636-600 Neural Networks

- Instructor: Yoonsuck Choe
  - Contact info: HRBB 322B, 845-5466, choe@tamu.edu
- Web page: http://faculty.cs.tamu.edu/choe

Textbook

- Simon Haykin. Neural networks and learning machines. Pearson Education. Upper Saddle River, NJ, 2009.
- Older edition:
  Simon Haykin, Neural Networks: A Comprehensive Foundation, Second edition, Prentice-Hall, Upper Saddle River, NJ, 1999.
- Code from the book:
  http://www.mathworks.com/books (click on Neural/Fuzzy and find the book title).
- Text and figures, etc. will be quoted from the textbook without repeated acknowledgment. Instructor's perspective will be indicated by "YC" where appropriate.

Other Textbooks and Books

- I. Goodfellow, Y. Bengio, and A. Courville, Deep Learning, MIT Press, 2016.
- J. Hertz, A. Krogh, and R. Palmer, Introduction to the Theory of Neural Computation, Addison-Wesley, 1991.
- C. M. Bishop, Neural Networks for Pattern Recognition, Oxford University Press, 1995.
- M. A. Arbib, The Handbook of Brain Theory and Neural Networks, 2nd edition, MIT Press, 2003.

Course Info

- Grading: Exams: 60% (midterm: 30%, final: 30%); Assignments: 4% (4 written+programming assignments, 10% each); No curving: ≥ 90 = A, ≥ 80 = B, etc.
- Academic integrity: individual work, unless otherwise indicated; proper references should be given in case online/offline resources are used.
- Students with disabilities: see the online syllabus.
- Lecture notes: check course web page for updates. Download, print, and bring to the class.
- Computer accounts: talk to CS helpdesk.
- Programming: Matlab, or better yet, Octave (http://www.octave.org). C/C++, Java, etc. (they should run on CS Unix or windows)

Relation to Other Courses

Some overlaps:

- Machine learning: neural networks
- Pattern analysis: PCA, support-vector machines, radial basis functions(?)
- (Relatively) unique to this course: in depth treatment of single/multilayer networks, neurodynamics, committee machines, information theoretic models, recurrent networks, etc.

Neural Networks in the Brain

- Human brain “computes” in an entirely different way from conventional digital computers.
- The brain is highly complex, nonlinear, and parallel.
- Orgnization of neurons to perform tasks much faster than computers. (Typical time taken in visual recognition tasks is 100–200 ms.)
- Key features of the biological brain: **experience** shapes the wiring through **plasticity**, and hence **learning** becomes the central issue in neural networks.

Neural Networks as an Adaptive Machine

A neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experimental knowledge and making it available for use.

Neural networks resemble the brain:

- Knowledge is acquired from the environment through a **learning process**.
- Inerneuron **connection strengths**, known as synaptic weights, are used to store the acquired knowledge.

Procedure used for learning: **learning algorithm**. Weights, or even the topology can be adjusted.

Benefits of Neural Networks

1. **Nonlinearity:** nonlinear components, distributed nonlinearity
2. **Input-output mapping**: supervised learning, nonparametric statistical inference (model-free estimation, no prior assumptions),
3. **Adaptivity:** either retain or adapt. Can deal with nonstationary environments. Must overcome *stability-plasticity dillema*.
4. **Evidential response:** decision plus *confidence* of the decision can be provided.
5. **Contextual information:** Every neuron in the network potentially influences every other neuron, so contextual information is dealt with naturally.

Benefits of Neural Networks (cont'd)

6. **Fault tolerance:** performance degrades gracefully.
7. **VLSI implementability:** network of simple components.
8. **Uniformity of analysis and design:** common components (neurons), sharability of theories and learning algorithms, and seamless integration based on modularity.
9. **Neurobiological analogy:** Neural nets motivated by neurobiology, and neurobiology also turning to neural networks for insights and tools.

Human Brain

Stimulus $\longrightarrow$ Receptors $\Leftrightarrow$ Neural Net $\Leftrightarrow$ Effectors $\longrightarrow$ Response

Arbib (1987)

- Pioneer: Santiago Ramón y Cajál, a Spanish neuroanatomist who introduced *neurons* as a fundamental unit of brain function.
- Neurons are slow: $10^{-3}s$ per operation, compared to $10^{-9}s$ of modern CPUs.
- Huge number of neurons and connections: 10^{10} (recent estimate is 10^{11}) neurons, 6×10^{13} connections in human brain.
- Highly energy efficient: $10^{-16}J$ per operation in the brain vs. $10^{-6}J$ in modern computers.

The Neuron and the Synapse

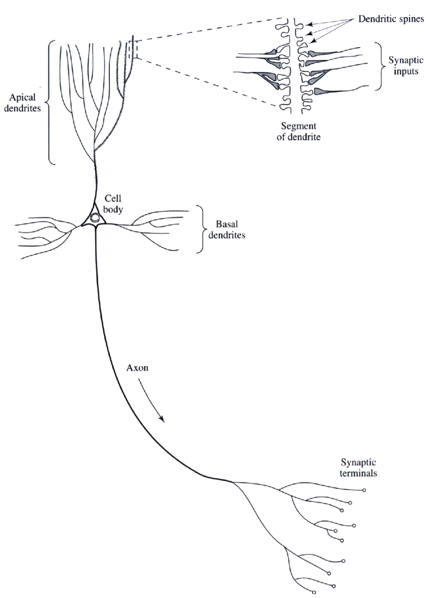
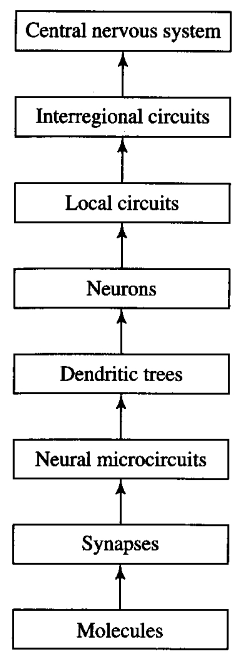


FIGURE 1.2 The pyramidal cell.

- Synapse: where two neurons meet.
- Presynaptic neuron: source
- Postsynaptic neuron: target
- Neurotransmitters: molecules that cross the synapse (positive, negative, or modulatory effect on postsynaptic activation)
- Dendrite: branch that receives input
- Axon: branch that sends out output (spike, or action potential traverses the axon and triggers neurotransmitter release at axon terminals).

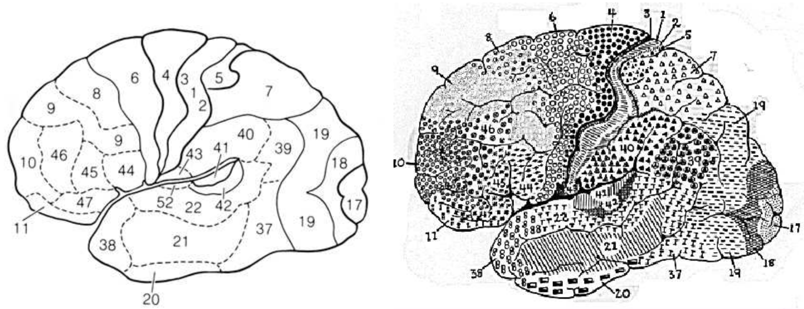
Structural Organization of the Brain



Small to large-scale organizations

- Molecules, Synapses, Neural microcircuits
- Dendritic trees, Neurons
- Local circuits
- Interregional circuits: pathways, columns, topographic maps
- Central nervous system

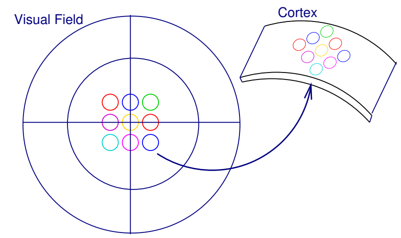
Cytoarchitectural Map of the Cerebral Cortex

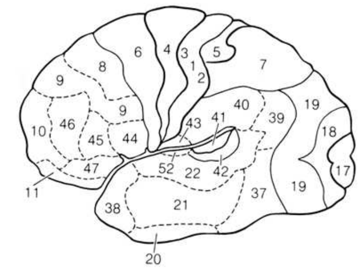


Map-like organization:

- Brodmann's cytoarchitectural map of the cerebral cortex.
- Area 17, 18, 19: visual cortices
- Area 41, 42: auditory cortices
- Area 1, 2, 3: somatosensory cortices (bodily sensation)

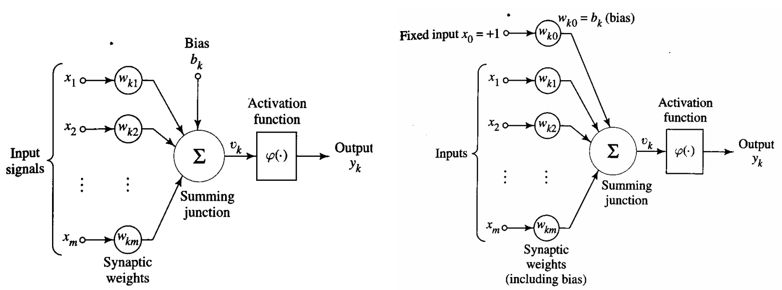
Topographic Maps in the Cortex





- Nearby location in the stimulus space are mapped to nearby neurons in the cortex.
- Thus, it is like a map of the sensory space, thus the term *topographic* organization.
- Many regions of the cortex are organized this way: visual (V1), auditory (A1), and somatosensory (S1) cortices.

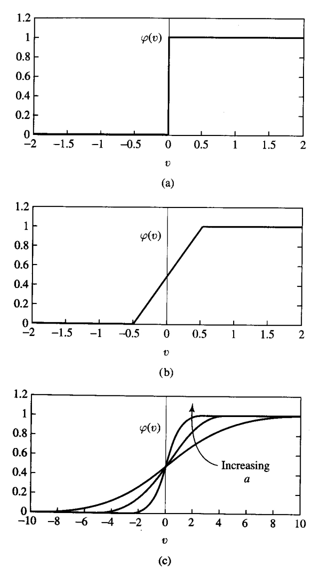
Models of Neurons



Neuron: information processing unit fundamental to neural network operation.

- Synapses with associated **weights**: j to k denoted w_{kj}.
- Summing junction: $u_k = \sum_{j=1}^{m} w_{kj} x_j$
- Activation function: $y_k = \phi(u_k + b_k)$
- Bias b_k: $v_k = u_k + b_k$, or $v_k = \sum_{j=0}^{m} w_{kj} x_j$ (in the right figure)

Activation Functions



- Threshold unit:

$$\phi(v) = \begin{cases} 1 \text{ if } v \geq 0 \\ 0 \text{ if } v < 0 \end{cases}$$

- Piece-wise linear:

$$\phi(v) = \begin{cases} 1 \text{ if } v \geq +\frac{1}{2} \\ v \text{ if } +\frac{1}{2} > v > -\frac{1}{2} \\ 0 \text{ if } v \leq -\frac{1}{2} \end{cases}$$

- Sigmoid: logistic function (a: slope parameter)

$$\phi(v) = \frac{1}{1 + \exp(-av)}$$

It is differentiable: $\phi'(v) = a\phi(v)(1 - \phi(v))$.

Other Activation Functions

- Signum function:

$$\phi(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{if } v = 0 \\ -1 & \text{if } v < 0 \end{cases}$$

- Sign function:

$$\phi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ -1 & \text{if } v < 0 \end{cases}$$

- Hyperbolic tangent function:

$$\phi(v) = \tanh(v)$$

Stochastic Models

- Instead of deterministic activation, stochastic activation can be done.
- x: state of neuron (+1 or -1); $P(v)$: probability of firing.

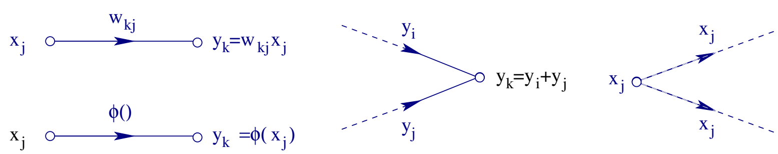
$$x = \begin{cases} +1 & \text{with probability } P(v) \\ -1 & \text{with probability } 1 - P(v) \end{cases}$$

- Typical choice of $P(v)$:

$$P(v) = \frac{1}{1 + \exp(-v/T)}$$

  where T is a pseudotemperature. When $T \to 0$, the neuron becomes deterministic.
- In computer simulations, use the **rejection method**.

Signal-flow Graphs



- Nodes and links
- Links: synaptic links and activation links.
- Incoming edges: summation
- Outgoing edges: replication

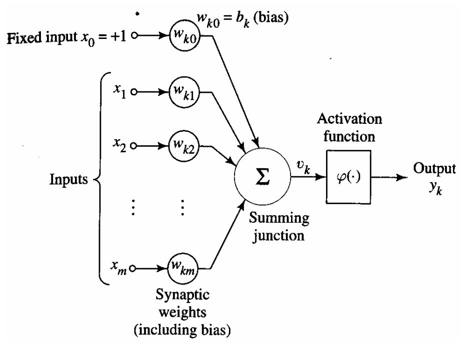
Architectureal graph simplifies the above and abstracts out internal neuronal function.

Definition of a Neural Network

An information processing system that has been developed as a generalization of **mathematical models of human cognition or neurobiology**, based on the assumptions that

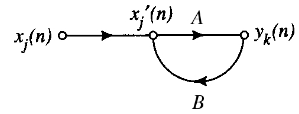
- Information processing occurs at many **simple elements called neurons**.
- Signals are passed between neurons over **connection links**.
- Each connection link has an **associated weight**, which typically multiplies the signal transmitted.
- Each neuron applies an **activation function** (usually non-linear) to its net input (sum of weighted input signals) to determine its output signal.

Signal-flow Graph Example: Exercise



- Turn the above into a signal-flow graph.

Feedback



Feedback gives dynamics (temporal aspect), and it is found in almost every part of the nervous system in every animal.

$$y_k(n) = A[x_j'(n)] (1)$$

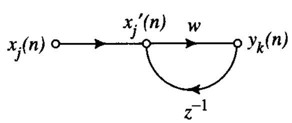
$$x_j'(n) = x_j(n) + B[y_k(n)] (2)$$

Substitute (2) into (1) and we get

$$y_k(n) = \frac{A}{1 - AB}[x_j(n)]$$

where $A/(1 - AB)$ is called the *closed-loop operator* and AB the *open loop operator*. Note that $BA \neq AB$.

Feedback (cont'd)



Substituting w for A and unit delay operator z^{-1} for B, we get

$$\frac{A}{1 - AB} = \frac{w}{1 - wz^{-1}} = w(1 - wz^{-1})^{-1}.$$

Using binomial expansion $(1 - x)^{-r} = \sum_{k=0}^{\infty} \frac{(r)_k}{k!} x^k$ & $r = 1$, we get [a]

$$\frac{A}{1 - AB} = w(1 - wz^{-1})^{-1} = w \sum_{l=0}^{\infty} w^l z^{-l}.$$

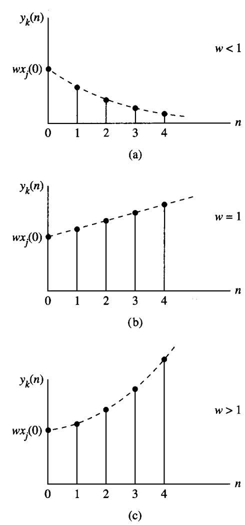
From this, we get

$$y_k(n) = w \sum_{l=0}^{\infty} w^l z^{-l} [x_j(n)].$$

With $z^{-l}[x_j(n)] = x_j(n - l)$, $y_k(n) = \sum_{l=0}^{\infty} w^{l+1} x_j(n - l)$.

[a] Pochhammer symbol $(r)_k = r(r+1)\ldots(r+k-1)$. Note: $(r)_k = k!$ when $r = 1$.

Feedback (cont'd)



$y_k(n) = \sum_{l=0}^{\infty} w^{l+1} x_j(n - l)$, so w determines the behavior. With a fixed $x_j(0)$, the output $y_k(n)$ either converges or diverges.

- $|w| < 1$: converge (*infinite memory*, *fading*)
- $|w| = 1$: linearly diverge
- $|w| > 1$: expontially diverge

Network Architectures

The **connectivity** of a neural network is intimately linked with the learning algorithm.

- **Single-layer feedforward** networks: one input layer, one layer of computing units (output layer), acyclic connections.
- **Multilayer feedforward** networks: one input layer, one (or more) hidden layers, and ont output layer. With more hidden layers, higher-order statistics can be processed.
- **Recurrent** networks: feedback loop exists.

Layers can be *fully connected* or *partially connected*.

Knowledge Representation

Knowledge refers to stored information or models used by a person or a machine to interpret, predict, and appropriately respond to the outside world.

- **What** information is actually **made explicit**.
- **How** the information is **physically encoded** for subsequent use.

Knowledge of the world consists of **two kinds** of information:

- The **known world state**: what is and what has been known – prior informaiton.
- **Observations** (measurements) of the world, obtained by sensors (they can be noisy). They provide *examples*. Examples can be *labeled* or *unlabeled*.

Design of Neural Networks

- Select architecture, and gather input samples and train using a learning algorithm (**learning** phase).
- Test with data not seen before (**generalization** phase).
- So, it is *data-driven*, unlike conventional programming.

Design of Representations

1. Similar inputs from similar classes should produce similar representations, leading to classification into the same category.
2. Items to be categorized as separate classes should be given widely different representations in the network.
3. If a particular feature is important, a larger number of neurons should be involved in the representation of the item in the network.
4. Prior information and invariances should be built into the design of a neural network with a *specialized structure*: biologically plausible, fewer free parameters, faster information transfer, and lower cost in building the network.

Similarity Measures

Similar inputs from similar classes should produce similar representations, leading to classification into the same category.

- Reciprocal of Euclidean distance $1/d(\mathbf{x}_i, \mathbf{x}_j)$:

$$\mathbf{x}_i = [x_{i1}, x_{i2}, ..., x_{im}]^T$$

$$d(\mathbf{x}_i, \mathbf{x}_j) = \|\mathbf{x}_i - \mathbf{x}_j\| = \left[\sum_{k=1}^{m}(x_{ik} - x_{jk})^2\right]^{1/2}$$

- Dot product (inner product)

$$(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i^T\mathbf{x}_j = \sum_{k=1}^{m} x_{ik}x_{jk} = \|\mathbf{x}_i\|\|\mathbf{x}_j\|\cos\theta_{ij}.$$

The two are related, when $\|\mathbf{x}_i\| = \|\mathbf{x}_j\| = 1$:

$$d^2(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{m}(x_{ik} - x_{jk})^2 = (\mathbf{x}_i - \mathbf{x}_j)^T(\mathbf{x}_i - \mathbf{x}_j) = 2 - 2\mathbf{x}_i^T\mathbf{x}_j.$$

Similarity Measures (cont'd)

When two vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are drawn from two distributions:

- Mean vector: $\mu_i = E[\mathbf{x}_i]$
- Mahalanobis distance:

$$d_{ij}^2 = (\mathbf{x}_i - \mu_i)^T\Sigma^{-1}(\mathbf{x}_j - \mu_j).$$

- Covariance matrix is assumed to be the same:

$$\Sigma = E[(\mathbf{x}_i - \mu_i)(\mathbf{x}_i - \mu_i)^T] = E[(\mathbf{x}_j - \mu_j)(\mathbf{x}_j - \mu_j)^T]$$

Building Prior Information into Neural Network Design

- Restrict network architecture: receptive fields
- Constrain the choice of synaptic weights: weight sharing

Building Invariance into Neural Network Design

- Invariance by structure
- Invariance by training
- Invariant feature space