Overview

- Learning in general
- Inductive learning (chapter 18)
- Statistical learning: neural networks (chapter 20; old 19)

Learning

- Adapt through interaction with the world: rote memory to developing a complex strategy
- Types of learning:
  1. Supervised learning (dense feedback)
  2. Unsupervised learning (no feedback)
  3. Reinforcement learning (sparse feedback, environment altering), etc.
- Advantages (two, among many):
  1. Fault tolerance
  2. No need for a complete specification to begin with
- Becoming a central focus of AI.

Inductive Learning

- Given **example** pairs $(x, f(x))$, return a function h that approximates the function f:
  - **pure inductive inference,** or **induction**.
- The function h is called a **hypothesis**.

Training and Testing

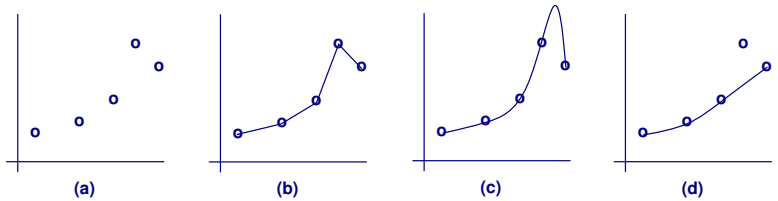
Different Types of Error

- Training error
- Validation error
- Test error

Issues

- Generalization
- Bias-Variance dillema
- Overfitting, underfitting
- Model complexity

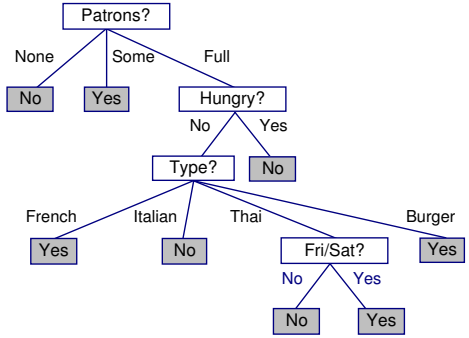
Inductive Learning and Inductive Bias



Given (a) as the training data, we can come up with several different hypotheses: (b) to (d)

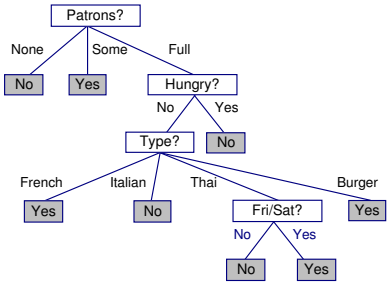
- selection of one hypothesis over another is called a **inductive bias** (don't confuse with other things called bias).
  - exact match to training data
  - prefer imprecise but smooth approximation
  - etc.

Decision Trees



- learn to approximate **discrete-valued** target functions.
- step-by-step decision making (disjunction of conjunctions)
- applications: medical diagnosis, assess credit risk of loan applicants, etc.

Decision Trees: What They Represent

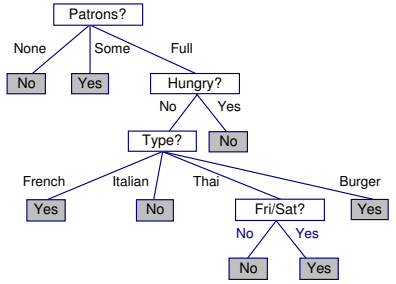


Wait or not (**Yes**/**No**)? The decision tree above corresponds to:

$$(Patrons = Some)$$
$$\vee(Patrons = Full \wedge Hungry = No \wedge Type = French)$$
$$\vee(Patrons = Full \wedge Hungry = No \wedge Type = Thai \wedge Fri/Sat = Yes)$$
$$\vee(Patrons = Full \wedge Hungry = No \wedge Type = Burger)$$

Decision trees represent disjunction of conjunctions.

Decision Trees: What They Represent (cont'd)



- In other words, for each instance (or example), there are attributes (Patrons, Hungry, etc.) and each instance have a full attribute value assignment.
- For a given instance, it is classified into different discrete classes by the decision tree.
- For training, many $(instance, class)$ pairs are used.

Constructing Decision Trees from Examples

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	*Yes*

- Given a set of examples (**training set**), both **positive** and **negative**, the task is to construct a decision tree that describes a concise decision path.
- Using the resulting decision tree, we want to **classify** new instances of examples (either as **yes** or **no**).

Constructing Decision Trees: Trivial Solution

- A trivial solution is to explicitly construct paths for each given example.
- The problem with this approach is that it is not able to deal with situations where, some attribute values are missing or new kinds of situations arise.
- Consider that some attributes may not count much toward the final classification.

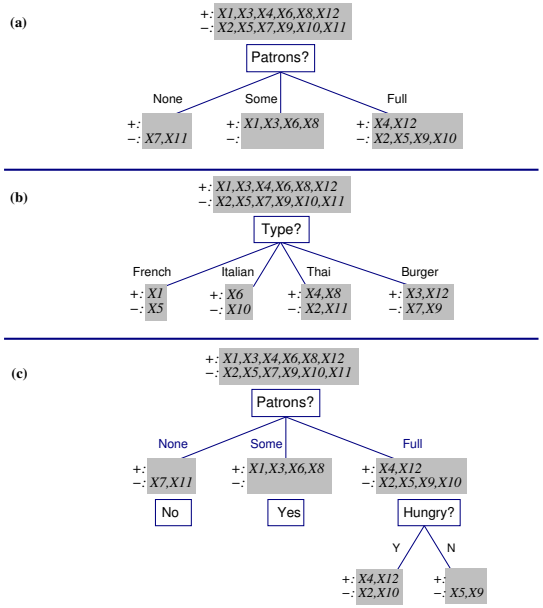
Finding a Concise Decision Tree

- Memorizing all cases may not be the best way.
- We want to extract a decision pattern that can describe a large number of cases in a **concise** way.
- Such an inductive bias is called **Ockham's razor**: *The most likely hypothesis is the simplest one that is consistent with all observations.*
- In terms of a decision tree, we want to make as few tests before reaching a decision, i.e. the depth of the tree should be shallow.

Finding a Concise Decision Tree (cont'd)

- Basic idea: pick up attributes that can clearly separate positive and negative cases.
- These attributes are more important than others: the final classification heavily depend on the value of these attributes.

Finding a Concise Decision Tree (cont'd)

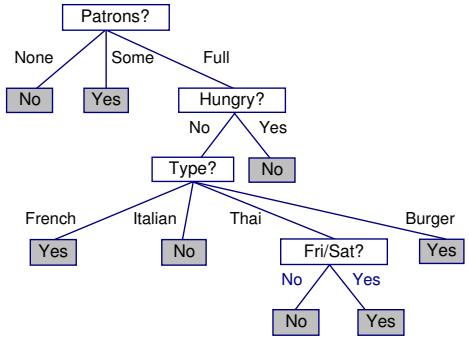


Decision Tree Learning Algorithm

function DECISION-TREE-LEARNING(*examples*, *attributes*, *default*) **returns** a decision tree
  inputs: *examples*, set of examples
    attributes, set of attributes
    default, default value for the goal predicate

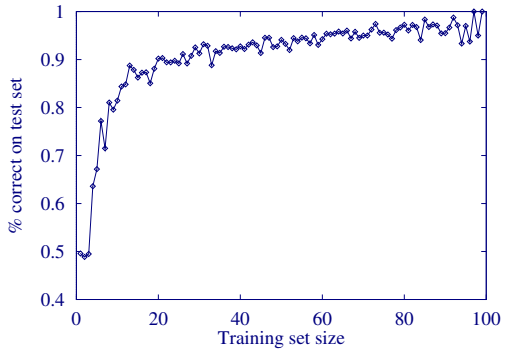
  if *examples* is empty **then return** *default*
  else if all *examples* have the same classification **then return** the classification
  else if *attributes* is empty **then return** MAJORITY-VALUE(*examples*)
  else
    $best \leftarrow$ CHOOSE-ATTRIBUTE(*attributes*, *examples*)
    tree $\leftarrow$ a new decision tree with root test *best*
    for each value v_i of *best* **do**
      $examples_i \leftarrow$ {elements of *examples* with $best = v_i$}
      subtree $\leftarrow$ DECISION-TREE-LEARNING($examples_i$, $attributes - best$,
                MAJORITY-VALUE(*examples*))
      add a branch to *tree* with label v_i and subtree *subtree*
    end
    return *tree*

Resulting Decision Tree



- Some attributes are not tested at all.
- Odd paths can be generated (Thai food branch).
- Sometimes the tree can be incorrect for new examples (exceptional cases).

Accuracy of Decision Trees



- Divide examples into training and test sets.
- Train using the training set.
- Measure accuracy of resulting decision tree on the test set.

Choosing the Best Attribute to Test First

Use Shannon's information theory to choose the attribute that give the maximum **information gain**.

- Pick an attribute such that the information gain (or entropy reduction) is maximized.
- Entropy measures the **average surprisal** of events. Less probable events are more surprising.

Entropy and Information Gain

$$Entropy(E) = \sum_{i \in C} -P_i log_2(P_i)$$

$$Gain(E, A) = Entropy(E) - \sum_{v \in Values(A)} \frac{|E_v|}{|E|} Entropy(E_v)$$

- E: set of examples
- A: a single attribute
- E_v: set of examples where attribute $A = v$.
- $|S|$: cardinality of set S.

Issues in Decision Tree Learning

- Noise and overfitting
- Missing attribute values from examples
- Multi-valued attributes with large number of possible values
- Continuous-valued attributes.

Key Points

Decision tree learning:

- What is the embodied principle (or bias)?
- How to choose the best attribute? Given a set of examples, choose the best attribute to test first.
- What are the issues? noise, overfitting, etc.

Neural Networks

Neural networks is one particular form of learning from data.

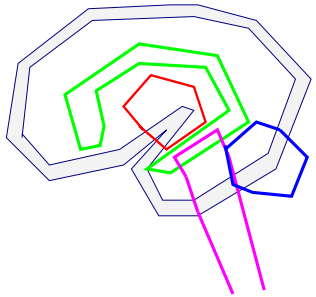
- simple processing elements: named units, or neurons
- connective structure and associated connection weights
- learning: adaptation of connection weights

Neural networks mimic the human (or animal) nervous system.

Many Faces of Neural Networks

- Abstract mathematical/statistical model
- Optimization algorithm
- Pattern recognition algorithm
- Tools for understanding the function of the brain
- Robust engineering application

The Central Nervous System



- Cortex: thin outer sheet where most of the neurons are.
- Sub-cortical nuclei: thalamus, hippocampus, basal ganglia, etc.
- Midbrain, pons, and medulla, connects to the spinal cord.
- Cerebellum (hind brain, or small brain)

Function of the Nervous System

Function of the nervous system:

- Perception
- Cognition
- Motor control
- Regulation of essential bodily functions

The Central Nervous System: Facts [a]

Facts: human neocortex

- Thickness: 1.6mm
- Area: 36cm $\times$ 36cm (about 1.4 ft^2)
- Neurons: 10 billion (10^{10})
- Connections: 60 trillion (6×10^{13}) to 100 trillion
- Connections per neuron: 10^4
- Energy usage per operation: 10^{-16}J (compare to 10^{-6}J in modern computers)

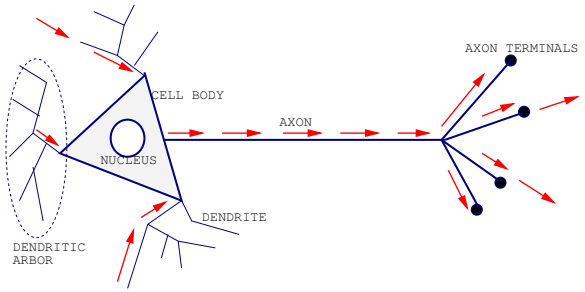
[a] *Neural networks: a comprehensive foundation* by Simon Haykin (1994), and *Foundations of Vision* by Brian Wandell (1995). May slightly differ from those in Russel & Norvig. No need to memorize these figures.

How the Brain Differs from Computers

- Densely connected.
- Massively parallel.
- Highly nonlinear.
- Asynchronous: no central clock.
- Fault tolerant.
- Highly adaptable.
- Creative.

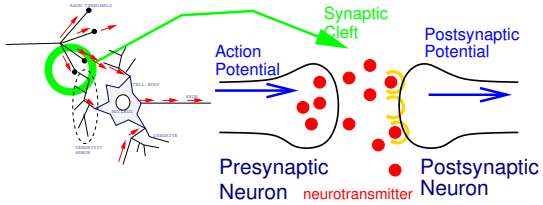
Why are these crucial?

Neurons: Basic Functional Unit of the Brain



- Dendrites receive input from upstream neurons.
- Ions flow in to make the cell positively charged.
- Once a firing threshold is reached, a spike is generated and transmitted along the axon.
- Axon terminals release neurotransmitters to relay the signal to the downstream neurons.

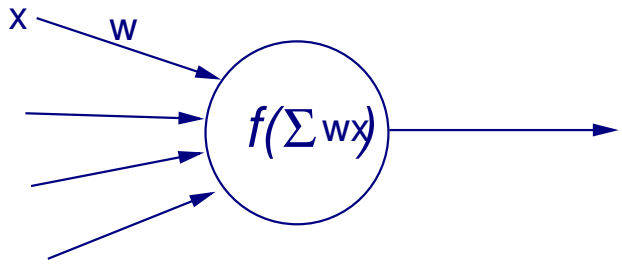
Propagation of Activation Across the Synapse



1. Action potential reaches axon terminal.
2. Neurotransmitters are released into synaptic cleft and bind to postsynaptic cell's receptors.
3. Binding allows ion channels to open (Na^+), and Na^+ ions flows in and makes the postsynaptic cell depolarize.
4. Once the membrane voltage reaches the threshold, an action potential is generated.

Lesson: neural activity propagation has a very complex cellular/molecular mechanism.

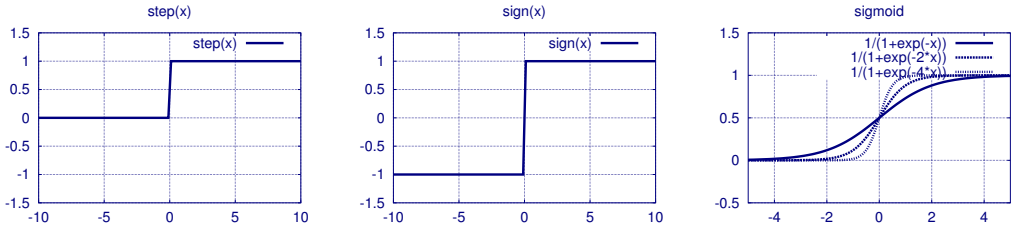
Abstraction of the Neuron in Neural Networks



- Input
- Connection weight
- Transfer function: $f(\cdot)$

Typical transfer functions: step-function or sigmoid.

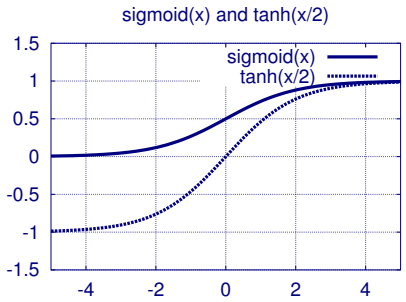
Typical Activation Functions



- $Step_t(x) = 1$ if $x \geq t, 0$ if $x < t$
- $Sign(x) = +1$ if $x \geq 0, -1$ if $x < 0$
- $Sigmoid(x) = \frac{1}{1+e^{-x}}$

Note that $Step_t(x) = Step_0(x - t)$, which we will simply call $Step(x - t)$.

More Activation Functions: $tanh(\frac{x}{2})$



- $Sigmoid(x) = \frac{1}{1+e^{-x}}$
- $tanh(\frac{x}{2}) = \frac{1-e^{-x}}{1+e^{-x}}$

Classification of Neural Networks

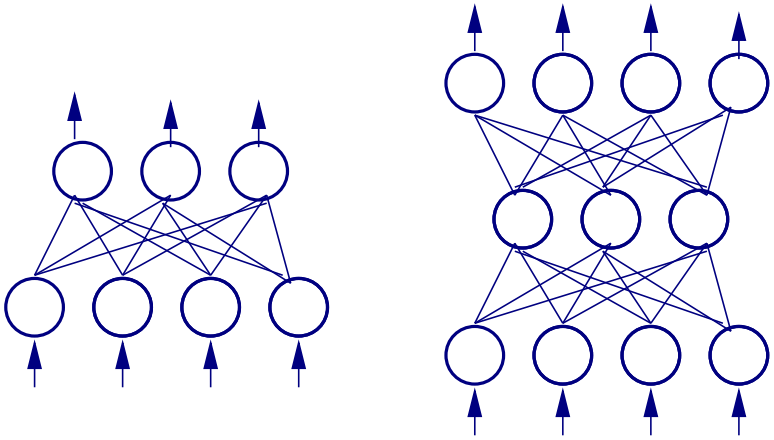
Teacher exists?

- Supervised (with teacher): perceptrons, backpropagation network, etc.
- Unsupervised (no teacher): self-organizing maps, etc.

Recurrent connections?

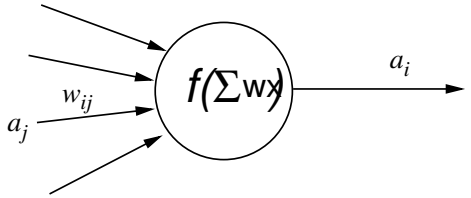
- Feed-forward: perceptrons, backpropagation network, etc.
- Recurrent: Hopfield network, Boltzmann machines, SRN (simple recurrent network), etc.

Feedforward Networks



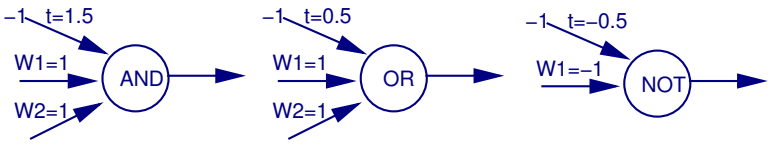
- Perceptrons: single layer, threshold-gated.
- Backpropagation networks: multiple layers, sigmoid (or tanh) activation function.

Perceptrons



$$
\begin{aligned}
a_i &= step_t(\sum_{j=1}^{n} W_{ij} a_j) \\
&= step_0(\sum_{j=1}^{n} W_{ij} a_j - t) \\
&= step_0(t \times (-1) + \sum_{j=1}^{n} W_{ij} a_j) \\
&= step_0(\sum_{j=0}^{n} W_{ij} a_j), \text{ where } W_{i0} = t \text{ and } a_0 = -1 \quad (1)
\end{aligned}
$$

Boolean Logic Gates with Perceptron Units [a]

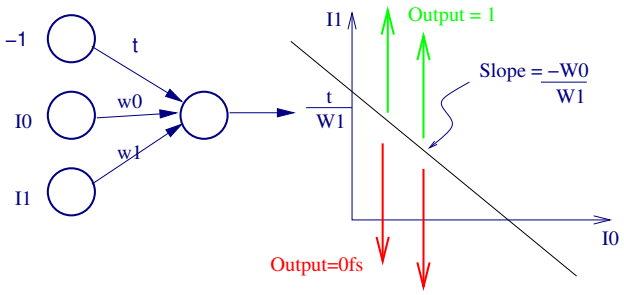


- Note that the activation function is the $step_0(\cdot)$ function.
- Perceptrons can represent basic boolean functions.
- Thus, a network of perceptron units can compute any Boolean function.

What about XOR or EQUIV?

[a] Same as Russel & Norvig p.570, figure 19.6

Limitation of Perceptrons



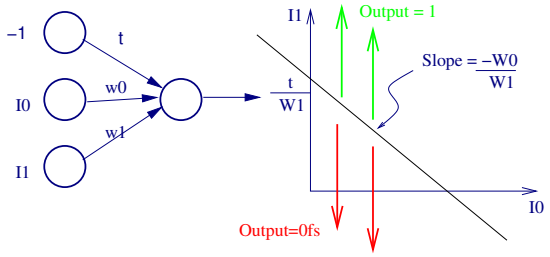
Perceptrons can only represent **linearly-separable** functions.

- Output of the perceptron:

$$W_0 \times I_0 + W_1 \times I_1 - t \geq 0, \text{ then output is } 1$$

$$W_0 \times I_0 + W_1 \times I_1 - t < 0, \text{ then output is } 0$$

Limitation of Perceptrons (cont'd)



- A geometrical interpretation of this is:

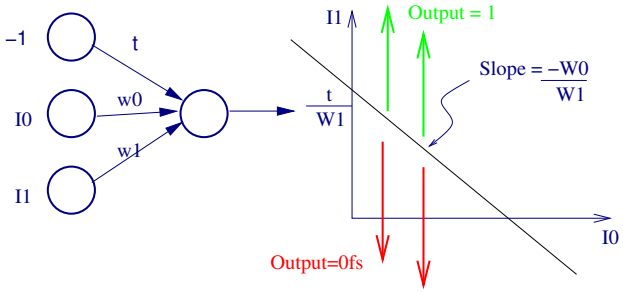
$$I_1 = \frac{-W_0}{W_1} \times I_0 + \frac{t}{W_1},$$

where points on or above the line, the output is 1, and 0 for those below the line (when W_1 is positive). Compare with

$$y = \frac{-W_0}{W_1} \times x + \frac{t}{W_1}.$$

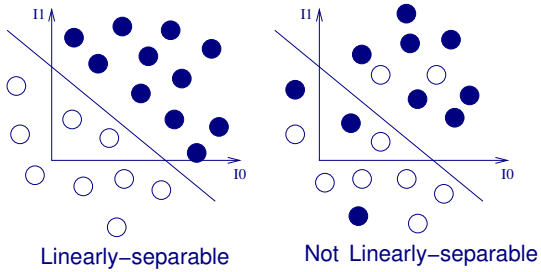
Note: When dividing both sides with W_1, depending on the sign, the inequality can flip its direction (see previous page).

Limitation of Perceptrons (cont'd)



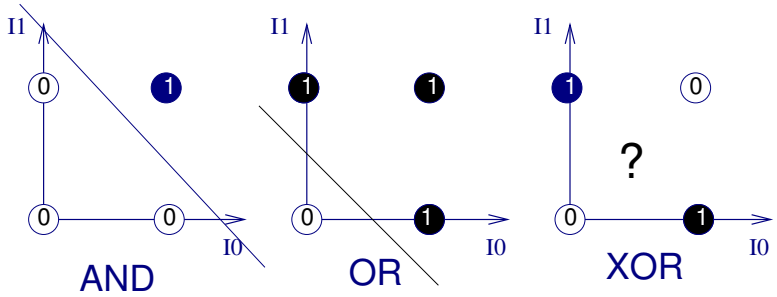
- Thus, only functions where the points that result in 0 and 1 as output can be separated by a line can be represented by perceptrons.
- **Note:** the previous result is generalizable to functions of n arguments, i.e. perceptron with n inputs plus one threshold (or bias) unit.

Linear Separability



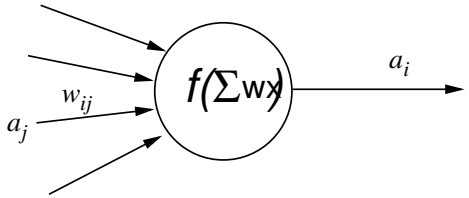
- For functions that take integer or real values as arguments and output either 0 or 1.
- Left: linearly-separable (i.e., can draw a straight line between the classes).
- Right: not linearly-separable (i.e., perceptrons cannot represent such a function)

Linear Separability (cont'd)



- Perceptrons cannot represent XOR!
- Minsky and Papert (1969)

Learning in Perceptrons



- The weights do not have to be calculated manually.
- We can train the network with (input,output) pair according to the following weight update rule:

$$W_{ij} \leftarrow W_{ij} + \alpha \times I_j \times Err,$$

where α is the learning rate parameter, I_j is the input (a_j in the figure), and $Err = DesiredOutput - NetworkOutput$.

Exercise: Implementing the Perceptron

- It is fairly easy to implement a perceptron.
- You can implement it in any programming language: C/C++, etc.
- Look for examples on the web, and JAVA applet demos.

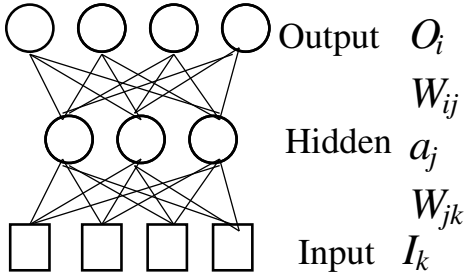
Key Points

- The central nervous system: how it differs from conventional computers.
- Basic mechanism of synaptic information transfer
- Types of neural networks
- Perceptrons: basic idea, and the geometric interpretation. What is the limitation? How to train?

Overview

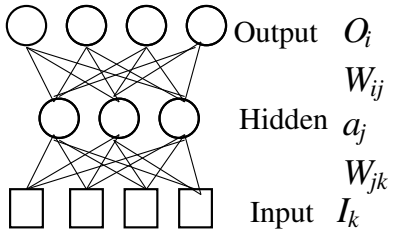
- Multilayer feed-forward networks
- Gradient descent search
- Backpropagation learning rule
- Evaluation of backpropagation
- Applications of backpropagation

Multilayer Feed-Forward Networks



- Proposed in the 1950s
- Proper procedure for training the network came later (1969) and became popular in the 1980s: **back-propagation**

Back-Propagation Learning Rule

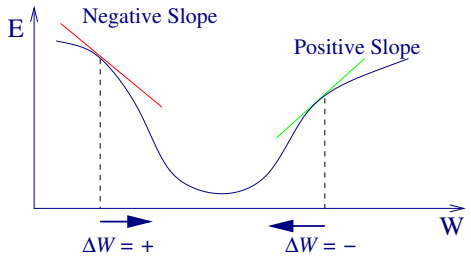


- Back-prop is basically a **gradient descent** algorithm.
- The tough problem: output layer has explicit error measure, so finding the error surface is trivial. However, for the hidden layers, how much error each connection eventually cause at the output nodes is hard to determine.
- Backpropagation determines how to distribute the **blame** to each connection.

Gradient Descent

- We want to minimize the total error E by tweaking the network weights.
- E depends on W_i, thus by adjusting W_i, you can reduce E.
- Figuring out how to simultaneously adjust weights W_i for all i at once is practically impossible, so use an iterative approach.
- A sensible way is to reduce E with respect to one weight W_i at a time, proportional to the gradient (or slope) at that point.

Gradient Descent (cont'd)



- For weight W_i and error function E, to minimize E, W_i should be changed according to $W_i \leftarrow W_i + \Delta W_i$:

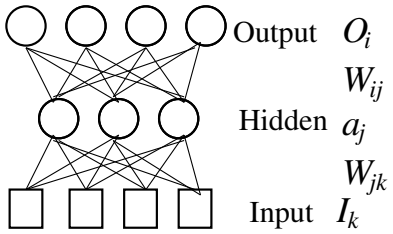
$$\Delta W_i = \alpha \times \left(-\frac{\partial E}{\partial W_i} \right),$$

where α is the learning rate parameter.

- E can be a function of many weights, thus the partial derivative is used in the above:

$$E(W_1, W_2, ..., W_i, ...W_n, ...)$$

Hidden to Output Weights

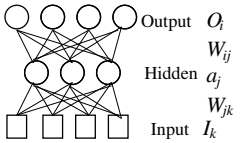


- Error function

$$\begin{aligned} E &= \frac{1}{2}\sum_i (E_i)^2 \\ E_i &= T_i - O_i \\ &= T_i - g\left(\sum_j W_{ij} a_j\right), \end{aligned}$$

where $g(\cdot)$ is the sigmoid activation function, and T_i the target.

Hidden to Output Weights (cont'd)



$$\begin{aligned} \frac{\partial E}{\partial W_{ij}} &= \frac{\partial\left(\frac{1}{2}\sum_i\left(T_i - g\left(\sum_j W_{ij}a_j\right)\right)^2\right)}{\partial W_{ij}} \\ &= \frac{\partial\left(\frac{1}{2}\left(T_i - g\left(\sum_j W_{ij}a_j\right)\right)^2\right)}{\partial W_{ij}} \\ &= (T_i - O_i) \times \left(-\frac{\partial g(\sum_j W_{ij}a_j)}{\partial W_{ij}}\right) \\ &= -(T_i - O_i) \times g'(\sum_j W_{ij}a_j) \times a_j \end{aligned}$$

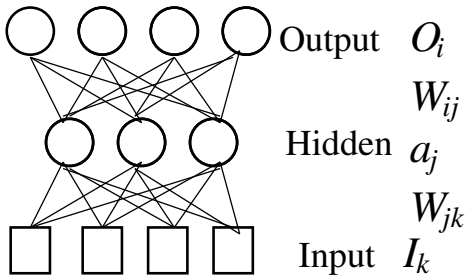
Hidden to Output Weights (cont'd)

- For easier calculation later on, we can rewrite:

$$\begin{aligned} \frac{\partial E}{\partial W_{ij}} &= -(T_i - O_i) \times g'(\sum_j W_{ij}a_j) \times a_j \\ &= -a_j \times \underbrace{(T_i - O_i) \times g'(\sum_j W_{ij}a_j)} \\ &= -a_j \times \Delta_i \end{aligned}$$

- It is easy to verify $g'(x) = g(x)(1 - g(x))$ from $g(x) = \frac{1}{1+e^{-x}}$, so we can reuse the $g(x)$ value from the feed-forward phase in the feedback weight update.

Hidden to Output Weight Update

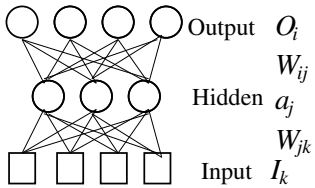


- From $\frac{\partial E}{\partial W_{ij}} = -a_j \times \Delta_i$, we get the update rule:

$$W_{ij} \leftarrow W_{ij} + \alpha \times a_j \times \Delta_i,$$

where $\Delta_i = (T_i - O_i) \times g'(\sum_j W_{ij}a_j)$.

Input to Hidden Weights

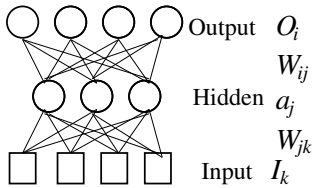


$$\frac{\partial E}{\partial W_{jk}} = \frac{\partial \left(\frac{1}{2} \sum_i \left(T_i - g\left(\sum_j W_{ij} a_j \right) \right)^2 \right)}{\partial W_{jk}}$$

$$= \frac{\partial \left(\frac{1}{2} \sum_i \left(T_i - g\left(\sum_j W_{ij} \left(g\left(\sum_k W_{jk} I_k \right) \right) \right) \right)^2 \right)}{\partial W_{jk}}$$

However, this is too complex.

Input to Hidden Weights (cont'd)



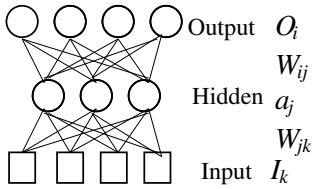
Use the chain rule for easier calculation of the partial derivative:

$$\frac{\partial E}{\partial W_{jk}} = \frac{\partial E}{\partial a_j} \times \frac{\partial a_j}{\partial W_{jk}}$$

$$= \underbrace{-\sum_i \left(\overbrace{(T_i - O_i) g'(\sum_j W_{ij} a_j)}^{\text{this is } \Delta_i} W_{ij} \right)}_{} \times \underbrace{g'(\sum_k W_{jk} I_k) I_k}$$

$$= \underbrace{-\sum_i \left(\Delta_i W_{ij} \right)}_{} \underbrace{g'(\sum_k W_{jk} I_k) I_k}$$

Input to Hidden Weight Update Rule



From $\frac{\partial E}{\partial W_{jk}}$, we can rename

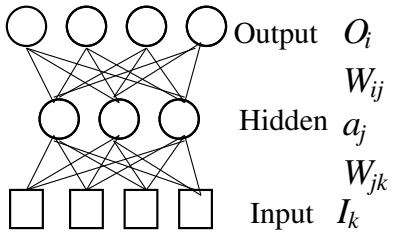
$\sum_i (\Delta_i W_{ij})\, g'(\sum_k W_{jk} I_k)$ to be Δ_j, then the whole equation becomes:

$$\frac{\partial E}{\partial W_{jk}} = -\Delta_j I_k$$

Thus the update rule becomes:

$$W_{jk} \leftarrow W_{jk} + \alpha \times \Delta_j \times I_k$$

Back-Propagation: Summary



- Weight update:

$$\Delta W_{yx} \propto \Delta_y \times Input_x$$

- The Δs:

$$\Delta_y = Error_y \times g'(WeightedSum_y)$$

Thus, each node has its own Δ and that is used to update the weights.
These Δs are backpropagated for weight updates further below.

General Case: More Than 2 Layers

- In general, the same rule for back-propagating Δs apply for multiple layer networks with more than two layers.
- That is, Δ for a deep hidden unit can be determined by the product of the weighted sum of feedback Δs and the first derivative of feedforward weighted sum at the current unit.

Backpropagation Algorithm

1. Pick $(Input, Target)$ pair.
2. Using input, activate hidden and output layers through feed-forward activation.
3. At the output node, calculate the error $(T_i - O_i)$, and from that calculate the Δs.
4. Update weights to the output layer, and backpropagate the Δs.
5. Successively update hidden layer weights until Input layer has been reached.
6. Repeat step1–5 until the total error goes below a set threshold.

Technical Issues in Training

- Batch vs. online training
  - Batch: accumulate weight updates for one epoch, and then update
  - Online: immediately apply weight updates after one input-output pair.
- When to stop training
  - Training set: use for training
  - Validation set: determine when to stop
  - Test set: use for testing performance

Problems With Backprop

- Learning can be extremely slow: introduce momentum, etc.
- Network can be stuck in local minima: this is a common problem for any gradient-based method.

Other issues are: how to introduce new batches of data after the training has been completed.

Backprop Application

- Speech generation: NetTALK (Sejnowski and Rosenberg, 1987)
- Character recognition: LeCun (1989)
- Driving a car: ALVINN, etc.

and many other Engineering applications – control, etc., especially nowadays in the form of deep neural networks (e.g., convolutional neural network).

Demo: NetTALK

- I want to I want to go to my grandmother's ....
- friend, sent, around, not, red, soon, doubt, key, attention, lost

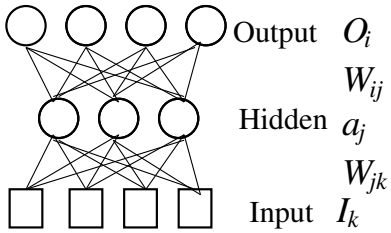
Key Points

- Basic concept of a multi-layer feed-forward network.
- How hidden units know how much error they caused.
- Backprop is a gradient descent algorithm.
- Drawbacks of backprop.

Overview

- More on backprop
- Self-organizing maps

Another Application of Backpropagation: Image Compression



- Image compression
  1. target output is the same as the input.
  2. hidden layer units are fewer than the output (and input) layer units.
  3. the hidden layer forms the compressed representation.

Improving Backpropagation [a]

To overcome the local minima problem:

- Adding momentum

$$\Delta W_{ij}(t) = \alpha \times \Delta_i \times I_j + \eta \times \Delta W_{ij}(t-1)$$

- Incremental update (as opposed to batch update) with **random input-target order**.
- Add a little bit of noise to the input.
- Allow increasing E with a small probability, as in Simulated Annealing.

[a] From Hertz et al., *Introduction to the Theory of Neural Computation*, Addison Wesley, 1991.

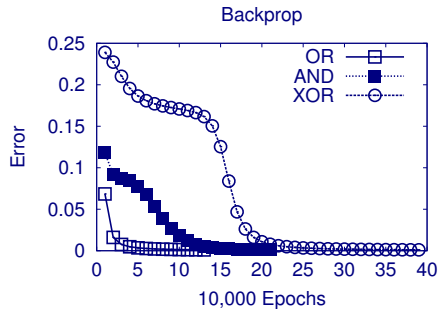
Backpropagation Exercise

- **URL:** http://www.cs.tamu.edu/faculty/choe/src/backprop-1.6.tar.gz
- Untar and read the `README` file:

```
gzip -dc backprop-1.6.tar.gz | tar
xvf -
```

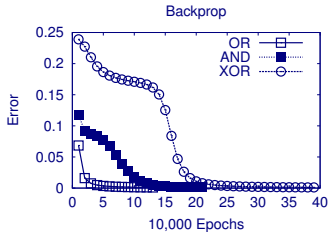
- Run `make` to build (on departmental unix machines).
- Run `./bp conf/xor.conf` etc.

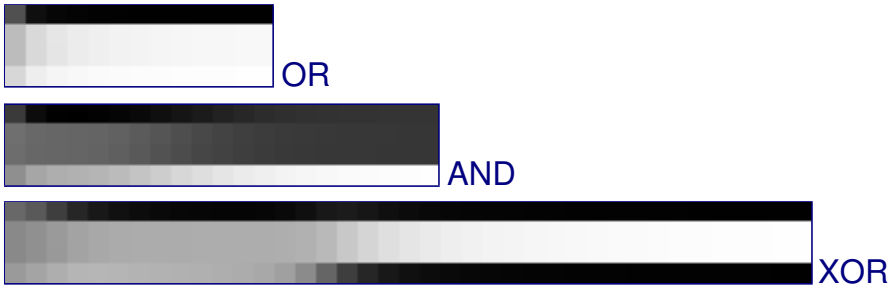
Backpropagation: Example Results



- Epoch: one full cycle of training through all training input patterns.
- `OR` was easiest, `AND` the next, and `XOR` was the most difficult to learn.
- Network had 2 input, 2 hidden and 1 output unit. Learning rate was 0.001.

Backpropagation: Example Results (cont'd)





Output to (0,0), (0,1), (1,0), and (1,1) form each row.

Backpropagation: Things to Try

- How does increasing the number of hidden layer units affect the (1) time and the (2) number of epochs of training?
- How does increasing or decreasing the learning rate affect the rate of convergence?
- How does changing the slope of the sigmoid affect the rate of convergence?
- Different problem domains: handwriting recognition, etc.

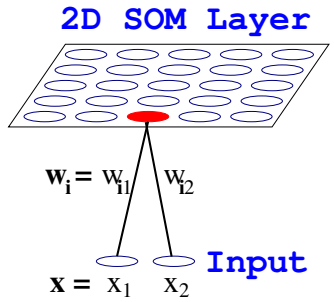
Unsupervised Learning

- No teacher signal (i.e. no feedback from the environment).
- The network must discover patterns, features, regularities, correlations, or categories in the input data and code them in the output.
- The units and connections must display some degree of **self-organization**.
- Unsupervised learning can be useful when there is **redundancy** in the input data.
- A data channel where the input data content is less than the channel capacity, there is redundancy.

What Can Unsupervised Learning Do?

- **Familiarity**: how similar is the current input to past inputs?
- **Principal Component Analysis**: find orthogonal basis vectors (or axes) against which to project high dimensional data.
- **Clustering**: n output class, each representing a distinct category. Each cluster of similar or nearby patterns will be classified as a single class.
- **Prototyping**: For a given input, the most similar output class (or **exemplar**) is determined.
- **Encoding**: application of clustering/prototyping.
- **Feature Mapping**: topographic mapping of input space onto output network configuration.

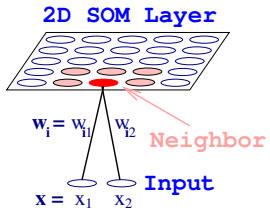
Self-Organizing Map (SOM)



Kohonen (1982)

- 1-D or 2-D layout of units.
- One reference vector for each unit.
- Unsupervised learning (no target output).

SOM Algorithm



1. Randomly initialize reference vectors $\mathbf{w}_i$
2. Randomly sample input vector $\mathbf{x}$
3. Find Best Matching Unit (BMU):

$$i(\mathbf{x}) = \text{argmin}_j \parallel \mathbf{x} - \mathbf{w}_j \parallel$$

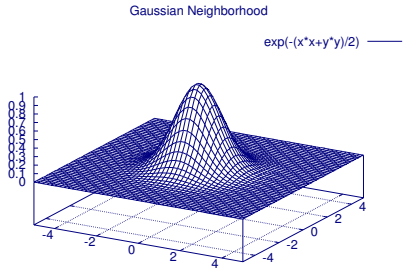
4. Update reference vectors:

$$\mathbf{w}_j \leftarrow \mathbf{w}_j + \alpha\Lambda(j, i(\mathbf{x}))(\mathbf{x} - \mathbf{w}_j)$$

α : learning rate
$\mathbf{\Lambda}(j, i(\mathbf{x}))$: neighborhood function of BMU.

5. Repeat steps 2 – 4.

Typical Neighborhood Functions

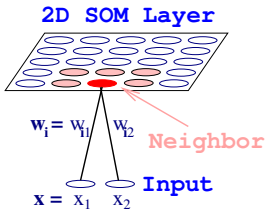


- Gaussian: $\Lambda(j, i(\mathbf{x})) = \exp(-|\mathbf{r}_j - \mathbf{r}_{i(\mathbf{x})}|^2/2\sigma^2)$
- Flat: $\Lambda(j, i(x)) = 1$ if $|\mathbf{r}_j - \mathbf{r}_{i(\mathbf{x})}| \leq \sigma$, and 0 otherwise.
- σ is called the **neighborhood radius**.

Training Tips

- Start with large neighborhood radius.
  Gradually decrease radius to a small value.
- Start with high learning rate α.
  Gradually decrease α to a small value.

Properties of SOM



- **Approximation of input space.**
  Maps continuous input space to discrete output space.
- **Topology preservation.**
  Nearby units represent nearby points in input space.
- **Density mapping.**
  More units represent input space that are more frequently sampled.

Performance Measures

- **Quantization Error**
  Average distance between each data vector and its BMU.

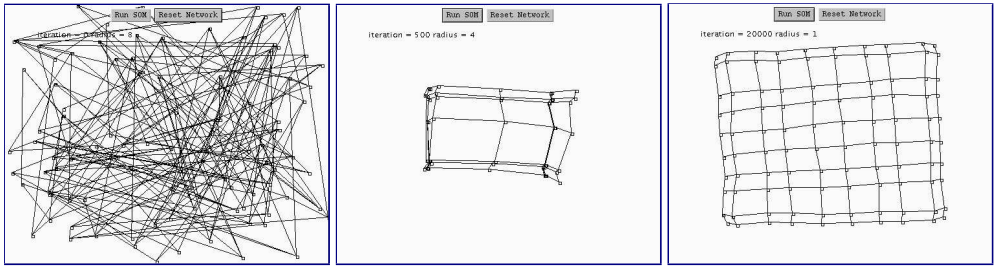
$$\epsilon_Q = \frac{1}{N}\sum_{j=1}^{N} \| \mathbf{x}_j - \mathbf{w}_{i(\mathbf{x}_j)} \|$$

- **Topographic Error**
  The proportion of all data vectors for which first and second BMUs are not adjacent units.

$$\epsilon_T = \frac{1}{N}\sum_{j=1}^{N} u(\mathbf{x}_j),$$

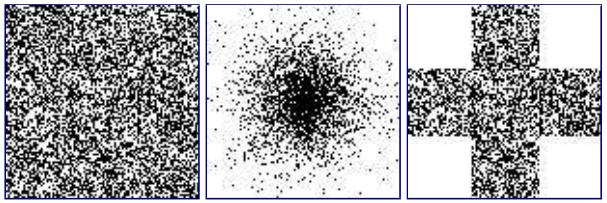
  $u(\mathbf{x})$ = 1 if the 1st and 2nd BMUs are not adjacent
  $u(\mathbf{x})$ = 0 otherwise.

Example: 2D Input / 2D Output



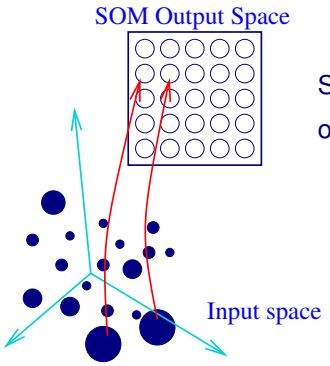
- Train with uniformly random 2D inputs.
  Each input is a point in Cartesian plane.
- Nodes: reference vectors (x and y coordinate).
- Edges: connect immediate neighbors on the map.

Different 2D Input Distributions



- What would the resulting SOM map look like?
- Why would it look like that?

High-Dimensional Inputs



SOM can be trained with inputs of arbitrary dimension.

- Dimensionality reduction: **N-D** to **2-D**.
- Extracts topological features.
- Used for visualization of data.

Applications

- **Data clustering and visualization.**
- **Optimization problems:**
  Traveling salesman problem.
- **Semantic maps:**
  Natural language processing.
- **Preprocessing for signal and image-processing.**
  1. Hand-written character recognition.
  2. Phonetic map for speech recognition.

Exercise

1. What happens when $\mathbf{N}_{i(\mathbf{x})}$ and α was reduced quickly vs. slowly?
2. How would the map organize if different input distributions are given?
3. For a fixed number of input vectors from real-world data, a different visualization scheme is required. How would you use the number of input vectors that best match each unit to visualize the property of the map?

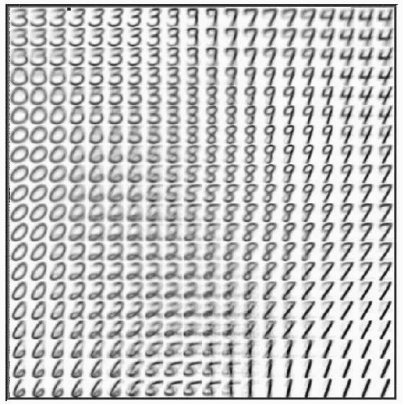
Key Points

- How can backprop be improved?
- What are the various ways to apply backprop?
- SOM basic algorithm
- What kind of tasks is SOM good for?

Overview

- SOM demo
- Recurrent networks
- Genetic Algorithms

SOM Example: Handwritten Digit Recognition



- Preprocessing for feedforward networks (supervised learning).
- Better representation for training.
- Better generalization.

SOM Demo

Jochen Fröhlich's *Neural Networks with JAVA* page:

`http://rfhs8012.fh-regensburg.de/ saj39122/jfroehl/diplom/e-index.html`

Check out the `Sample Applet` link.

SOM Demo: Traveling Salesman Problem

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 8.
- Stop when radius $<$ 0.001.
- Try 50 nodes, 20 input points.

Click on `[Parameters]` to bring up the config panel. After the parameters are set, click on `[Reset]` in the main applet, and then `[Start learning]`.

SOM Demo: Space Filling in 2D

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 100.
- Stop when radius $<$ 0.001.
- Try 1000 nodes, and 1000 input points.

SOM Demo: Space Filling in 3D

Using Fröhlich's SOM applet:

- 2D SOM map ($n \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 10.
- Stop when radius $<$ 0.001.
- Try 30×30 nodes, and 500 input points. Limit the y range to 15.

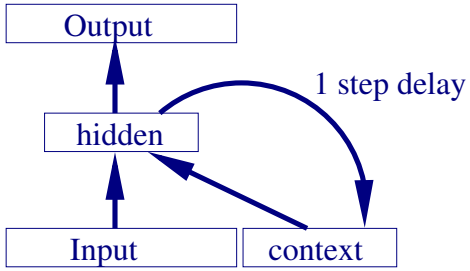
Also try 50×50, 1000 input points, and 16 initial radius.

Recurrent Networks

Connection graph can contain cycles, e.g. reciprocal connections: i.e. not strictly feed-forward.

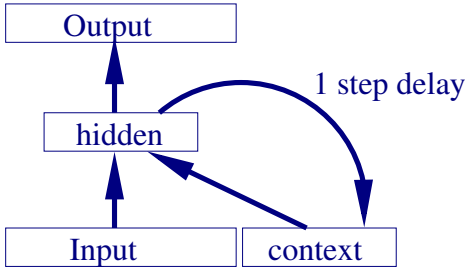
- Statistical mechanics based models (associative or content-addressable memory): Hopfield network, Boltzmann machines, etc.
- Sequence encoding: Simple Recurrent Network, etc.
- Other biologically motivated networks: laterally connected self-organizing maps, etc.

Simple Recurrent Network (Elman Network)



- Sequence encoding.
- Hidden layer activation from previous time step used as input.
- Use standard back-propagation learning.

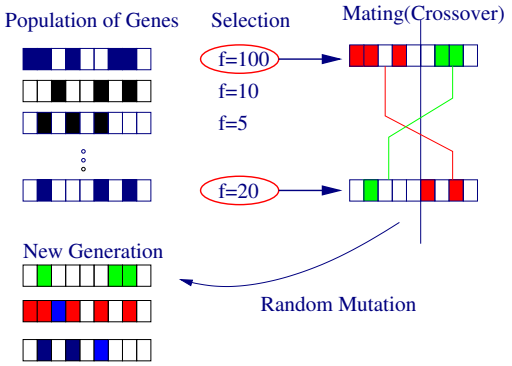
SRN Example



Example input and target sequence: output 1 when two 1s appear in a row.

Time:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Input:	1	0	1	0	1	1	0	0	0	1	1	1	1	0	1	0	1	1	0	1
Target:	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	1	0	0

Genetic Algorithms



Evolution as a problem solving strategy:

- population of solutions, where each chromosome represent an individual
- selection based on fitness function: survival of the fittest
- mating (cross-over) and reproduction
- random mutation

Properties of Genetic Algorithms

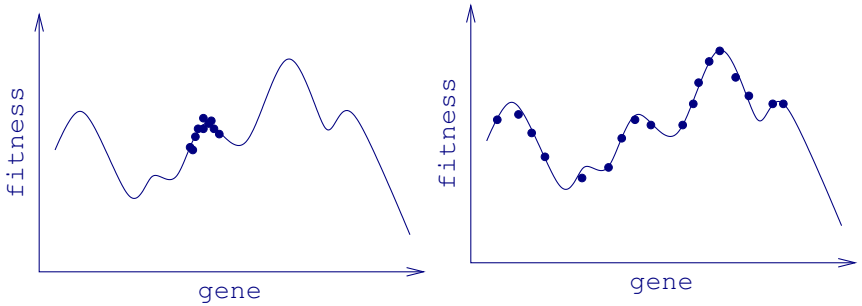
- each chromosome encodes a solution
- similar to hill-climbing search
- parallel search
- works for both immediate or delayed reward

Designing a GA Solution to a Problem

There are many different issues:

- What is the fitness function?
- How is an individual chromosome represented (how to encode) and what does it represent?
- How are individuals selected?
- How do individuals reproduce?

Issues in GA: Diversity



How to maintain diversity:

- Letting only successful ones to reproduce can seriously reduce the gene pool and an epidemic can wipe out the whole population: solution can not generalize in new and unexpected conditions.
- Converged population can often times be found at local minima, not at the global optimum.

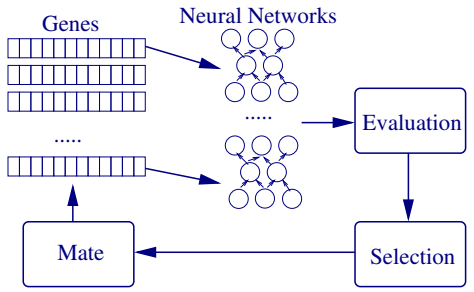
More Issues in GA

- Cross-over strategy: success depends on how genes are encoded (or represented).
- Not too much theoretical understanding about why it works so well.
- Crevices in fitness landscape: similar to spikes in hill climbing.
- How to combine learning with evolution.
- How to use cultural leverage.

GA as a Learning Algorithm

- An individual chromosome may not seem to learn, but when we look at the evolution of individuals over time, they can be seen as adapting, and thus learning to cope with the environment.
- If each individual encodes a function rather than a simple solution, the above point becomes clearer. At each generation, the parameters in the function can be seen as being adapted.
- Fitness can then be measured by using the function with the given parameters in specific tasks.

GA as a Learning Algorithm: Neuroevolution

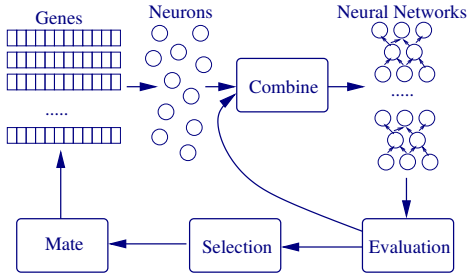


Evolving neural networks:

- Genes encode neural networks (connection topology and connection weights).
- Evaluate, select, and reproduce new population of neural networks.

Problem: individual neurons performing good work may get lost.

Neuroevolution: Evolving Individual Neurons



SANE: Moriarty and Miikkulainen

- Genes encode individual neurons.
- Neurons solve sub-problems and the ones that solve the problem well gets a chance to participate in a network in the next generation.
- Better diversity is maintained.

GA Demo

Neuroevolution:

`http://www.cs.utexas.edu/users/nn/`

- Generation of melodies (Chen and Miikkulainen)
- Gaming AI; harvesters and predators (Stanley and Miikkulainen)
- Non-markovian control (Gomez and Miikkulainen)

Key Points

- SOM: Try out the effects of different parameters, network size, 1D or 2D map, neighborhood radius, etc.
- Simple recurrent networks: how can it encode sequences, how is it different from standard backprop and who similar is it?
- Genetic algorithms basics.
- What are the issues to be solved in genetic algorithms.