# Reinforcement Learning

- Blue slides: Mitchell
- Green slides: Alpaydin

# Reinforcement Learning (RL)

- How an **autonomous agent** that **sense** and **act** in the environment can **learn to choose optimal actions** to achieve its **goals**.
- Examples: mobile robot, optimization in process control, board games, etc.
- Ingredients: **reward/penalty** for each action, where the reinforcement signal can be significantly **delayed**.
- One approach: $Q$ **learning**

# Introduction: Agent

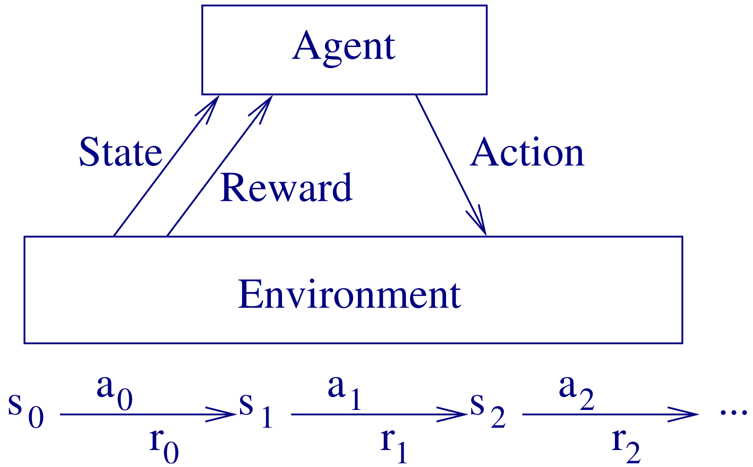
Terminology:

- **State**: state of the environment, obtained through sensors
- **Action**: alter the state
- **Policy**: choosing actions that achieve a particular goal, based on the current state.
- **Goal**: desired configuration (or state).

Desired policy:

- From any initial state, choose actions that **maximize the reward accumulated over time** by the agent.

## RL Task



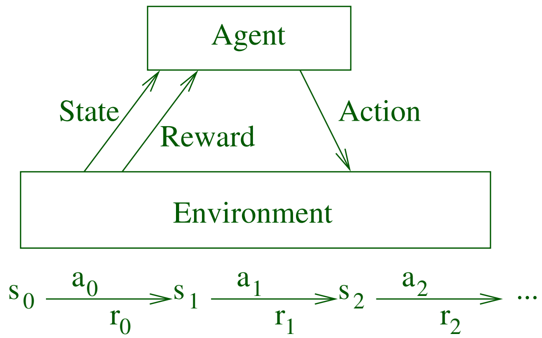

- Goal: learn to choose actions that maximize **discounted, cumulative award**:

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots, \text{ where } 0 \leq \gamma < 1.$$

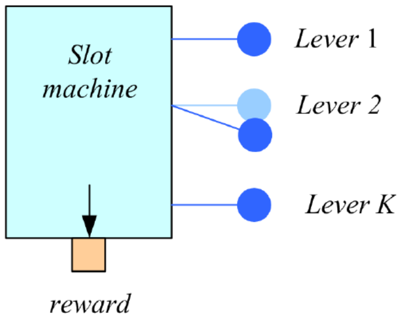
- That is, we want to learn a policy $\pi : S \rightarrow A$ that maximizes the above, where $S$ is the set of states, and $A$ that of actions.

## Applications and Issues (ALP)

- Game-playing : Sequence of moves to win a game
- Robot in a maze: Sequence of actions to find a goal
- Play video games (Atari 2600, Starcraft, Dota)
- Control robot arms, etc.
- Issue: Rewards are sparse; Credit assignment



## Single-State RL: $K$-armed bandit (ALP)




- Among $K$ levers, choose the one that pays the best.
- $Q(a)$ = value of action $a$, Reward $r_a$; Choose $a^*$ if $Q(a^*) = max_a Q(a)$
- When reward is deterministic: Set $Q(a) = r_a$.
- When reward is stochastic:

$$Q_{t+1}(a) \leftarrow Q_t(a) + \eta \left[ r_{t+1}(a) - Q_t(a) \right]$$

# Variations of RL Tasks

- Deterministic vs. nondeterministic action outcomes.
- With or without prior knowledge about the effect of action on environmental state.
- Partially or fully known environmental state (e.g., Partially Observable Markov Decision Process [POMDP]).

## RL Compared to Other Learning Algorithms

- Planning (in AI)
- Function approximation: $\pi : S \rightarrow A$.
- Differences:
  - Delayed reward
  - Exploration vs. exploitation
  - Partially observable states
  - Life-long learning: leveraging on existing knowledge, to make learning of a new complex task easier.

## The Learning Task

Markov Decision Process: only immediate state matters.

- State $s_t$, action $a_t$ at time step $t$.
- Reward from environment: $r_t = r(s_t, a_t)$
- State transition by environment: $s_{t+1} = \delta(s_t, a_t)$
- $r(\cdot, \cdot)$ and $\delta(\cdot, \cdot)$ may be **unknown** to the agent!
- Task: learn $\pi : S \rightarrow A$ to select $a_t = \pi(s_t)$.
- Question: how to specify which $\pi$ to learn?

## RL as Markov Decision Process (ALP)

- $s_t$: state of agent at time $t$
- $a_t$: action taken at time $t$
- In $s_t$, action $a_t$ is taken, clock ticks and reward $r_{t+1}$ is received, and state advances to $s_{t+1}$.
- Next state probability: $P(s_{t+1}|s_t, a_t)$
- Reward probability: $P(r_{t+1}|s_t, a_t)$
- Initial state and goal given.
- Episode (trial) of actions from initial state to goal.

Sutteon and Barto 1998, Kaelbling et al. 1996.

## Discounted Cumulative Reward: $V^{\pi}(s_t)$

- Obvious approach is to find $\pi$ that maximizes the cumulative reward when $\pi$ is executed:

$$\begin{aligned} V^{\pi}(s_t) &\equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots \\ &\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}, \end{aligned}$$

  where $0 \leq \gamma < 1$ is the discount rate.

- $\pi$ is repeatedly executed: $a_t = \pi(s_t), a_{t+1} = \pi(s_{t+1}), \ldots$
- When $\gamma = 0$, only the current reward is used.
- When $\gamma \to 1$, future rewards become more important.

# Choosing a Policy

- Optimal policy $\pi^*$

$$\pi^* = \underset{\pi}{\operatorname{argmax}}\, V^{\pi}(s), \forall s$$

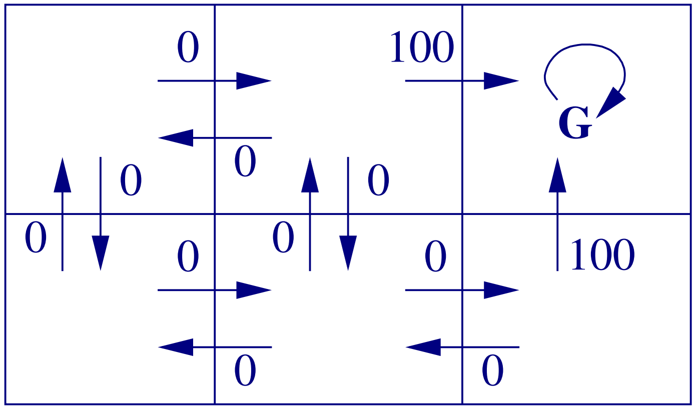
- Want a policy that does its best for all states.

- Cumulative reward under optimal policy $\pi^*$:

$$V^*(s) \equiv V^{\pi^*}(s),$$

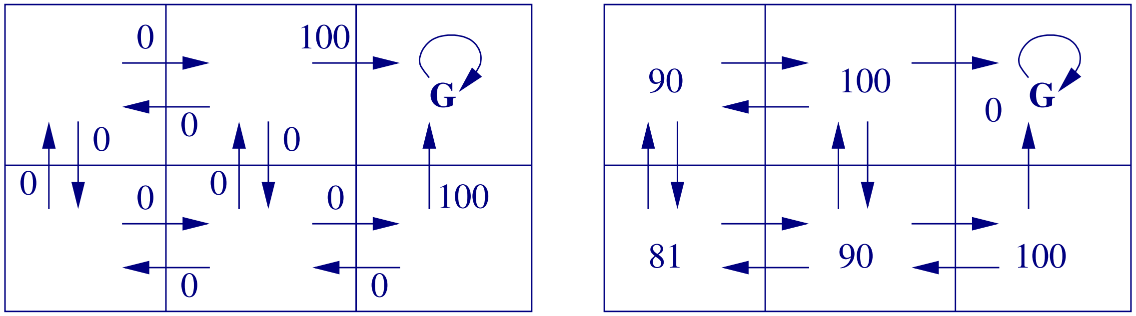
for short.

## Example: Grid World




- Immediate reward given only when entering the goal state $G$.
- Given any initial state, we want to generate an action sequence to maximize $V$.

## Grid World: $V^*(s)$ Values



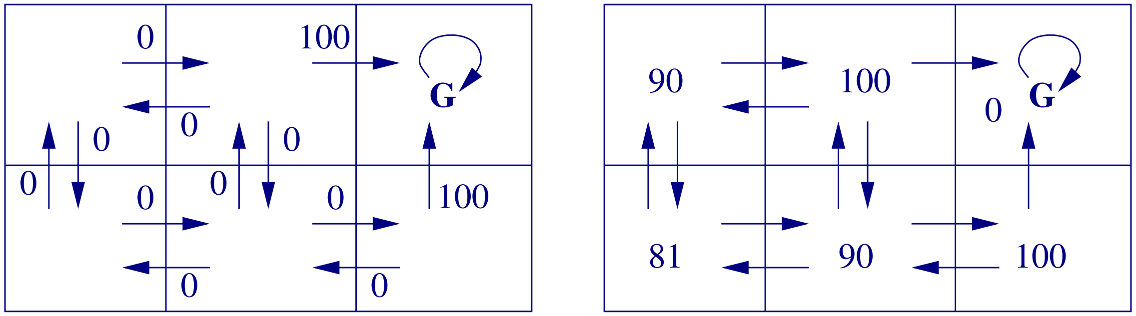

$(a)$ $r(s,a)$ values  $(b)$ $V^*(s)$ values

- Discount rate: $\gamma = 0.9$.
- Top middle: $100 + \gamma 0 + \gamma^2 0 + ... = 100$
- Top left: $0 + \gamma 100 + \gamma^2 0 + ... = 90$
- Bottom left: $0 + \gamma 0 + \gamma^2 100 + ... = 81$
- Note that these values are supposed to be **obtained using the optimal policy** $\pi^*$.

# $Q$ Learning

- Policy is hard to learn directly, because training experience does not provide $< s, a >$ pairs.
- Only available info: sequence of immediate rewards $r(s_i, a_i)$ for $i = 0, 1, 2, ....$
- In this case, it is easier to learn an **evaluation function** and construct a policy based on that.

## Optimal Policy using $V^*(s)$




$(a)$ $r(s, a)$ values $\quad$ $(b)$ $V^*(s)$ values

- If reward $r(s, a)$, state transition $\delta(s)$, and evaluation function $V^*(s)$ are known the following gives an optimal policy:

$$\pi^*(s) = \underset{a}{\operatorname{argmax}} \left[ r(s, a) + \gamma V^*(\delta(s, a)) \right]$$

- For example, top middle state: move right = $100 + \gamma 0 = 100$, move left = $0 + \gamma 90 = 81$, move down = $0 + \gamma 90 = 81$.

## Model-based Learning (ALP)

- If environment, $P(s_{t+1}|s_t, a_t)$ and $P(r_{t+1}|s_t, a_t)$ are known,
- There is no need for exploration.
- Can be solved directly using dynamic programming.
- Solve for

$$V^*(s_t) = \max_{a_t}\left( E[r_{t+1}] + \gamma \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a_t) V^*(s_{t+1}) \right)$$

- Optimal policy:

$$\pi^*(s_t) = \arg\max_{a_t}\left( E[r_{t+1} \mid s_t, a_t] + \gamma \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a_t) V^*(s_{t+1}) \right)$$

Note: $E[X]$ is expected value of $X$.

## Problems with Policy Based on $V^*(s)$

- Requires perfect knowledge of $r(s, a)$ and $\delta(s, a)$, to exactly predict the outcome and reward of a particular action.
- In practice, the above is impossible.
- Thus, even when $V^*(s)$ is known, $\pi^*(s)$ cannot be found. Refer to:

$$\pi^*(s) = \underset{a}{\operatorname{argmax}} \left[ r(s, a) + \gamma V^*(\delta(s, a)) \right]$$

- Solution: use a surrogate – the $Q$ function.

# The $Q$ Function

Can we get by without explicit knowledge of $r(s,a)$ and $\delta(s,a)$?

- $Q(s,a)$: evaluation function whose value is the **maximum discounted cumulative reward** obtainable when action $a$ is taken in state $s$:

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

- The derived policy is then:

$$\pi^*(s) = \operatorname*{argmax}_a Q(s,a)$$

  Note that if $Q(s,a)$ can be learned without any reference to $r(s,a)$ and $\delta(s,a)$, we have solved our problem.

- Further problem: how to **estimate** $Q(s,a)$?

## Learning the $Q$ Function: Getting Rid of $V^*(\delta(s, a))$

- $Q(s, a)$ is defined over all possible actions $a$ from state $s$. But note that one of these actions is optimal for state $s$, and thus:

$$V^*(s) = \max_{a'} Q(s, a')$$

- With the above,

$$Q(s, a) \equiv r(s, a) + \gamma V^*(\delta(s, a))$$

can be rewritten as:

$$Q(s, a) \equiv r(s, a) + \gamma \max_{a'} Q(\delta(s, a), a'),$$

thus getting rid of $V^*(\delta(s, a))$.

## Learning the $Q$ Function: Getting Rid of $r$ and $\delta$

In state $s$, execute action $a$, and observe immediate reward $r$ and resulting state $s'$. Then, simply use those $r$ and $s'$ you got without worrying about $r(s, a)$ or $\delta(s, a)$.

- Initialize the estimate $\hat{Q}(s, a)$ to zero.
- Iteratively update, with estimated function $\hat{Q}(s, a)$:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a').$$

## The $Q$ Learning Algorithm

1. For each $s$, $a$, initialize the table entry $\hat{Q}(s, a)$ to zero.
2. Observe the current state $s$.
3. Do forever:
   - Select action $a$ and execute.
   - Receive immediate reward $r$.
   - Observe resulting state $s'$.
   - Update table entry for $\hat{Q}(s, a)$ as:

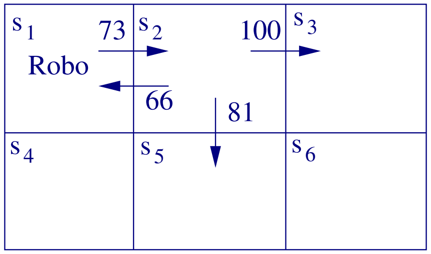
     $$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a').$$

   - $s \leftarrow s'$

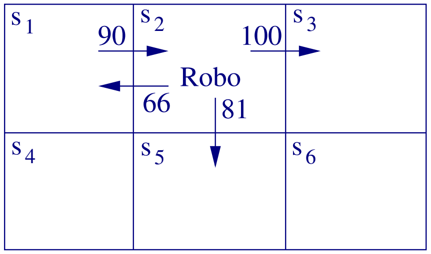
## $Q$ Learning Properties

- For deterministic Markov decision processes
- $\hat{Q}$ converges to $Q$, when
  - process is deterministic **MDP**,
  - $r$ **is bounded** (and non-negative), and
  - actions are chosen so that every state-action pair is visited **infinitely often**.

# Example




$(a)$ Initial state, in $s_1$




$(b)$ Next state, in $s_2$

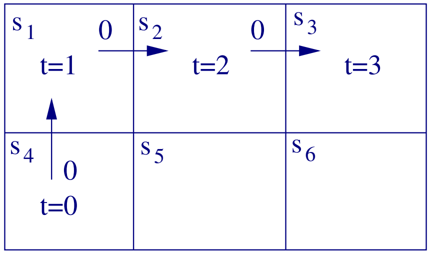
Arrows represent the $\hat{Q}$ values.

- Move right ($a = a_{right}$) and get immediate reward $r = 0$, with discount rate $\gamma = 0.9$:

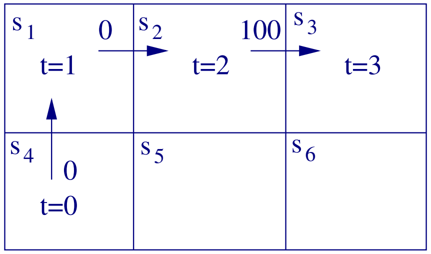
$$\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \max\{66, 81, 100\} \\
&\leftarrow 90
\end{aligned}$$

- Note that in $(b)$, the $\hat{Q}(s_1, a_{right})$ value is updated from 73 to 90.

# Exercise, from scratch



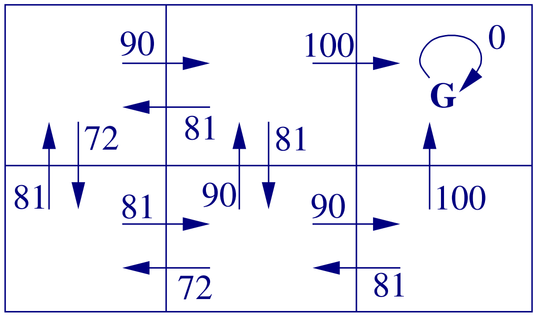

$(a)$ Initial state $Q(s,a) = 0$




$(b)$ After one iteration

- Robot moved from $s_4 \to s_1 \to s_2 \to s_3$.
- How do the various $Q(s,a)$ values get updated?
  - For the first iteration?
  - For the next iteration of $s_4 \to s_1 \to s_2 \to s_3$?

# Final learned $\hat{Q}$




- For this domain, following actions that have max $Q(s, a)$ will lead you to the goal through an optimal path.

## Convergence of $\hat{Q}$ to $Q$

- Properties (for non-negative rewards):

$$\forall s, a, n : \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

$$\forall s, a, n : 0 \leq \hat{Q}_n(s, a) \leq Q_n(s, a)$$

- In general, convergence is guaranteed under three conditions:

1. The system is a deterministic MDP.
2. The reward is bounded $(\forall s, a) \ \ |r(s, a)| < c$ for a fixed constant $c$.
3. All $(s, a)$ pairs are visited infinitely often.

## Proof of Convergence: Sketch

- The table entry $\hat{Q}(s,a)$ with the largest error must have its error reduced by a factor of $\gamma$ whenever it is updated.
- The updated $\hat{Q}(s,a)$ will be based on the error-prone $\hat{Q}(s,a)$ only partially. The accurate immediate reward $r$ used in the $Q$ update rule will help reduce the error.
- *Proof*: Define a full interval to be an interval during which each table entry $\langle s,a \rangle$ is visited. During each full interval the largest error in $\hat{Q}$ table is reduced by factor of $\gamma$.

# Convergence of $Q$

Let $\hat{Q}_n$ be table after $n$ updates, and $\Delta_n$ be the maximum error in $\hat{Q}_n$; that is

$$\Delta_n = \max_{s,a} |\hat{Q}_n(s,a) - Q(s,a)|$$

For any table entry $\hat{Q}_n(s,a)$ updated on iteration $n+1$, the error in the revised estimate $\hat{Q}_{n+1}(s,a)$ is

$$\begin{aligned}
|\hat{Q}_{n+1}(s,a) - Q(s,a)| &= |(r + \gamma \max_{a'} \hat{Q}_n(s',a')) \\
&\quad -(r + \gamma \max_{a'} Q(s',a'))| \\
&= \gamma |\max_{a'} \hat{Q}_n(s',a') - \max_{a'} Q(s',a')| \\
&\leq \gamma \max_{a'} |\hat{Q}_n(s',a') - Q(s',a')| \\
&\leq \gamma \max_{s'',a'} |\hat{Q}_n(s'',a') - Q(s'',a')| \\
|\hat{Q}_{n+1}(s,a) - Q(s,a)| &\leq \gamma \Delta_n
\end{aligned}$$

# Convergence in $Q$

- Main result:

$$|\hat{Q}_{n+1}(s,a) - Q(s,a)| \leq \gamma \Delta_n$$

- That is, error in the updated $\hat{Q}(s,a)$ is less than $\gamma$ times the max error in the table before the update.

- Note that $\gamma < 1.0$.

- Given initial $\Delta_0$, after $k$ visits to $\langle s,a \rangle$, the error will be at most $\gamma^k \Delta_0$, and as $k \to \infty$, $\Delta_k \to 0$.

## Constructing the Policy from the Learned $Q$

1. Greedy: given state $s$, pick $\operatorname{argmax}_a Q(s,a)$.
   - May cause the agent to **exploit** early successes and ignore interesting possibilities.
   - This would prevent the agent from visiting all $(s,a)$ pairs infinitely often.
2. Probabilistic: pick action $a_i$ with probability:

$$P(a_i|s) = \frac{k^{\hat{Q}(s,a_i)}}{\sum_j k^{\hat{Q}(s,a_j)}}$$

   where $k > 0$ controls **exploration (low $k$)** vs. **exploitation (high $k$, greedy)**.

## Updating Sequence

No specific order of $(s, a)$ visit is necessary for convergence. However, this can be inefficient.

1. Perform update in reverse order, once the goal has been reached.

2. Store past state-action transitions.

## Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine $V, Q$ by taking expected values

$$
\begin{aligned}
V^{\pi}(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\
&\equiv E\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right]
\end{aligned}
$$

$$Q(s,a) \equiv E[r(s,a) + \gamma V^{*}(\delta(s,a))]$$

## Nondeterministic Case

$Q(s, a)$ can be redefined as follows:

$$\begin{aligned} Q(s,a) &\equiv E[r(s,a) + \gamma V^*(\delta(s,a))] \\ &= E[r(s,a)] + \gamma E[V^*(\delta(s,a))] \\ &= E[r(s,a)] + \gamma \sum_{s'} P(s'|s,a) V^*(s') \end{aligned}$$

Finally, rewriting it recursively, we get:

$$Q(s,a) = E[r(s,a)] + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q(s',a')$$

## Nondeterministic Case: Learning

Using the original learning rule can result in oscillation in $\hat{Q}(s,a)$, and thus no convergence. Taking a decaying weighted average can solve the problem:

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n \left[ r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a') \right]$$

where

$$\alpha_n = \frac{1}{1 + visits_s(s,a)}$$

and $\alpha$ determines how much the old and new $\hat{Q}$ values will be used. The $\alpha_n$ formula above is known to allow convergence (there can be other formulas).

# Temporal Difference Learning

$Q$ learning reduces the difference between $\hat{Q}$ of a state and its immediate successor (one-step look ahead). This can be generalized to include more distant successors.

$Q$ learning reduces the difference between $\hat{Q}$ of a state

- $\hat{Q}(s_t, a_t)$ is estimated based $\hat{Q}(s_{t+1}, \cdot)$, where $s_{t+1} = \delta(s_t, a_t)$.
- One-step look ahead:

$$Q^{(1)}(s_t, a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1}, a)$$

- Two-step look ahead:

$$Q^{(2)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2}, a)$$

- $n$-step look ahead:

$$Q^{(n)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + ... + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n}, a)$$

# Learning in TD

TD($\lambda$) for learning $Q$ using various lookaheads ($0 \leq \lambda \leq 1$):

$$Q^{\lambda}(s_t, a_t) \equiv (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \ldots\right]$$

which can be rewritten recursively:

$$\begin{aligned}
&Q^{\lambda}(s_t, a_t) \\
&= (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \ldots\right] \\
&= \ldots \\
&= r_t + \gamma(1-\lambda)\max_a \hat{Q}(s_{t+1}, a) + \gamma\lambda\left[r_{t+1} + \gamma(1-\lambda)\max_a \hat{Q}(s_{t+2}, a) + \ldots\right] \\
&= r_t + \gamma\left[(1-\lambda)\max_a \hat{Q}(s_{t+1}, a) + \lambda Q^{\lambda}(s_{t+1}, a_{t+1})\right]
\end{aligned}$$

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Note: there's a typo in Mitchell's book. $r_t + \gamma\left[(1-\lambda)\max_a \hat{Q}(\underbrace{s_t}_{\text{typo}}, a) + \lambda Q^{\lambda}(s_{t+1}, a_{t+1})\right]$

## TD($\lambda$) Properties

$$Q^{\lambda}(s_t, a_t) = r_t + \gamma \left[ (1-\lambda) \max_a \hat{Q}(s_{t+1}, a_t) + \lambda Q^{\lambda}(s_{t+1}, a_{t+1}) \right]$$

- TD(0): same as $Q^{(1)}$.
- TD(1): only observed $r_{t+i}$ values are considered.
- When $Q = \hat{Q}$, $Q^{\lambda}$ values are the same for any $0 \leq \lambda \leq 1$.

## Curious Properties of TD($\lambda$)

Why is TD($\lambda$) not 0 when $\lambda = 1$? Note that TD(0)$= Q^{(1)}$.

$$Q^{\lambda}(s_t, a_t) = (1 - \lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \ldots\right]$$

It's because of the infinite sum that involve $\lambda$:

$$\begin{aligned}
Q^{\lambda} &= (1-\lambda)Q^{(1)} + (1-\lambda)\lambda Q^{(2)} + (1-\lambda)\lambda^2 Q^{(3)} + \ldots \\
&= (1-\lambda)(r_t + ..) + (1-\lambda)\lambda(r_t + \gamma r_{t+1}..) + (1-\lambda)\lambda^2(r_t + \gamma r_{t+1} + \gamma^2 r_{t+2}..) + \\
&= (1-\lambda)r_t + (1-\lambda)\lambda r_t + (1-\lambda)\lambda^2 r_t + \ldots \\
&= (1-\lambda)\sum_{n=0}^{\infty} \lambda^n r_t + \ldots \\
&= (1-\lambda)\frac{1}{1-\lambda} r_t + \ldots \\
&= r_t + \ldots
\end{aligned}$$

## TD($\lambda$) Properties

- Sometimes converges faster than $Q$ learning
- Converges for learning $V^*$ for any $0 \leq \lambda \leq 1$ (Dayan, 1992)
- Tesauro's TD-Gammon uses this algorithm

## Q-learning and variants (ALP)

- Q-learning : use max $a$ from next state
- SARSA : randomly pick $a$ from next state
- SARSA($\lambda$) : uses eligibility traces

## Q-learning (ALP)

Initialize all $Q(s,a)$ arbitrarily
For all episodes
    Initalize $s$
    Repeat
        Choose $a$ using policy derived from $Q$, e.g., $\epsilon$-greedy
        Take action $a$, observe $r$ and $s'$
        Update $Q(s,a)$:
            $Q(s,a) \leftarrow Q(s,a) + \eta(r + \gamma \boxed{\max_{a'} Q(s',a')} - Q(s,a))$
        $s \leftarrow s'$
    Until $s$ is terminal state

Use $\max_{a'} Q(s',a')$

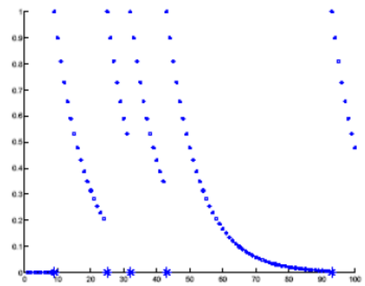
## SARSA (ALP)

Initialize all $Q(s,a)$ arbitrarily
For all episodes
  Initalize $s$
  Choose $a$ using policy derived from $Q$, e.g., $\epsilon$-greedy
  Repeat
    Take action $a$, observe $r$ and $s'$
    $\boxed{\text{Choose } a' \text{ using policy derived from } Q\text{, e.g., } \epsilon\text{-greedy}}$
    Update $Q(s,a)$:
      $Q(s,a) \leftarrow Q(s,a) + \eta(r + \gamma \boxed{Q(s',a')} - Q(s,a))$
    $s \leftarrow s', \ a \leftarrow a'$
  Until $s$ is terminal state

Use $Q(s', a')$ of $a'$ picked from current policy.

# Eligibility Traces (ALP)

$$e_t(s,a) = \begin{cases} 1 & \text{if } s = s_t \text{ and } a = a_t \\ \gamma\lambda e_{t-1}(s,a) & \text{otherwise} \end{cases}$$



$$\delta_t = r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)$$
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta \delta_t e_t(s,a), \forall s, a$$

- Needed in SARSA($\lambda$)
- Update *ALL* $(s, a)$ pairs after each action!
- Weight the update with *recency information*
  $\rightarrow$ If $(s, a)$ was more recently taken, give more weight.

# SARSA($\lambda$) (ALP)

Initialize all $Q(s,a)$ arbitrarily, $e(s,a) \leftarrow 0, \forall s, a$
For all episodes
  Initalize $s$
  Choose $a$ using policy derived from $Q$, e.g., $\epsilon$-greedy
  Repeat
    Take action $a$, observe $r$ and $s'$
    Choose $a'$ using policy derived from $Q$, e.g., $\epsilon$-greedy
    $\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$
    $e(s,a) \leftarrow 1$
    For all $s, a$:
      $Q(s,a) \leftarrow Q(s,a) + \eta \delta e(s,a)$
      $e(s,a) \leftarrow \gamma \lambda e(s,a)$
    $s \leftarrow s', \;\; a \leftarrow a'$
  Until $s$ is terminal state

Use eligibility trace. $\lambda$ controls how fast the e.t. degrades.

# Partially Observable States (ALP)

- The agent does not know its state but receives an observation $o_t$ with probability $P(o_{t+1}|s_t, a_t)$, which can be used to infer a belief about the true state.
- Partially Observable MDP = POMDP

## Subtleties and Ongoing Research

- Replace $\hat{Q}$ table with neural net or other generalizer : Deep RL!
- Make state and action space continuous : Deep RL!
- Handle case where state is only partially observable (partially observable MDP, or POMDP).
- Design optimal exploration strategies.
- Extend to continuous action, state.
- Learn and use $\hat{\delta} : S \times A \rightarrow S$.
- Relationship to dynamic programming.
- Multi-task learning, Meta learning: Deep RL!