Slide07

Haykin Chapter 9: Self-Organizing Maps

CPSC 636-600

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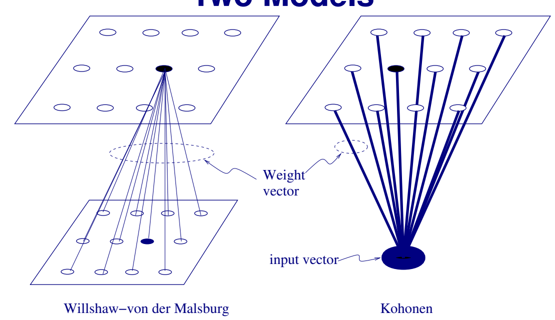
Introduction

- Self-organizing maps (SOM) is based on **competitive learning**, where output neurons compete with each other to be activated (Kohonen, 1982).
- The output neuron that activates is called the **winner-takes-all neuron**.
- **Lateral inhibition** is one way to implement competition for map formation (von der Malsburg 1973).
- In SOM, neurons are placed on a **lattice**, on which a meaningful coordinate system for different **features** is created (**feature map**).
- The lattice thus forms a **topographic map** where the spatial location on the lattice is indicative of the input features.

SOM and the Cortical Maps

- The development of SOM as a neural model is motivated by the **topographical nature** of cortical maps.
- Visual, tactile, and acoustic inputs are mapped in a topographical manner.
  - **Visual**: retinotopy (position in visual field), orientation, spatial frequency, direction, ocular dominance, etc.
  - **Tactile**: somatotopy (position on skin)
  - **Acoustic**: tonotopy (frequency)

Two Models



- **Willshaw-von der Malsburg model**: input neurons arranged in 2D lattice, output in 2D lattice. Lateral excitation/inhibition (Mexican hat) gives rise to soft competition. Normalized Hebbian learning. Biological motivation.
- **Kohonen model**: input of any dimension, output neurons in 1D, 2D, or 3D lattice. Relaxed winner-takes-all (neighborhood). Competetive learning rule. Computational motivation.

SOM Overview

SOM is based on three principles:

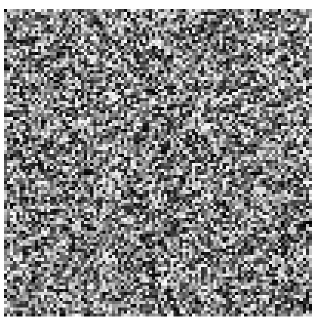
- **Competition**: each neuron calculates a discriminant function. The neuron with the highest value is declared the winner.
- **Cooperation**: Neurons near-by the winner on the lattice get a chance to adapt.
- **Adaptation**: The winner and its neighbors increase their discriminant function value relative to the current input. Subsequent presentation of the current input should result in enhanced function value.

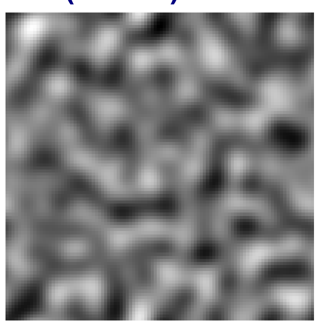
Redundancy in the input is needed!

Redundancy, etc.

- Unsupervised learning such as SOM require redundancy in the data.
- The following are intimately related:
  - Redundancy
  - Structure (or organization)
  - Information content relative to channel capacity

Redundancy, etc. (cont'd)

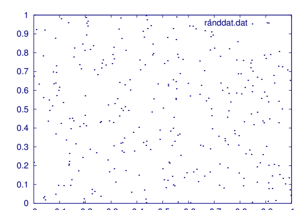


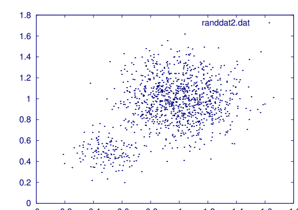


	Left	Right
Structure	No	Yes
Redundancy	No	Yes
Info $<$ Capacity	No	Yes

Consider each pixel as one random variable.

Redundancy, etc. (cont'd)

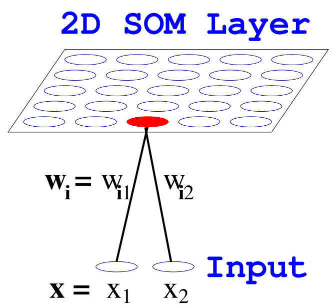




	Left	Right
Structure	No	Yes
Redundancy	No	Yes
Info $<$ Capacity	No	Yes

Consider each axis as one random variable.

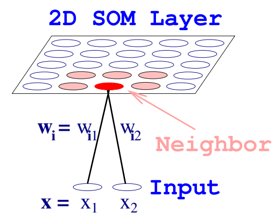
Self-Organizing Map (SOM)



Kohonen (1982)

- 1-D, 2-D, or 3-D layout of units.
- One weight vector for each unit.
- Unsupervised learning (no target output).

SOM Algorithm



1. Randomly initialize weight vectors $\mathbf{w}_i$
2. Randomly sample input vector $\mathbf{x}$
3. Find Best Matching Unit (BMU):

$$i(\mathbf{x}) = \text{argmin}_j \|\mathbf{x} - \mathbf{w}_j\|$$

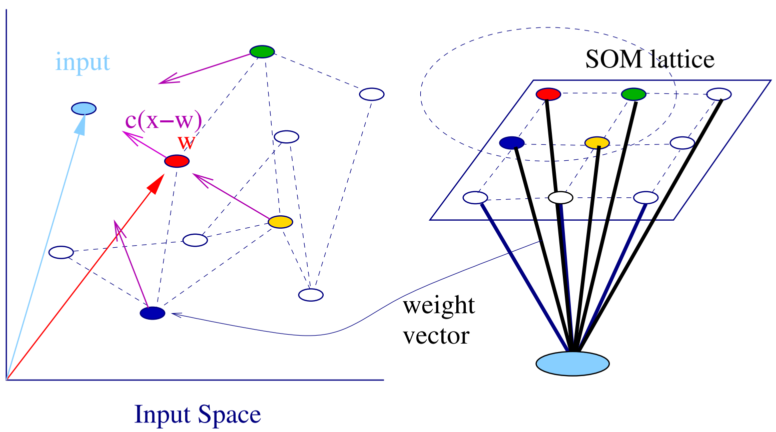
4. Update weight vectors:

$$\mathbf{w_j} \leftarrow \mathbf{w}_j + \eta h(j, i(\mathbf{x}))(\mathbf{x} - \mathbf{w}_j)$$

η : learning rate
$h(j, i(x))$: neighborhood function of BMU.

5. Repeat steps 2 – 4.

SOM Learning



- Weight vectors can be plotted in the input space.
- Weight vectors move, not according to their proximity to the input in the input space, but according to their proximity in the lattice.

Is This Hebbian Learning?: Sort of

- SOM learning can be viewed as Hebbian learning with a **forgetting term** to check unbounded growth.
- Original Hebb's rule:

$$\Delta \mathbf{w}_j = \eta y_j \mathbf{x},$$

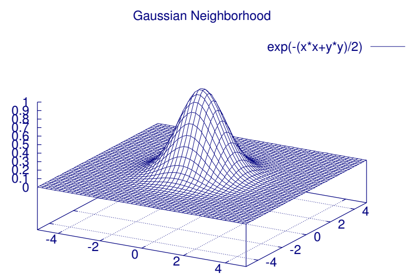
where $\mathbf{w}_j$ is the weight vector, η the learning rate, y_j the output response, and $\mathbf{x}$ the input vector.

- Hebb's rule plus a **forgetting term**:

$$\begin{aligned}\Delta \mathbf{w}_j &= \eta y_j \mathbf{x} - g(y_j)\mathbf{w}_j \\ &= \eta y_j \mathbf{x} - \eta y_j \mathbf{w}_j \\ &= \eta h_{j,i(\mathbf{x})}(\mathbf{x} - \mathbf{w}_j),\end{aligned}$$

assuming $g(y) = \eta y$ and $y_j = h_{j,i(\mathbf{x})}$.

Typical Neighborhood Functions



- Gaussian: $h(j, i(x)) = \exp(-\|\mathbf{r}_j - \mathbf{r}_{i(x)}\|^2 / 2\sigma^2)$
- Flat: $h(j, i(x)) = 1$ if $\|\mathbf{r}_j - \mathbf{r}_{i(x)}\| \leq \sigma,$ and 0 otherwise.
- σ is called the **neighborhood radius**.
- $\mathbf{r}_j$ is the location of unit j on the lattice.

Training Tips

- Start with large neighborhood radius.
  Gradually decrease radius to a small value.

$$\sigma(n) = \sigma_0 \exp\left(\frac{n}{\tau_1}\right)$$

- Start with high learning rate η.
  Gradually decrease η to a small value.

$$\eta(n) = \eta_0 \exp\left(\frac{n}{\tau_2}\right)$$

Two Phases of Adaptation

- **Self-organization or ordering phase**: High learning rate, large neighborhood radius (entire map).
- **Convergence phase**: Low learning rate, small neighborhood radius (one or zero).

Performance Measures

- **Quantization Error**
  Average distance between each data vector and its BMU.

$$\epsilon_Q = \frac{1}{N} \sum_{j=1}^{N} \| \mathbf{x}_j - \mathbf{w}_{i(x_j)} \|$$

- **Topographic Error**
  The proportion of all data vectors for which first and second BMUs are not adjacent units.

$$\epsilon_T = \frac{1}{N} \sum_{j=1}^{N} u(\mathbf{x}_j),$$

$u(\mathbf{x})$ = 1 if the 1st and 2nd BMUs are not adjacent
$u(\mathbf{x})$ = 0 otherwise.

SOM Summary

Essential ingredients of SOM: Hebbian learning rule (with forgetting term)

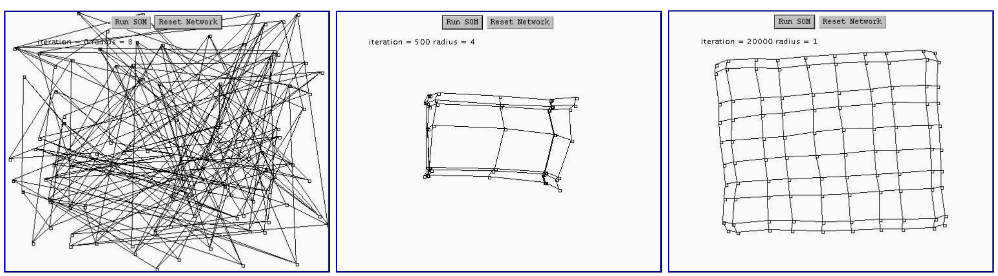
- Input generated according to a certain probability distribution on a continuous input space.
- Topology of network form on the discrete lattice.
- Time-varying neighborhood function around the winner.
- Time-varying leanring rate.

SOM Summary (cont'd)

Properties of SOM

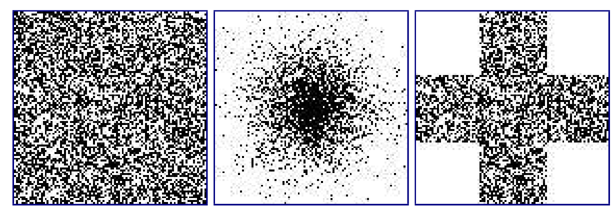
- **Approximation of the input space**: The collection of weight vectors provides a good approximation of the input space.
- **Topological ordering**: Spatial location on the lattice correspond to a certain feature of input patterns. Near-by neurons on the lattice represent similar input features.
- **Density matching**: More neurons are recruited to represent dense area in the input space.
- **Feature selection**: Select best features to approximate the underlying distribution.

Example: 2D Input / 2D Output



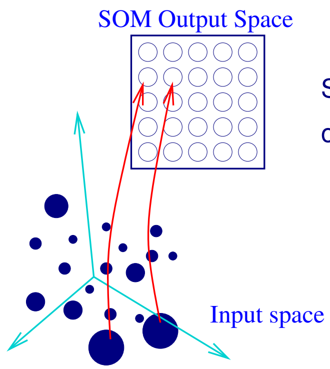
- Train with uniformly random 2D inputs.
  Each input is a point in Cartesian plane.
- Nodes: weight vectors (x and y coordinate).
- Edges: connect immediate neighbors on the map.

Different 2D Input Distributions



- What would the resulting SOM map look like?
- Why would it look like that?

High-Dimensional Inputs



SOM can be trained with inputs of arbitrary dimension.

- Dimensionality reduction: **N-D** to **2-D**.
- Extracts topological features.
- Used for visualization of data.

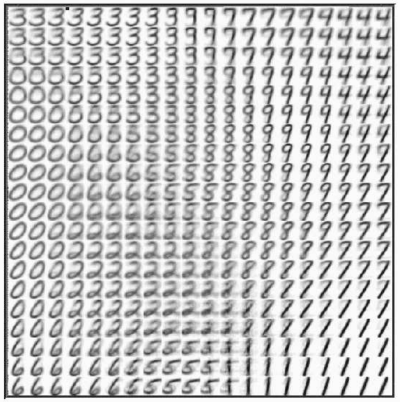
Applications

- **Data clustering and visualization.**
- **Optimization problems:** Traveling salesman problem.
- **Semantic maps:** Natural language processing.
- **Preprocessing for signal and image-processing.**
  1. Hand-written character recognition.
  2. Phonetic map for speech recognition.

Exercise

1. What happens when $h_{j,i(x)}$ and η was reduced quickly vs. slowly?
2. How would the map organize if different input distributions are given?
3. For a fixed number of input vectors from real-world data, a different visualization scheme is required. How would you use the number of input vectors that best match each unit to visualize the property of the map?

SOM Example: Handwritten Digit Recognition



- Preprocessing for feedforward networks (supervised learning).
- Better representation for training.
- Better generalization.

SOM Demo

Jochen Fröhlich's *Neural Networks with JAVA* page:

`http://fbim.fh-regensburg.de/~saj39122/jfroehl/diplom/e-index.html`

Check out the `Sample Applet` link.

SOM Demo: Traveling Salesman Problem

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 8.
- Stop when radius $<$ 0.001.
- Try 50 nodes, 20 input points.

Click on `[Parameters]` to bring up the config panel. After the parameters are set, click on `[Reset]` in the main applet, and then `[Start learning]`.

SOM Demo: Space Filling in 2D

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 100.
- Stop when radius $<$ 0.001.
- Try 1000 nodes, and 1000 input points.

SOM Demo: Space Filling in 3D

Using Fröhlich's SOM applet:

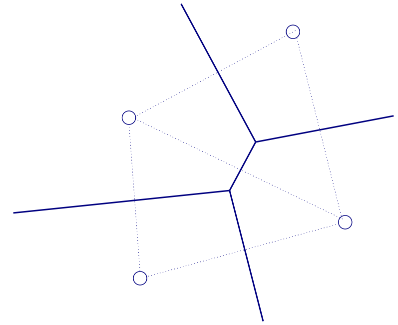
- 2D SOM map ($n \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 10.
- Stop when radius $<$ 0.001.
- Try 30×30 nodes, and 500 input points. Limit the y range to 15.

Also try 50×50, 1000 input points, and 16 initial radius.

Vector Quantization

- **Vector quantization** exploits the structure in the input distribution for the purpose of data compression.
- In vector quantization, the input space is partitioned into a number of distinct regions and for each region a **reconstruction vector** is defined.
- A new input is then represented by the reconstruction vector representing the region it falls into.
- Since only the **index** of the reconstruction vector need to be stored or transmitted, significant saving is possible in terms of storage space and bandwidth.
- The collection of reconstruction vectors is called the **code book**.

Vector Quantization (cont'd)



- A vector quantizer that minimizes encoding distortion is called a **Voronoi** or **nearest-neighbor quantizer**.
- SOM provides an approximate method for calculating the Voronoi tessellation.

Learning Vector Quantization

- Train with SOM in unsupervised mode.
- Then, tune the weight vectors in a supervised mode:
  - If class of the input vector and the class of the best matching weight vector **match**,

$$\mathbf{w}_c(n+1) = \mathbf{w}_c(n) + \alpha_n[\mathbf{x}_i - \mathbf{w}_c(n)]$$

  - If class of the input vector and the class of the best matching weight vector **do not match**,

$$\mathbf{w}_c(n+1) = \mathbf{w}_c(n) - \alpha_n[\mathbf{x}_i - \mathbf{w}_c(n)]$$

Other Topics

- Different ways of visualization using SOM.
- Contextual map (or semanics map).
- SOM viewed as
  - Abstract neuroscientific model of the cortex
  - Vector quantizer
- Difficulty of analysis (convergence, etc.)
- Use in modeling cortical map formation.