# CPSC 633-600 Homework 2, part I of II (Total 50 points)
# Reinforcement Learning

See course web page for the **due date**.

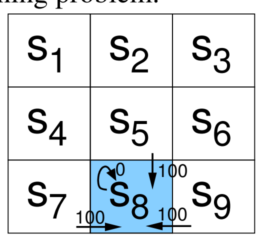
Use **ecampus.tamu.edu** to submit your assignments, or submit a hard copy, but not to both.

Instructor: Yoonsuck Choe

February 19, 2014

## 1 Deterministic Case

Consider the following reinforcement learning problem.




- There are 9 states, and the actions are $\{up, down, left, right\}$. Legal actions are those that go to the immediate neighbor, horizontally or vertically (but not diagonally). Treat State 8 ($s_8$) as having no legal action.
- The rewards for all action are 0, except for all actions that lead into $s_8$, which are 100.
- In all cases, assume $\gamma = 0.9$.

**Problem 1 (Program: 10 pts):** Program a Q-learning algorithm to learn the $Q(s, a)$ values for the above example. Use the algorithm in slide04.pdf, Mitchell slide page 18 (pdf page 22). Stop learning when change in the Q table is 0 for the past 50 Q updates or so. Note: use a random policy to select action $a$ given current state $s$ (take care to check if the random action chosen is a legal one).

(1) Include your code.

(2) Show resulting Q table ($9 \times 4$ matrix).

- Rows represent state and columns represent action.
- Row ordering should be $s_1, s_2, ..., s_9$.
- Column ordering should be $up, down, left, right$.
- Set $Q(s, a) = -99$ to mark illegal moves. Don't use this value during your calculations.

(3) Show a plot showing sum(abs($Q_{t+1} - Q_t$)) over the iterations $t$.

**Problem 2 (Program: 10 pts):** Modify the program from problem 1 so that the exploration policy is $\epsilon$-greedy. Initialize your Q table with a very small random number to break the initial tie (rand * 0.0001).

(1) Include your code.

(2) Test $\epsilon \in \{0.0, 0.2, 0.5, 1.0\}$. Note: $\epsilon = 1.0$ is the greedy policy, and $\epsilon = 0.0$ is the random policy.

If `rand() > epsilon`, choose random action. Otherwise, choose `[val, a] = max(Q(s,:))`.

(3) Show resulting Q tables for all 4 cases ($9 \times 4$ matrix).

(4) Show plots showing sum(abs($Q_{t+1} - Q_t$)) over the iterations $t$ for all four cases.

(5) Discuss the effect of $\epsilon$ on the quality of the learned Q-table.

## 2 Stochastic Case

Consider a stochastic version of the reinforcement learning problem posed in Section 1. Modify the rules so that:

- $\delta(s, a)$ is stochastic: The probability of landing in the intended direction is $0.70$. The probability of landing in one of $n$ unintended legal direction is $\frac{0.30}{n}$.
- Example 1 : If you are in $s_5$ and $a$ was right, probability of landing in $s_6$ is 0.70, and ending up in $s_2$, $s_4$, or $s_8$ is 0.10 each.
- Example 2: If you are in $s_1$ and $a$ was down, probability of landing in $s_2$ is 0.70, and ending up in $s_4$ is 0.30.
- Reward $r(s, a)$ depends on where you landed based on the above. All rewards are 0 unless the resulting state was the goal state $s_8$. For example, if you were in $s_5$ and the action was $a = left$, with 10% chance you will land in $s_9$, the goal state. In this case $r(s_5, left) = 100$. In a different run, if you landed in $s_4$, then $r(s_5, left) = 0$.

**Problem 3 (Program: 10 pts):** Repeat problem 1, with the stochastic version of the task (random policy). In addition to all the requirements, keep a running estimate of $E[r(s, a)]$ for states $s_5$, $s_7$, and $s_9$ and report their **final** values. Use the learning rule in slide04.pdf, Mitchell slide page 31 (pdf page 35).

**Estimating $E[r(s, a)]$ throughout the learning run:**

$$E[r(s, a)] = \frac{\sum_{\text{for all visits to } (s,a)} r}{visits(s, a)}$$

**Problem 4 (Program: 10 pts):** Repeat problem 2, with the stochastic version of the task ($\epsilon$-greedy policy with the four different $\epsilon$ values). In addition to all the requirements, keep a running estimate of $E[r(s, a)]$ for states $s_5$, $s_7$, and $s_9$ and report the values. Use the learning rule in slide04.pdf, Mitchell slide page 31 (pdf page 35).

**Problem 5 (Written: 5 pts):** For states $s_5$, $s_7$, and $s_9$, manually compute $E[r(s, a)]$ (using the exact probabilities **[note: it relates with $P(s'|s, a)$ and the reward depending on state outcome $s'$]**) and compare those to the estimated values from problem 3 and problem 4. Are the results similar?

**Problem 6 (Written: 5 pts):** For states $s_5$, $s_7$, and $s_9$, using the estimated $E[r(s,a)]$ and all the estimated $\hat{Q}(s,a)$ values from your result in problem 3 above, see if the following holds:

$$\hat{Q}(s,a) = E[r(s,a)] + \gamma \sum_{s'} P(s'|s,a) \max_{a'} \hat{Q}(s',a')$$