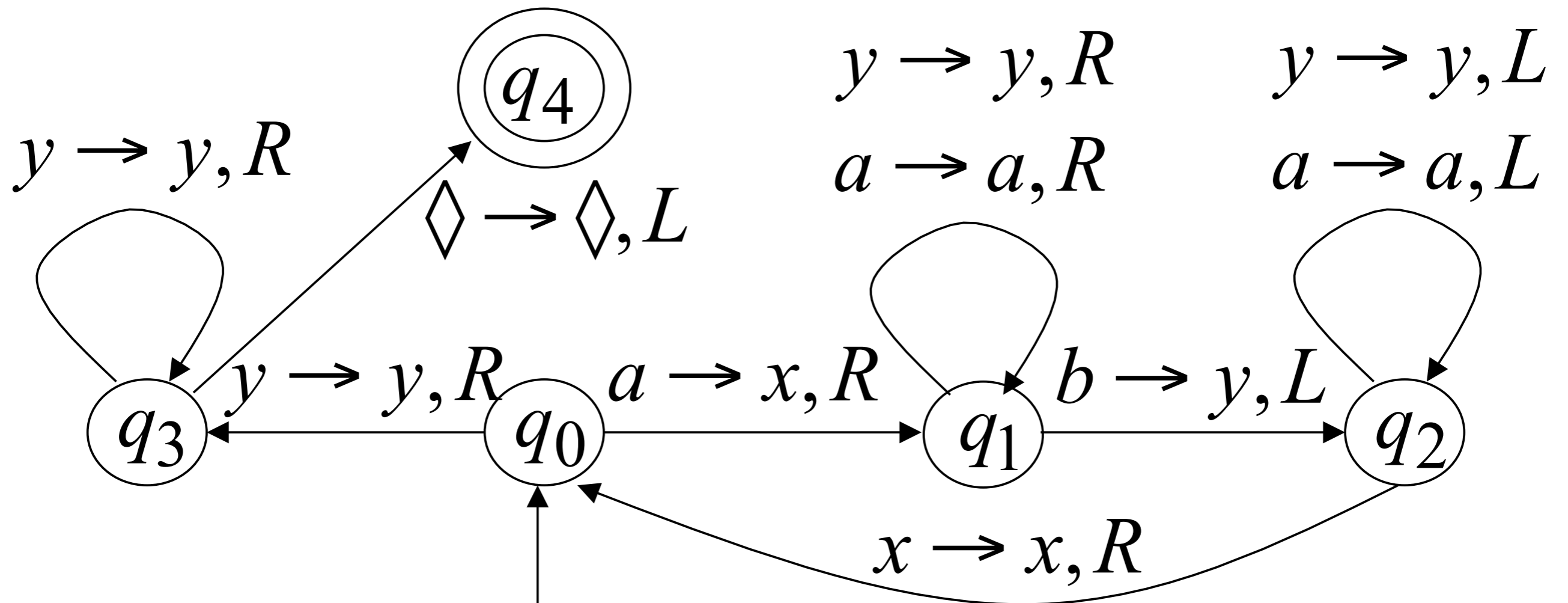
TM that accepts a^* (Cont.)

Time 4



Turing Machine Example 2

Turing machine for the language $\{a^n b^n \mid n \geq 1\}$



Turing Machine for $\{a^n b^n \mid n \geq 1\}$

Basic idea:

Match **a**'s with **b**'s:

Repeat

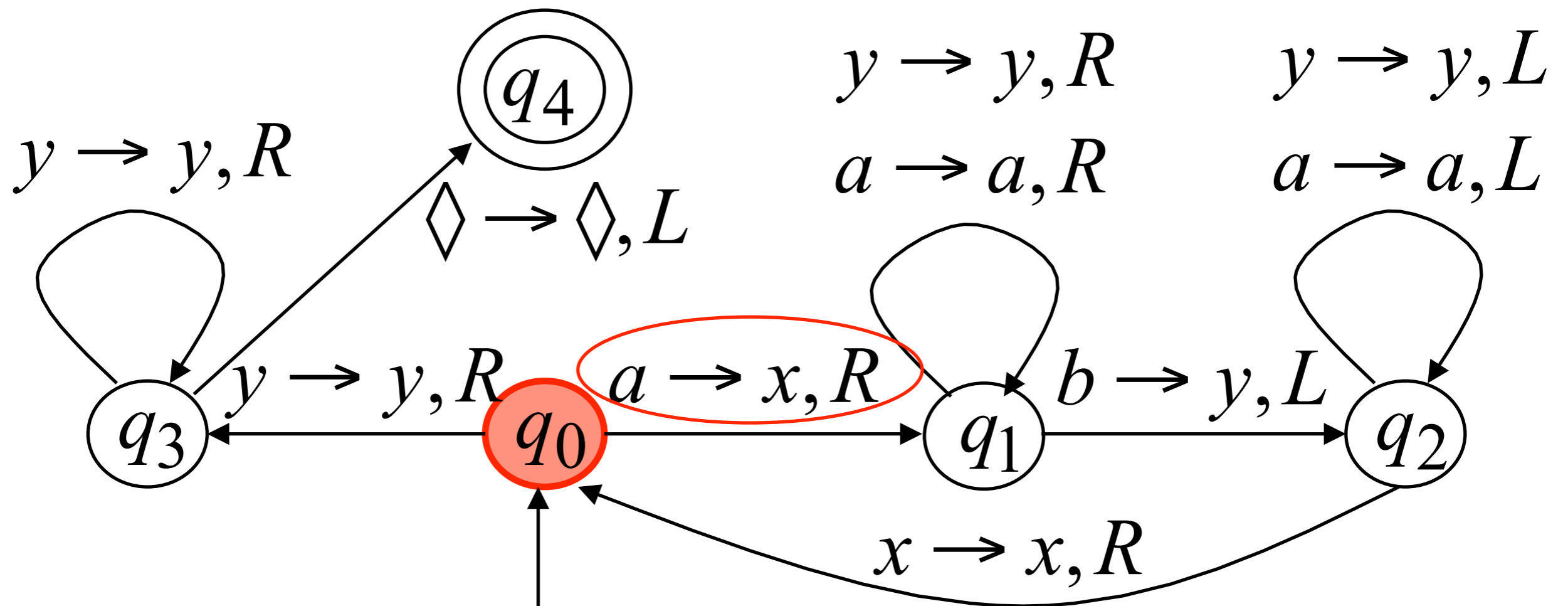
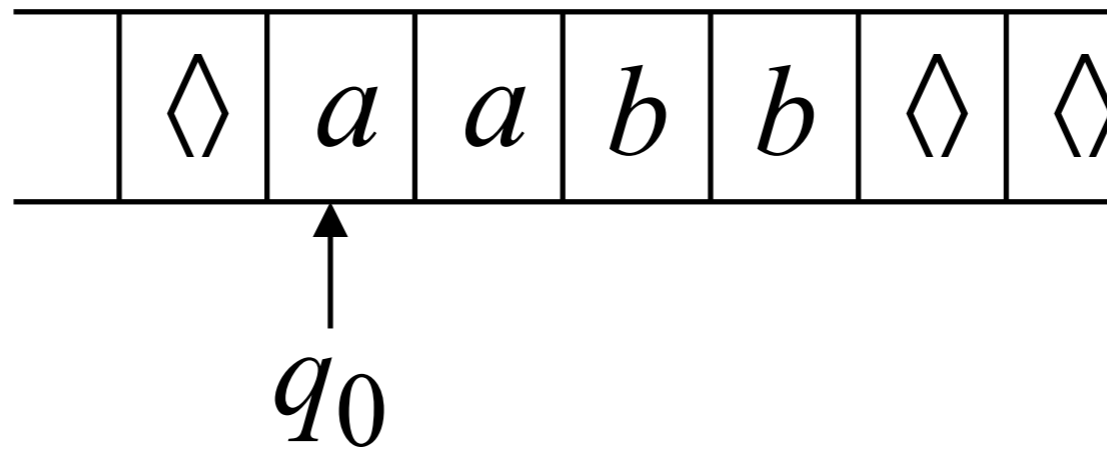
replace leftmost **a** with **x**

find leftmost **b** and replace it with **y**

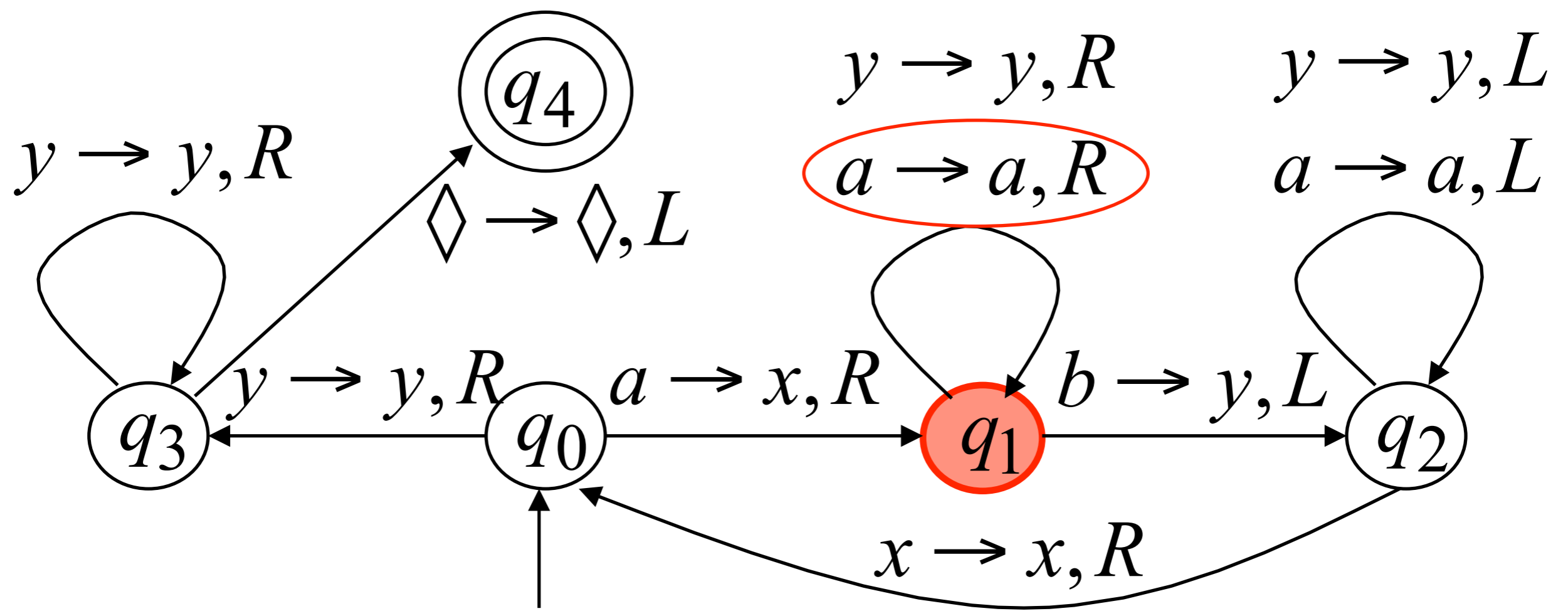
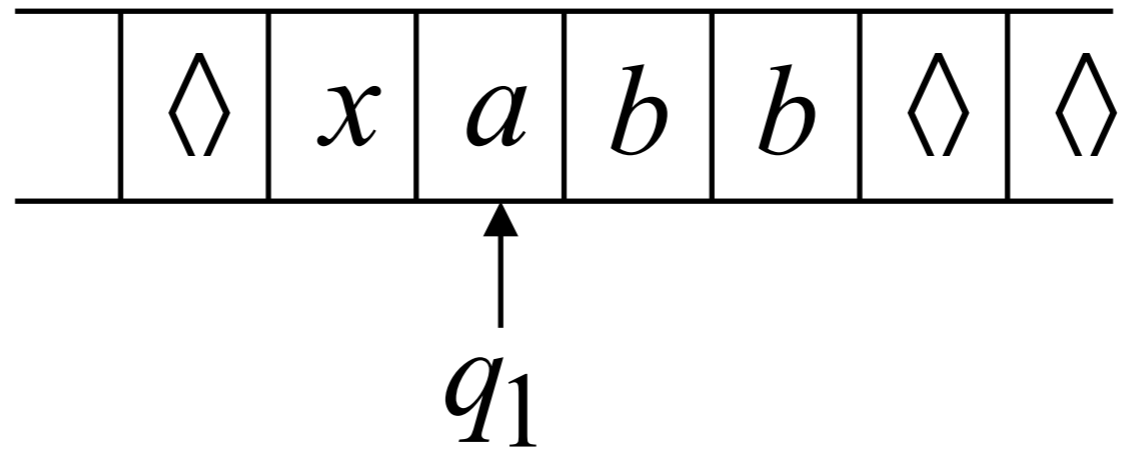
Until there are no more **a**'s or **b**'s

If there is a remaining **a** or **b**, then reject

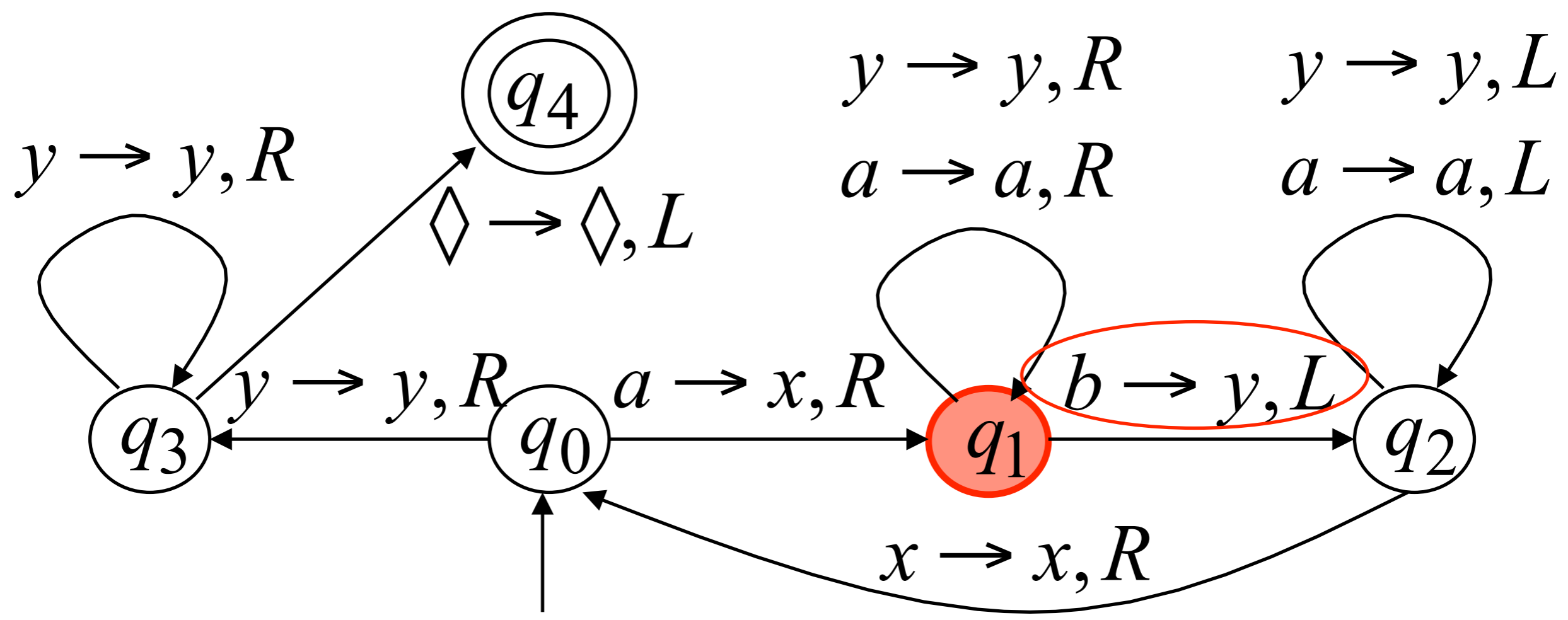
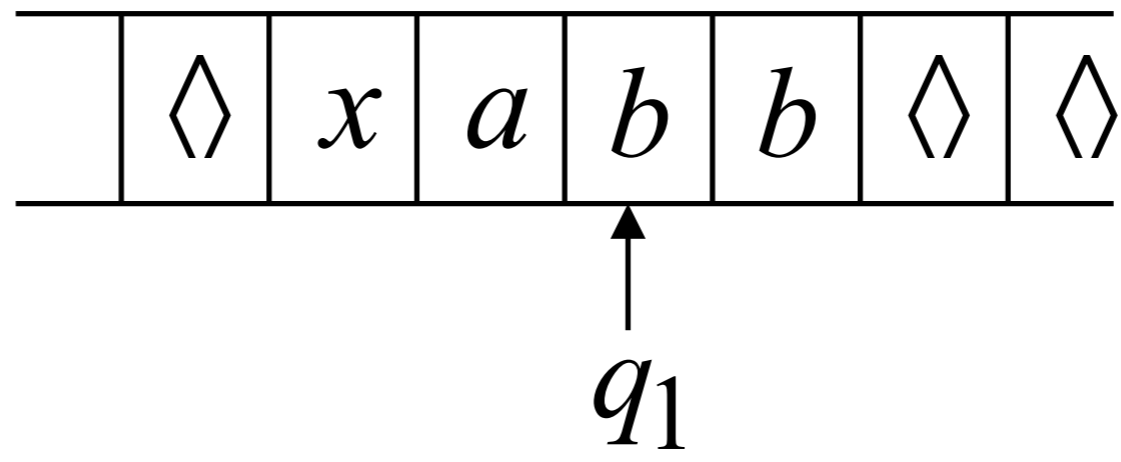
Time 0



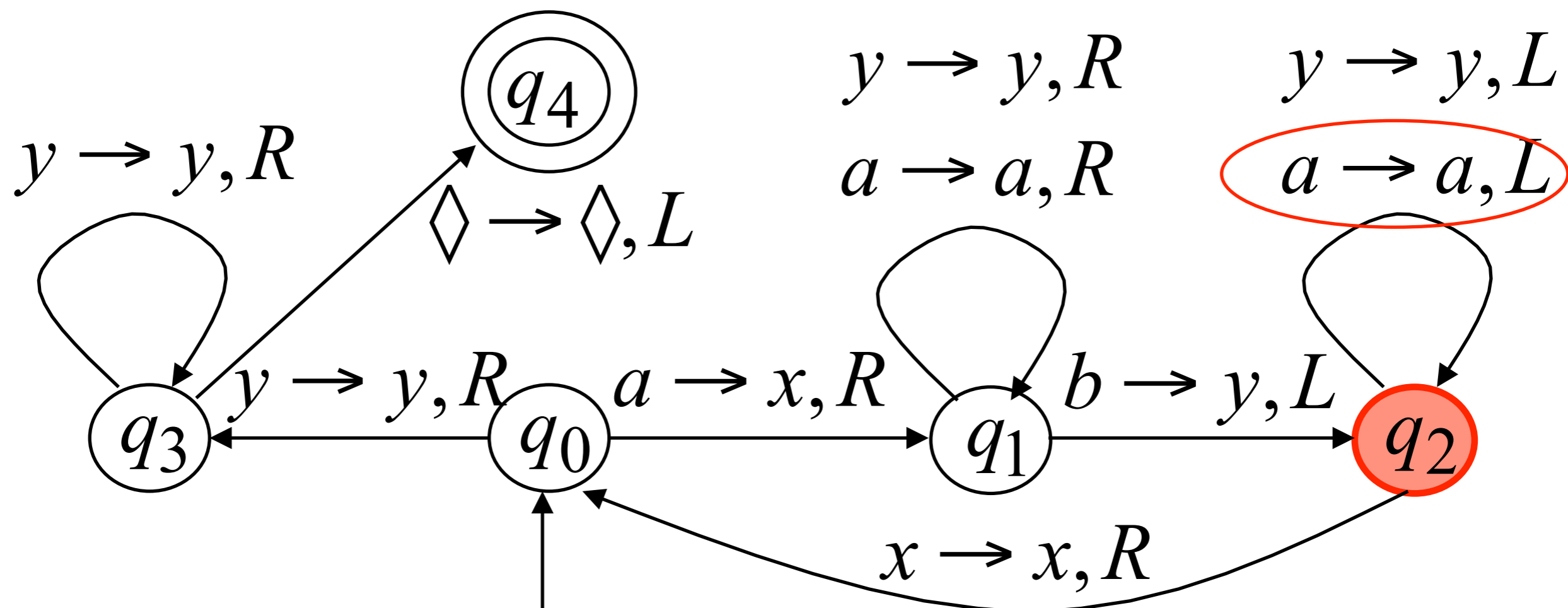
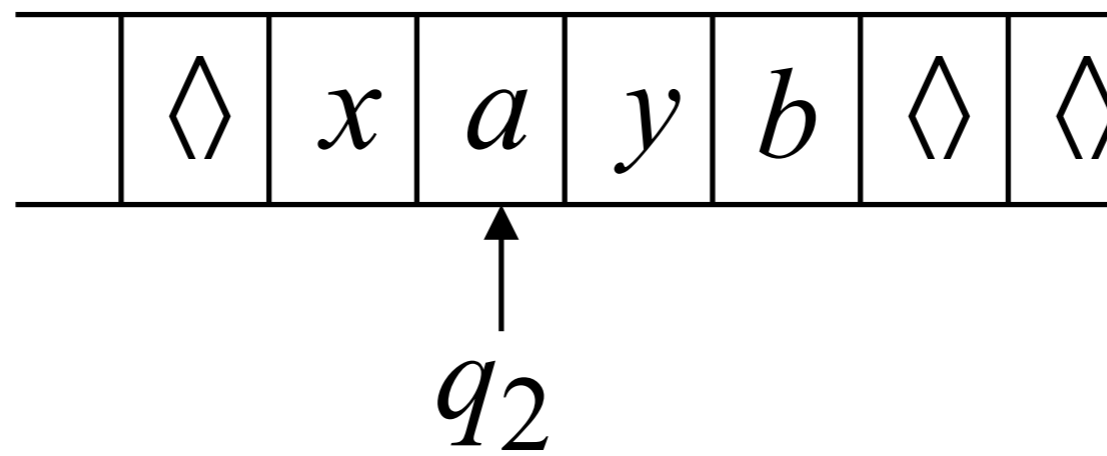
Time 1



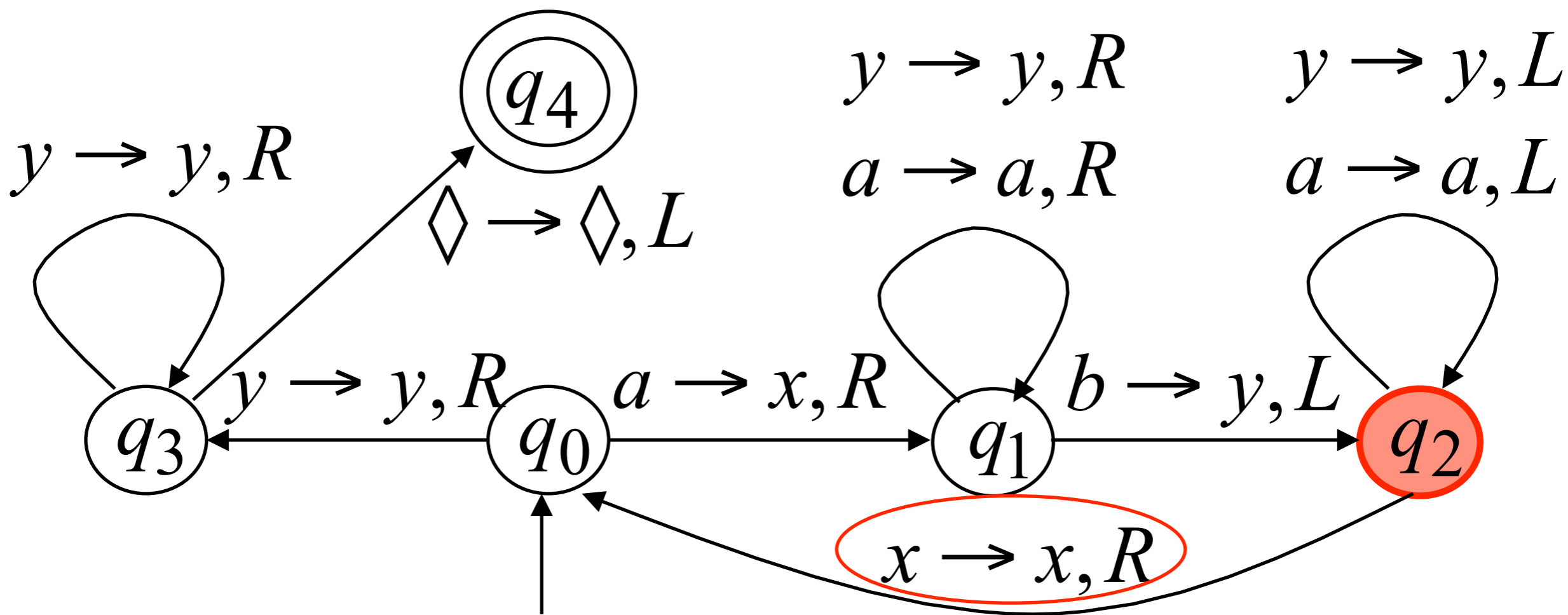
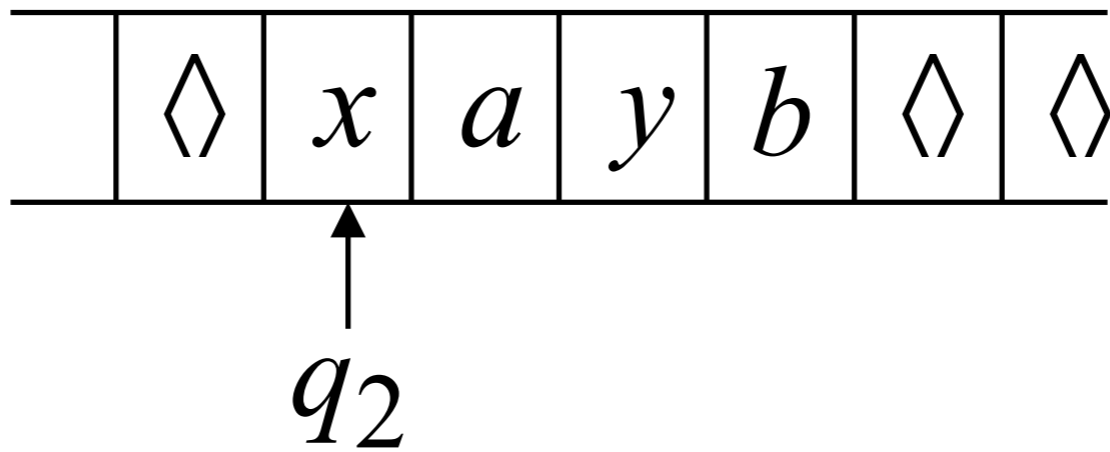
Time 2



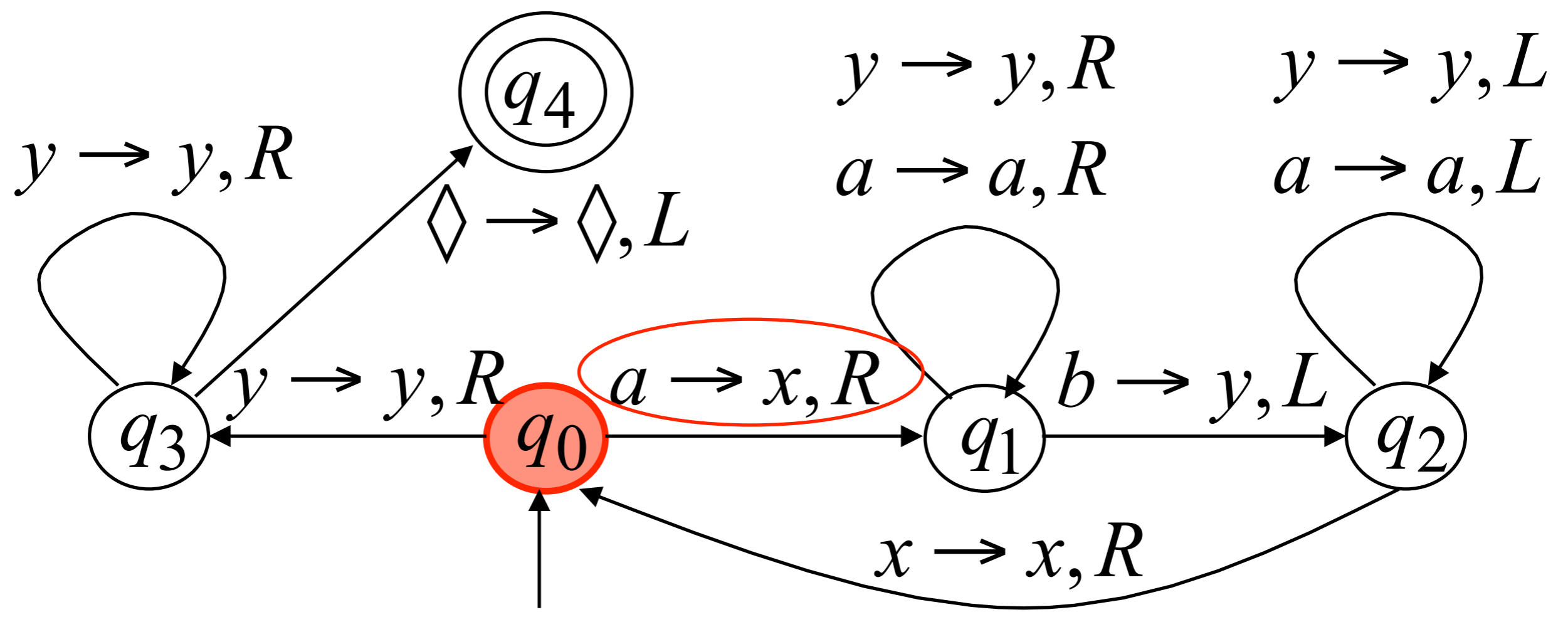
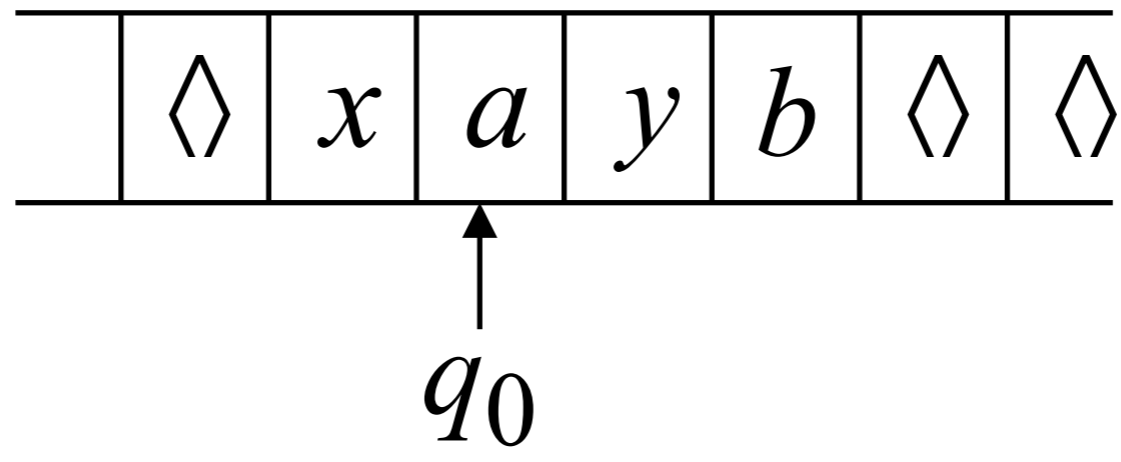
Time 3



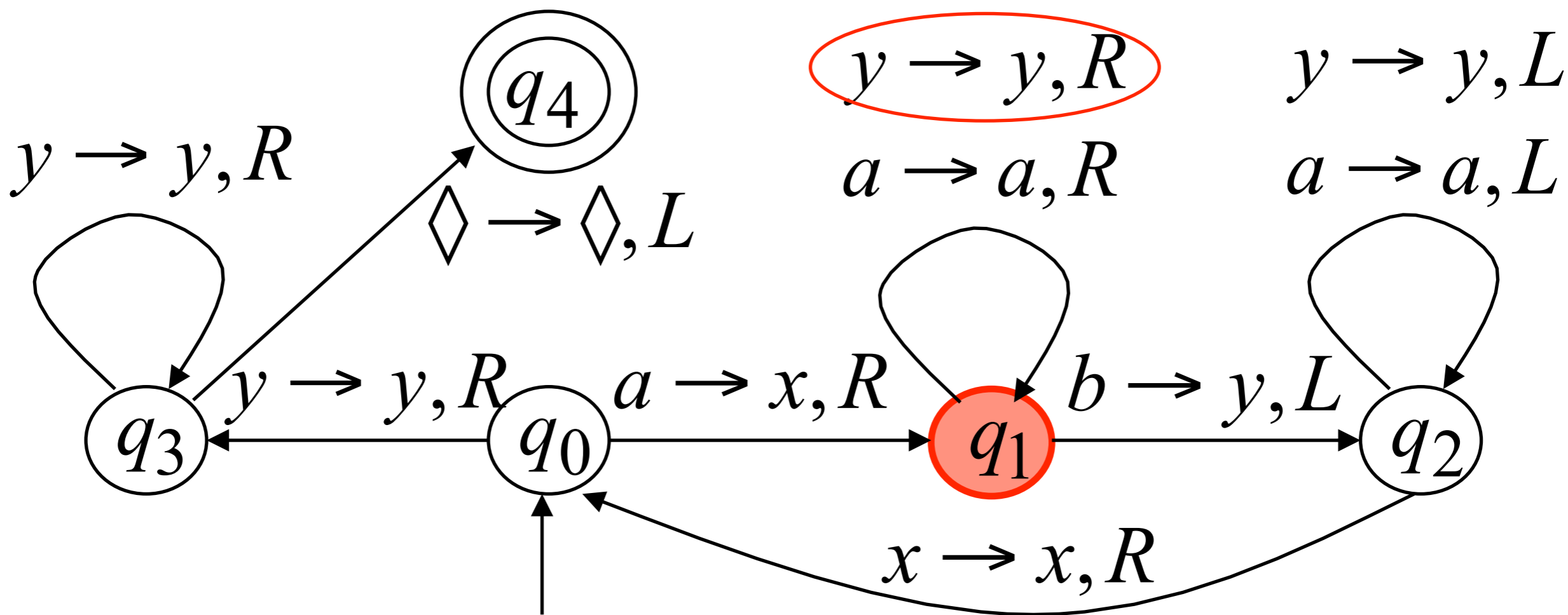
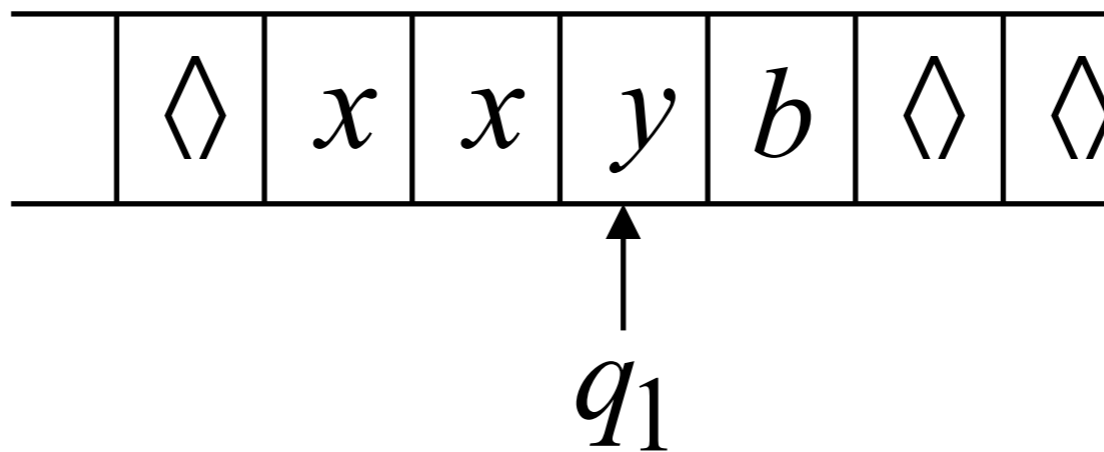
Time 4



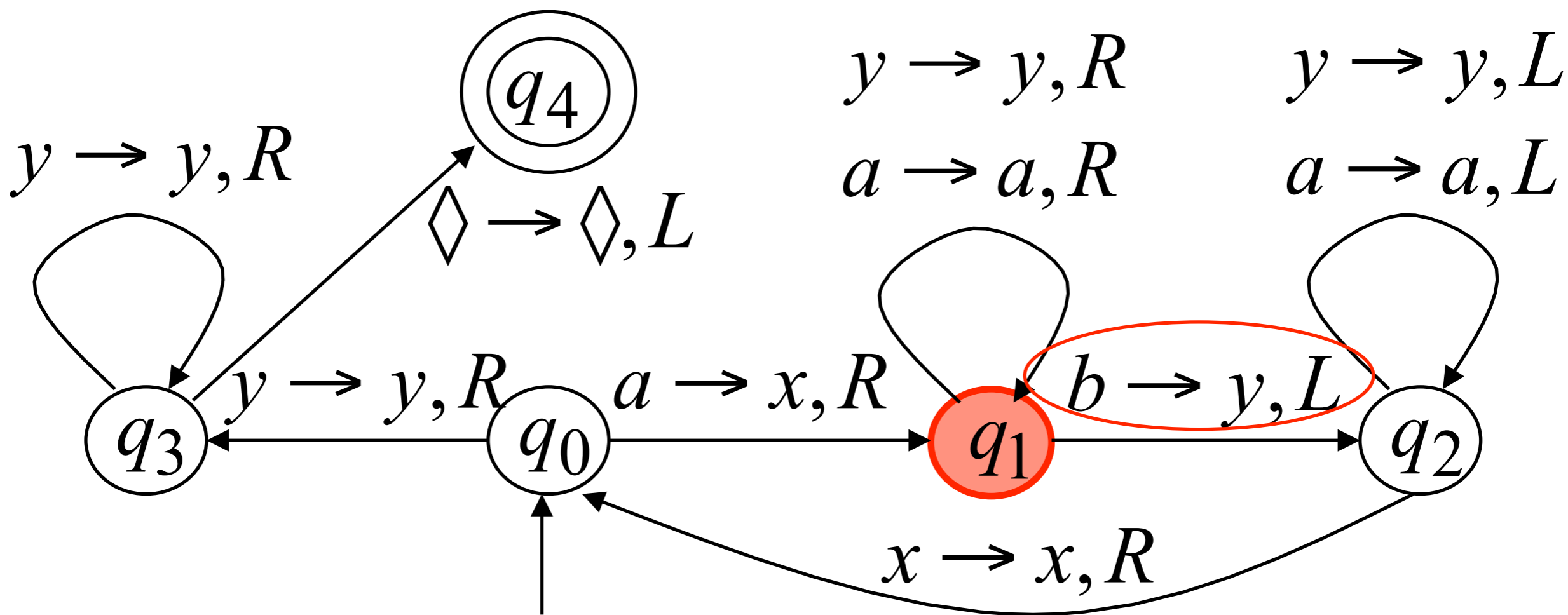
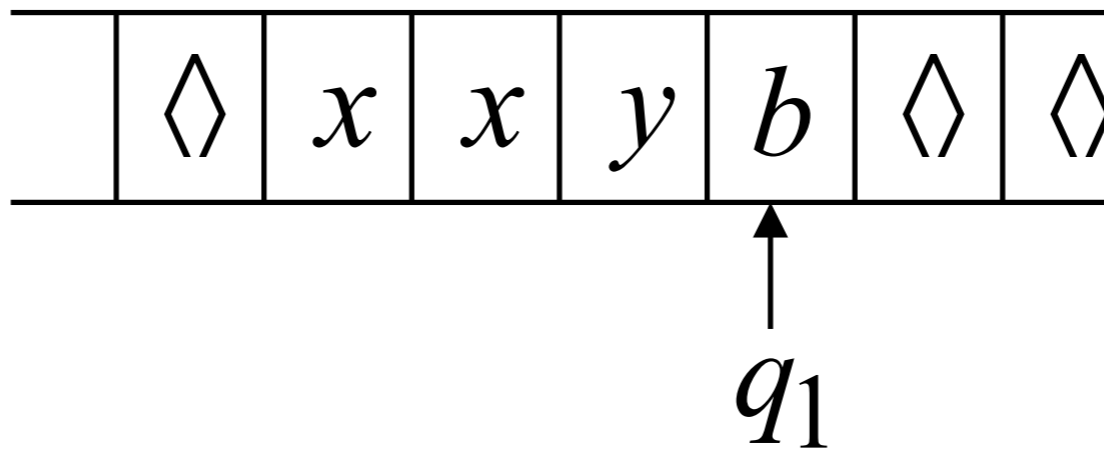
Time 5



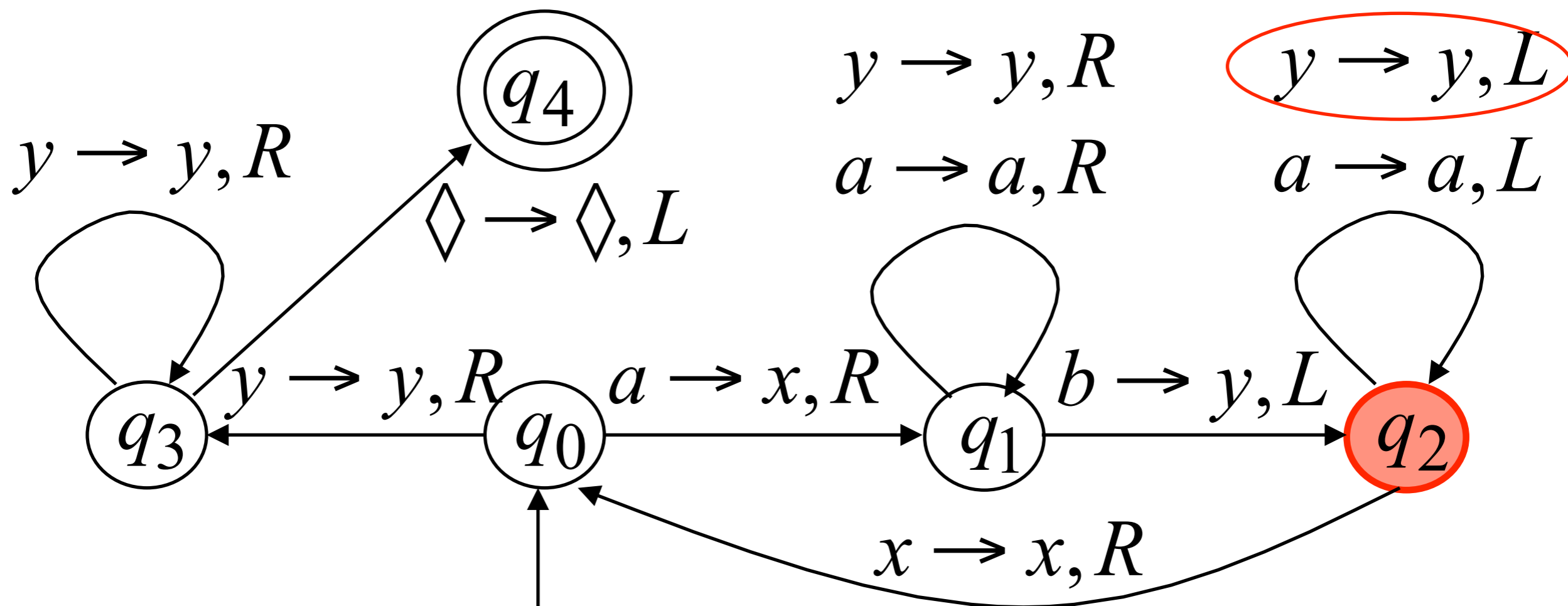
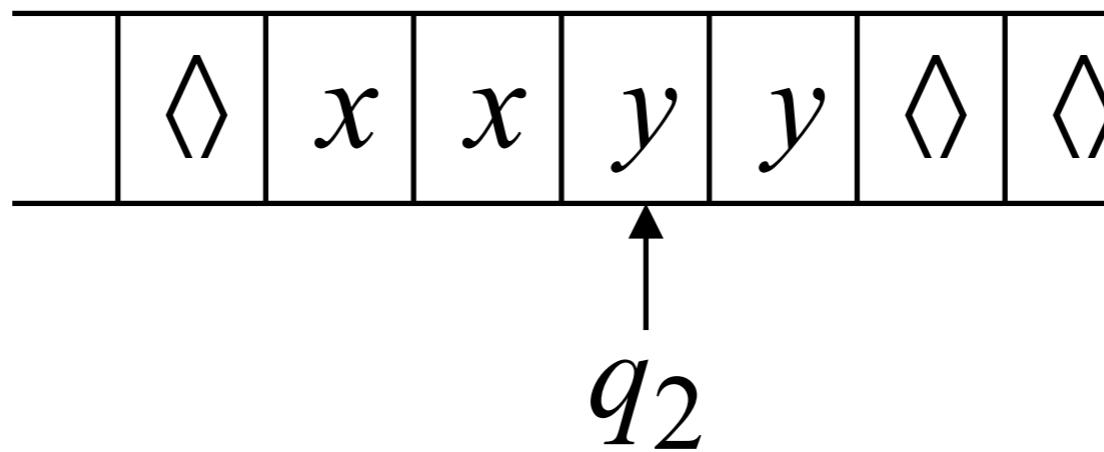
Time 6



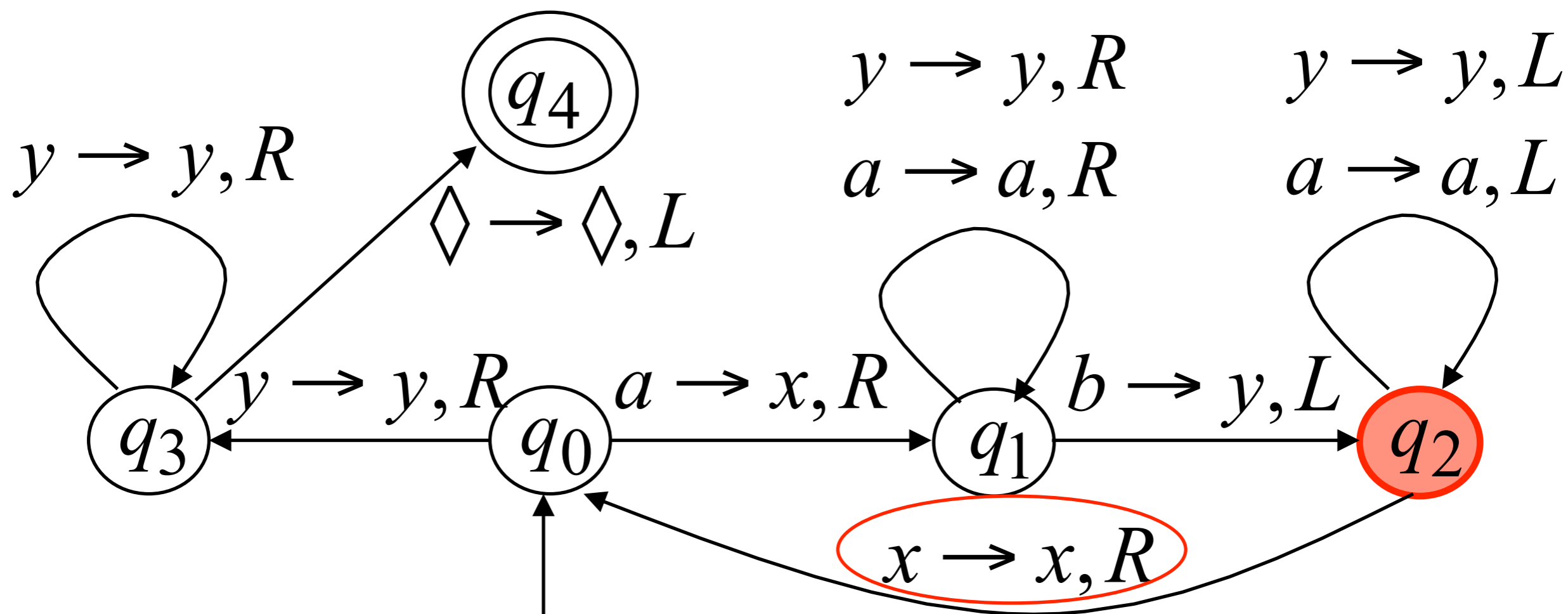
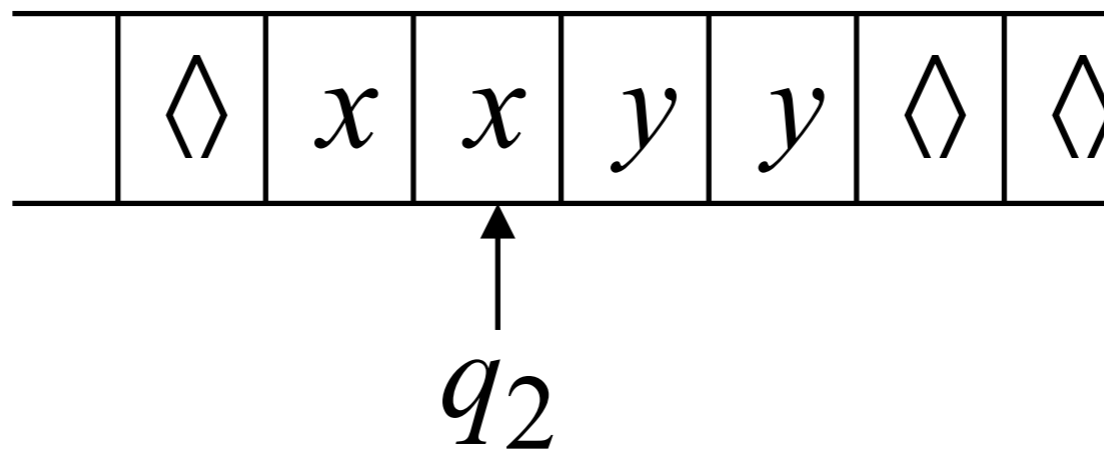
Time 7



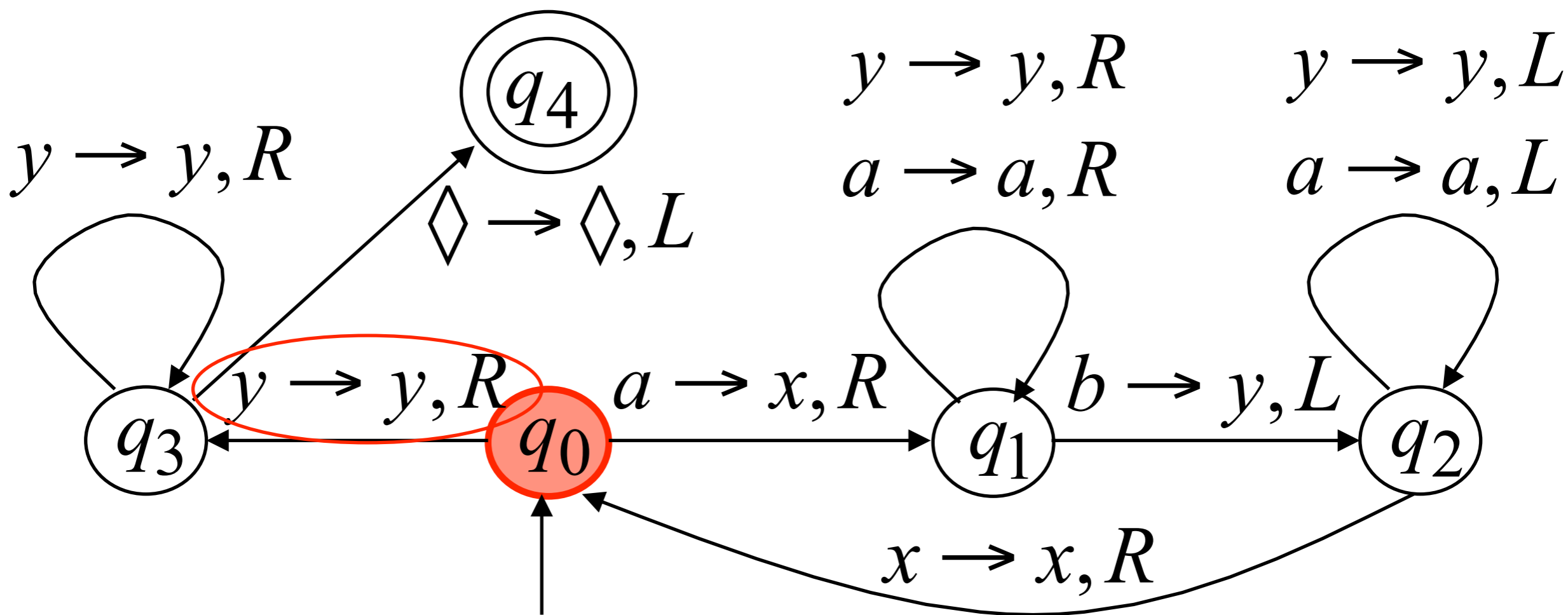
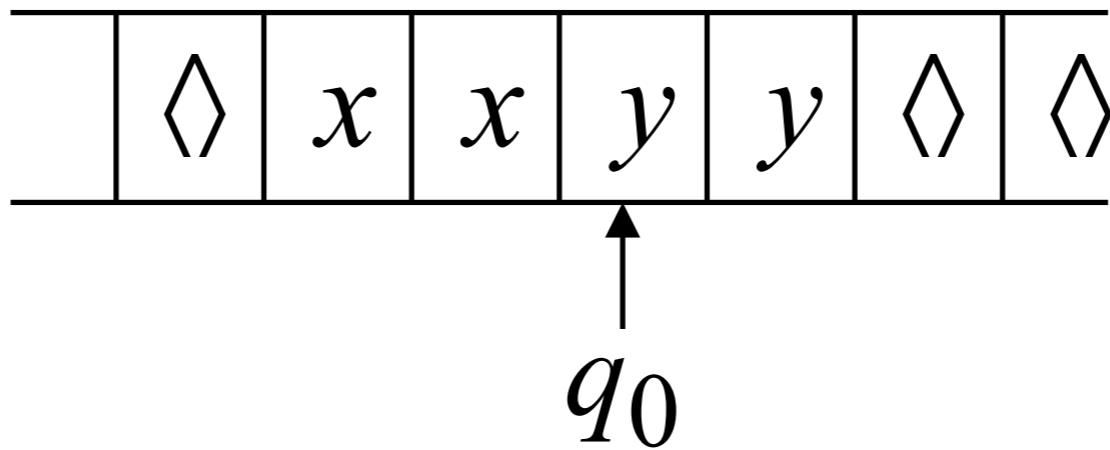
Time 8



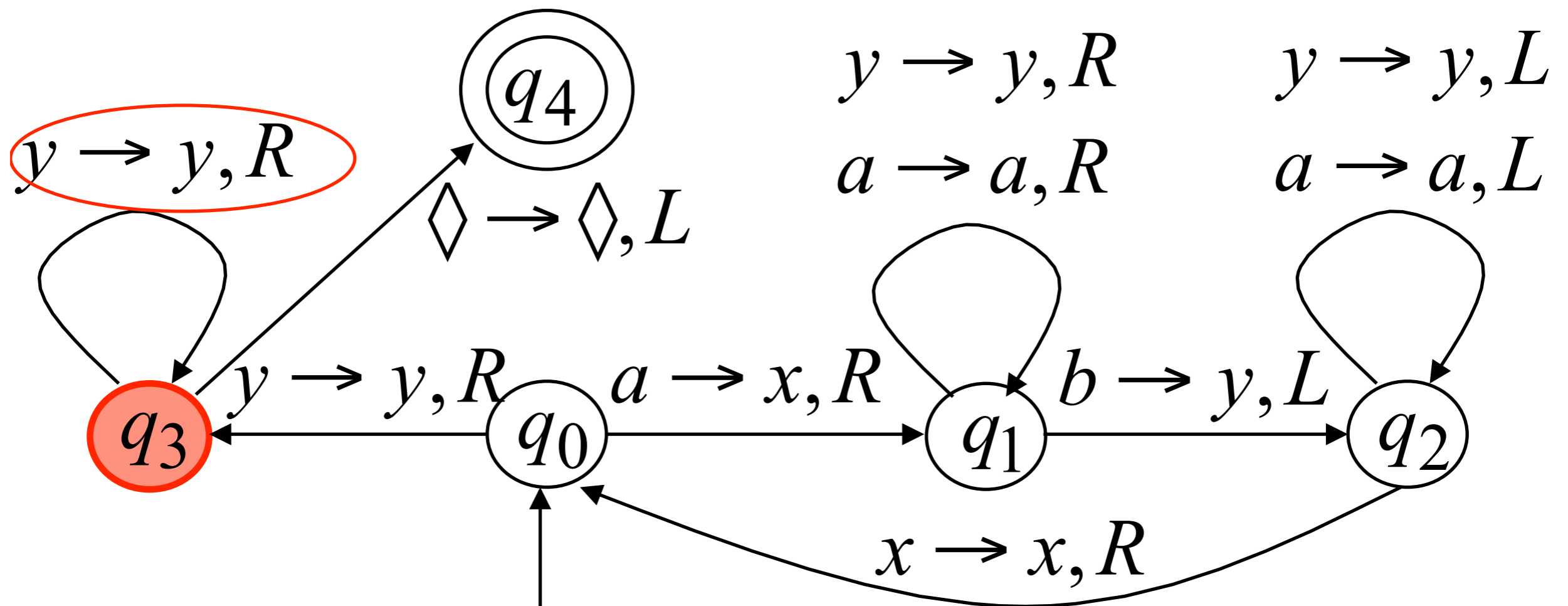
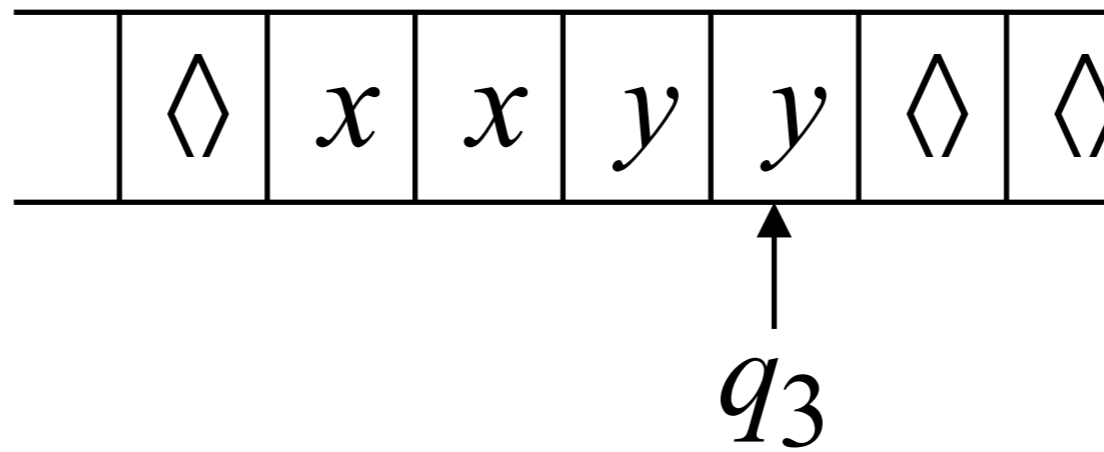
Time 9



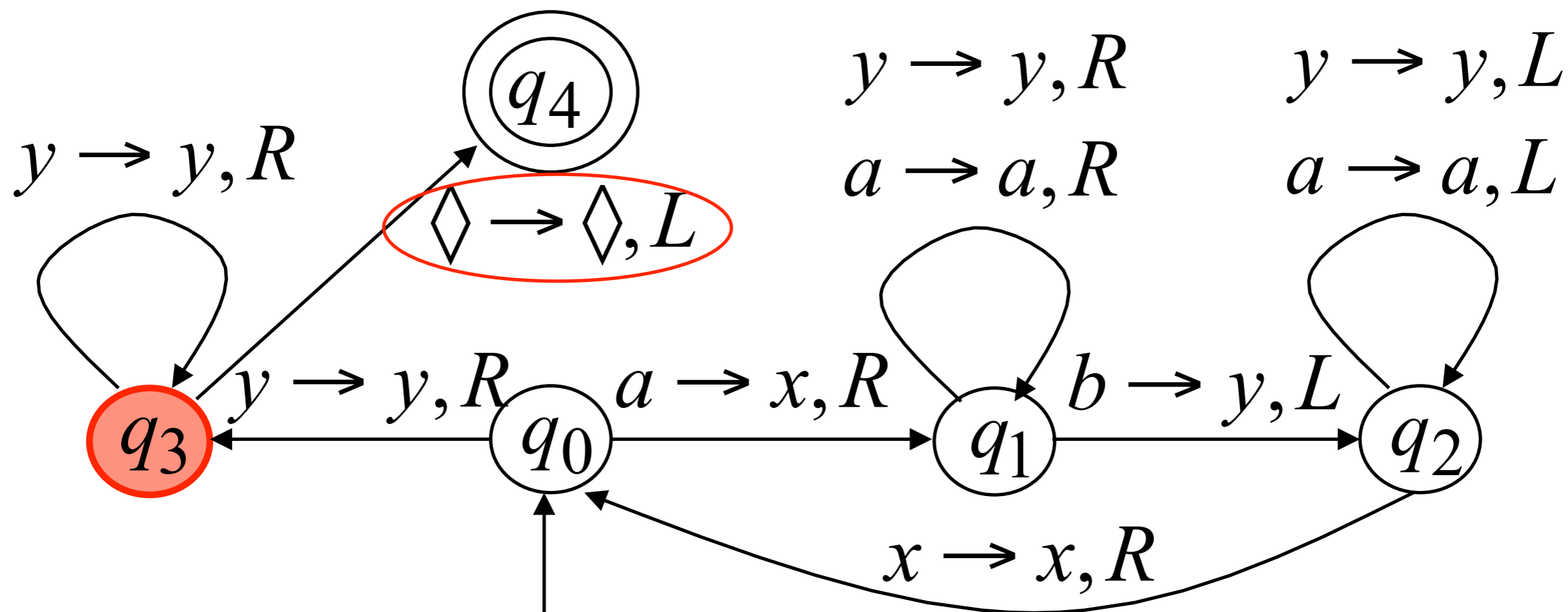
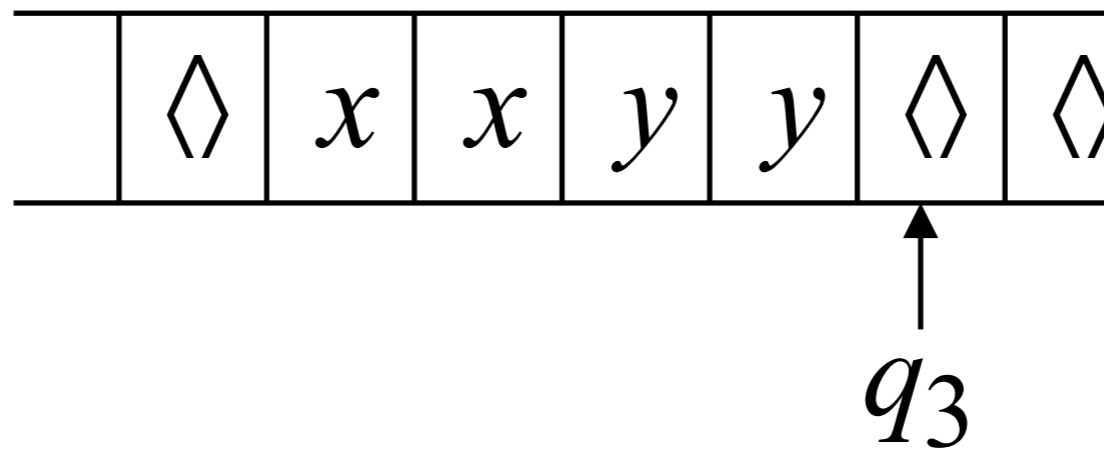
Time 10



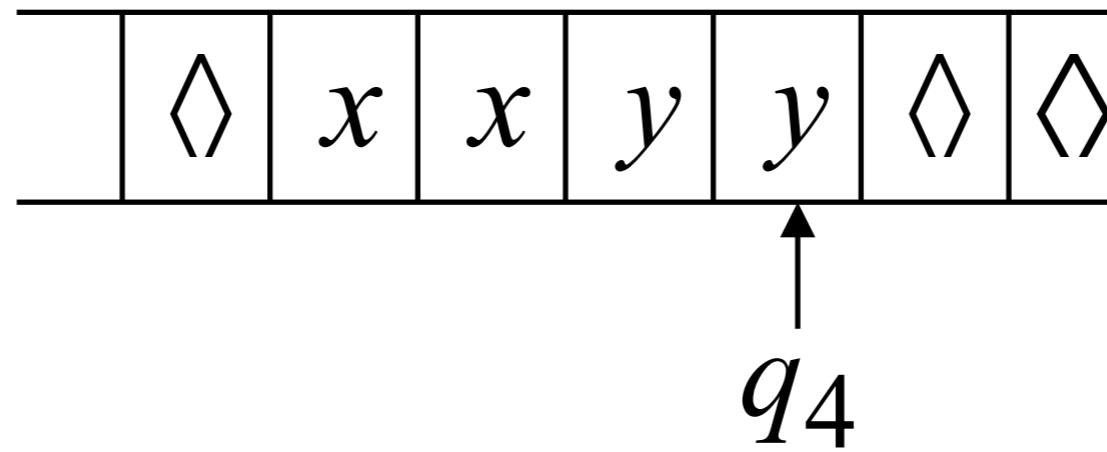
Time 11



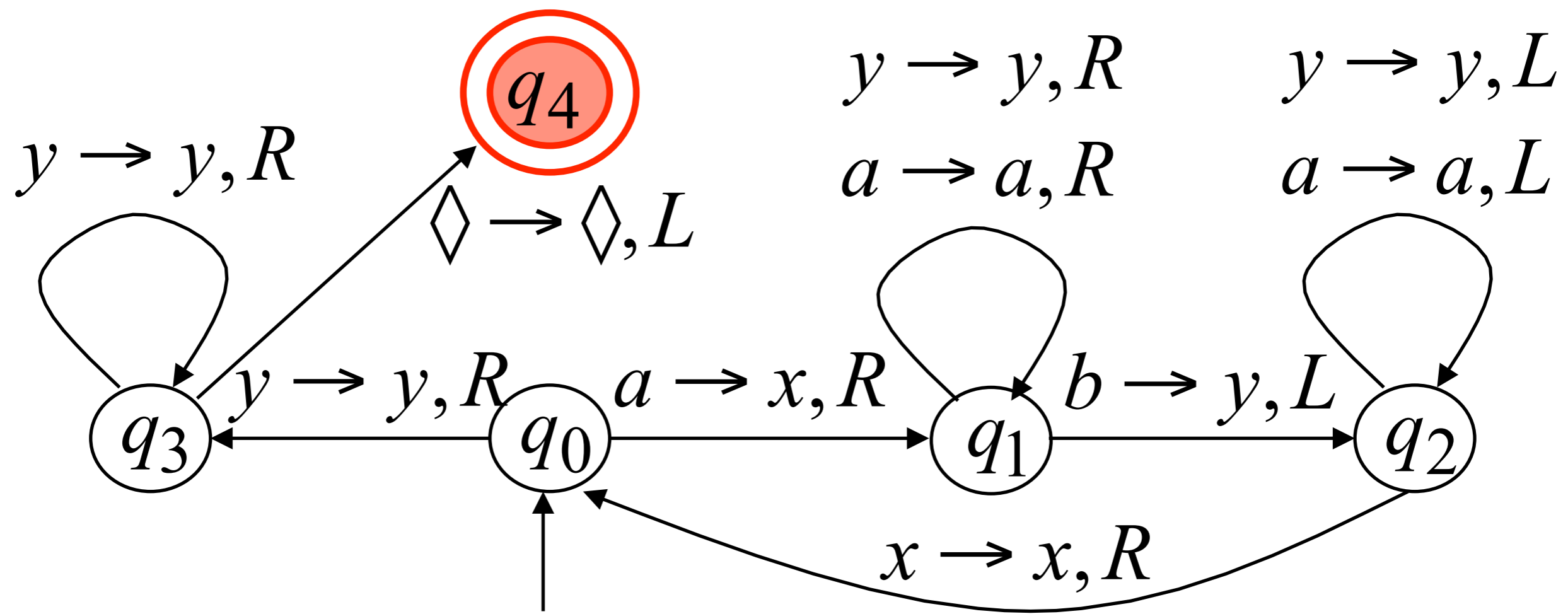
Time 12



Time 13



Halt & Accept



Observation

The machine for the language $\{a^n b^n \mid n \geq 1\}$ can be modified to construct a machine for the language $\{a^n b^n c^n \mid n \geq 1\}$.