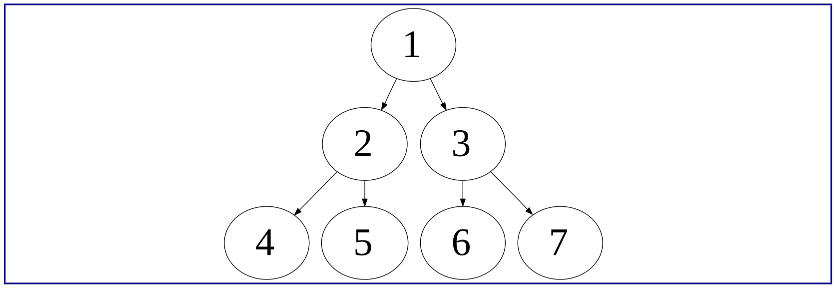
Search and Game Playing

- CSCE 315 Programming Studio
- Material drawn from Gordon Novak's AI course, Yoonsuck Choe's AI course, and Russell and Norvig's *Artificial Intelligence, A Modern Approach*, 2nd edition.

Overview

- Search problems: definition
- Example: 8-puzzle
- General search
- Evaluation of search strategies
- Strategies: breadth-first, uniform-cost, depth-first
- More uninformed search: depth-limited, iterative deepening, bidirectional search

Search Problems: Definition



Search = $<$ initial state, operators, goal states $>$

- Initial State: description of the current situation as given in a problem
- Operators: functions from any state to a set of successor (or neighbor) states
- Goal: subset of states, or test rule

Variants of Search Problems

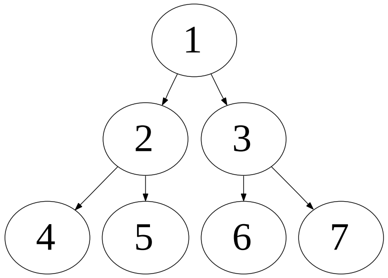
Search = $<$ state space, initial state, operators, goal states $>$

- State space: set of all possible states reachable from the current initial state through repeated application of the operators (i.e. path).

Search = $<$ initial state, operators, goal states, path cost $>$

- Path cost: find **the best** solution, not just **a** solution. Cost can be many different things.

Types of Search



- Uninformed: systematic strategies
- Informed: Use domain knowledge to narrow search
- Game playing as search: minimax, state pruning, probabilistic games

Search State

State as Data Structure

- examples: variable assignment, properties, order in list, bitmap, graph (vertex and edges)
- captures all possible ways world could be
- typically static, discrete (symbolic), but doe snot have to be

Choosing a Good Representation

- concise (keep only the relevant features)
- explicit (easy to compute when needed)
- embeds constraints

Operators

Function from state to subset of states

- drive to neighboring city
- place piece on chess board
- add person to meeting schedule
- slide tile in 8-puzzle

Characteristics

- often requires instantiation (fill in variables)
- encode constraints (only certain operations are allowed)
- generally discrete: continuous parameters $\longrightarrow$ infinite branching

Goals: Subset of states or test rules

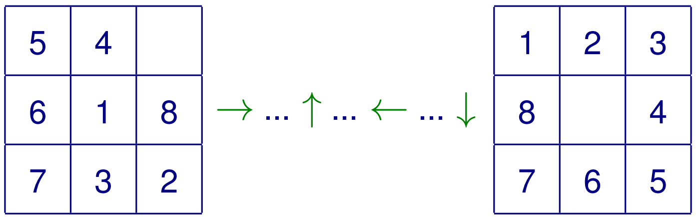
Specification:

- set of states: enumerate the eligible states
- partial description: e.g. a certain variable has value over x.
- constraints: or set of constraints. Hard to enumerate all states matching the constraints, or very hard to come up with a solution at all (i.e. you can only verify it; P vs. NP).

Other considerations:

- space, time, quality (exact vs. approximate trade-offs)

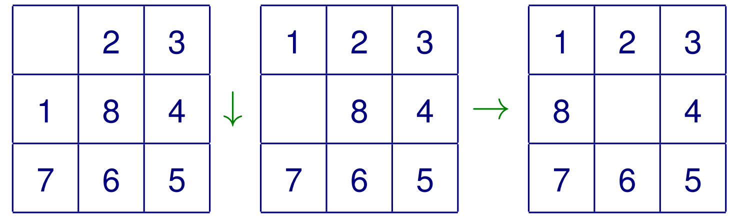
An Example: 8-Puzzle



- **State**: location of 8 number tiles and one blank tile
- **Operators**: blank moves left, right, up, or down
- **Goal test**: state matches the configuration on the right (see above)
- **Path cost**: each step cost 1, i.e. path length, or search tree depth

Generalization: 15-puzzle, ..., $(N^2 - 1)$-puzzle

8-Puzzle: Example

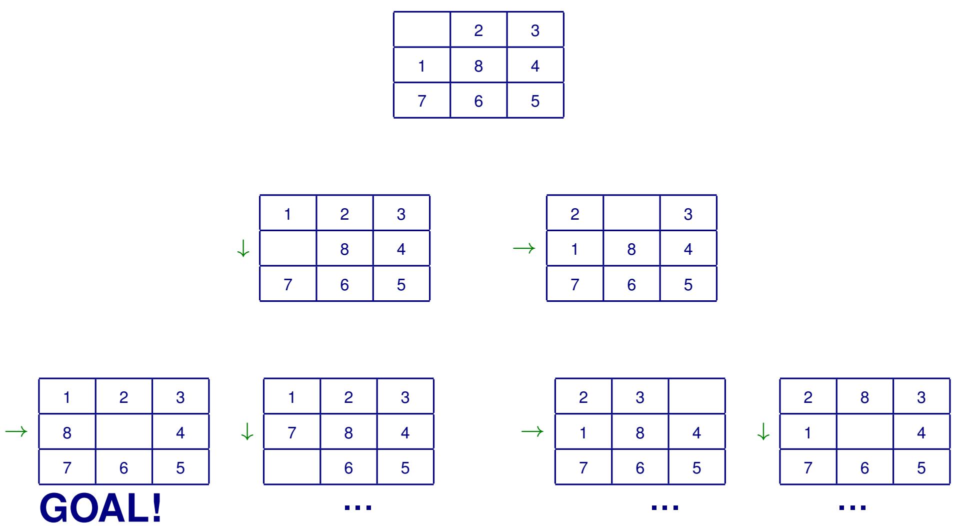


Possible state representations in LISP (0 is the blank):

- `(0 2 3 1 8 4 7 6 5)`
- `((0 2 3) (1 8 4) (7 6 5))`
- `((0 1 7) (2 8 6) (3 4 5))`
- or use the `make-array`, `aref` functions.

How easy to: (1) compare, (2) operate on, and (3) store (i.e. size).

8-Puzzle: Search Tree



General Search Algorithm

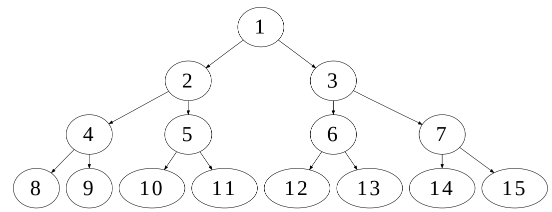
Pseudo-code:

```
function General-Search (problem, Que-Fn)
  node-list := initial-state
  loop begin
      // fail if node-list is empty
      if Empty(node-list) then return FAIL
      // pick a node from node-list
      node := Get-First-Node(node-list)
      // if picked node is a goal node, success!
      if (node == goal) then return as SOLUTION
      // otherwise, expand node and enqueue
      node-list := Que-Fn(node-list, Expand(node))
  loop end
```

Evaluation of Search Strategies

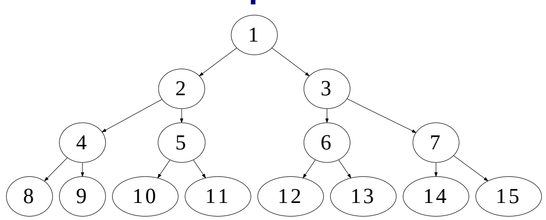
- time-complexity: how many nodes expanded so far?
- space-complexity: how many nodes must be stored in node-list at any given time?
- completeness: if solution exists, guaranteed to be found?
- optimality: guaranteed to find the best solution?

Breadth First Search



- node visit order (goal test): 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
- queuing function: enqueue at end (add expanded node at the end of the list)
- Important: A node taken out of the node list for inspection counts as a single visit!

BFS: Expand Order



Evolution of the queue (**bold**= expanded and added children):

1. **[1]** : initial state
2. **[2][3]** : dequeue 1 and enqueue 2 and 3
3. `[3]` **[4][5]** : dequeue 2 and enqueue 4 and 5
4. `[4][5]` **[6][7]** : **all** depth 3 nodes

...

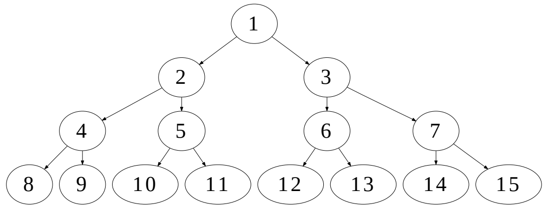
8. `[8][9][10][11][12][13]` **[14][15]** : **all** depth 4 nodes

BFS: Evaluation

branching factor b, depth of solution d:

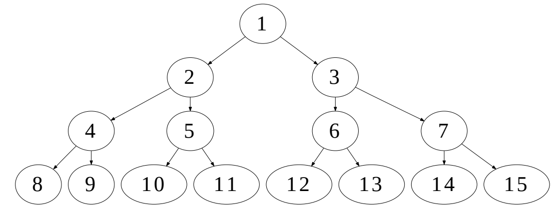
- complete: it will find the solution if it exists
- time: $1 + b + b^2 + ... + b^d$
- space: $O(b^{d+1})$ where d is the depth of the shallowest solution
- space is more problem than time in most cases (p 75, figure 3.12).
- time is also a major problem nonetheless (same as time)

Depth First Search



- node visit order (goal test): 1 2 4 8 9 5 10 11 3 6 12 13 7 14 15
- queuing function: enqueue at left (stack push; add expanded node at the beginning of the list)

DFS: Expand Order



Evolution of the queue (**bold**=expanded and added children):

1. `[1]` : initial state
2. **[2][3]** : pop 1 and push expanded in the front
3. **[4][5]** `[3]` : pop 2 and push expanded in the front
4. **[8][9]** `[5] [3]` : pop 4 and push expanded in the front

DFS: Evaluation

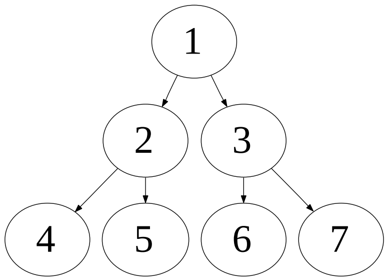
branching factor b, depth of solutions d, max depth m:

- incomplete: may wander down the wrong path
- time: $O(b^m)$ nodes expanded (worst case)
- space: $O(bm)$ (just along the current path)
- good when there are many shallow goals
- bad for deep or infinite depth state space

Key Points

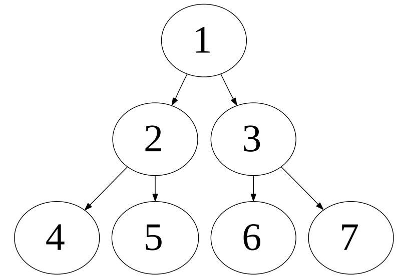
- Description of a search problem: initial state, goals, operators, etc.
- Considerations in designing a representation for a state
- Evaluation criteria
- BFS, DFS: time and space complexity, completeness
- When to use one vs. another
- Node visit orders for each strategy
- Tracking the stack or queue at any moment

Depth Limited Search (DLS): Limited Depth DFS



- node visit order for each depth limit l:
  1 ($l = 1$); 1 2 3 ($l = 2$); 1 2 4 5 3 6 7 ($l = 3$);
- queuing function: enqueue at front (i.e. stack push)
- **push the depth of the node as well**:
  ($<$depth$>$ $<$node$>$)

DLS: Expand Order



Evolution of the queue (**bold**=expanded and then added):
(<depth>,<node>)); Depth limit = 3

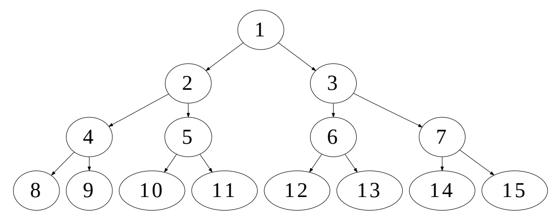
1. [(d1,1)] : initial state
2. **[(d2,2)][(d2,3)]** : pop 1 and push 2 and 3
3. **[(d3,4)][(d3,5)]** [(d2,3)] : pop 2 and push 4 and 5
4. [(d3,5)][(d2,3)]: pop 4, cannot expand it further
5. [(d2,3)]: pop 5, cannot expand it further
6. **[(d3,6)][(d3,7)]**: pop 3, and push 6, 7

DLS: Evaluation

branching factor b, depth limit l, depth of solution d:

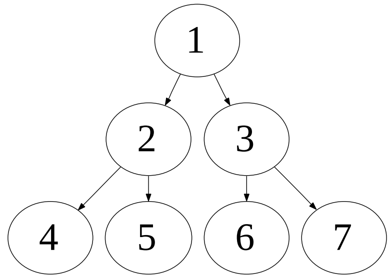
- complete: if $l \geq d$
- time: $O(b^l)$ nodes expanded (worst case)
- space: $O(bl)$ (same as DFS, where $l = m$ (m: max depth of tree in DFS)
- good if solution is within the limited depth.
- non-optimal (same problem as in DFS).

Iterative Deepening Search: DLS by Increasing Limit



- node visit order:
  1 ; 1 2 3; 1 2 4 5 3 6 7; 1 2 4 8 9 5 10 11 3 6 12 13 7 14 15; ...
- revisits already explored nodes at successive depth limit
- queuing function: enqueue at front (i.e. stack push)
- **push the depth of the node as well**: **($<$depth$>$ $<$node$>$)**

IDS: Expand Order



Basically the same as DLS: Evolution of the queue (**bold**=expanded and then added): `(<depth>,<node>)`); e.g. Depth limit = 3

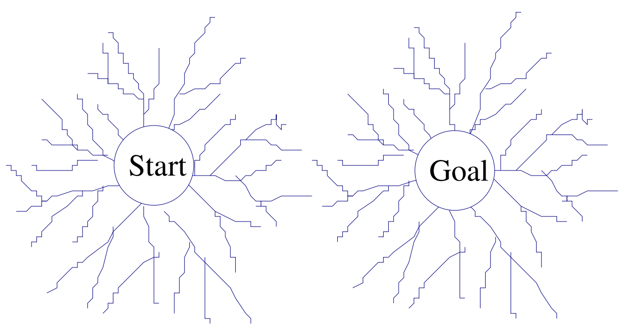
1. `[(d1,1)]` : initial state
2. **[(d2,2)][(d2,3)]** : pop 1 and push 2 and 3
3. **[(d3,4)][(d3,5)]** `[(d2,3)]` : pop 2 and push 4 and 5
4. `[(d3,5)][(d2,3)]`: pop 4, cannot expand it further
5. `[(d2,3)]`: pop 5, cannot expand it further
6. **[(d3,6)][(d3,7)]**: pop 3, and push 6, 7

IDS: Evaluation

branching factor b, depth of solution d:

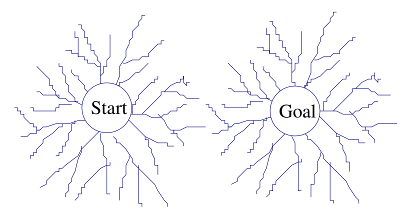
- complete: cf. DLS, which is conditionally complete
- time: $O(b^d)$ nodes expanded (worst case)
- space: $O(bd)$ (cf. DFS and DLS)
- **optimal!**: unlike DFS or DLS
- good when search space is huge and the depth of the solution is not known (*)

Bidirectional Search (BDS)



- Search from both initial state and goal to reduce search depth.
- $O(b^{d/2})$ of BDS vs. $O(b^{d+1})$ of BFS.

BDS: Considerations



1. how to back trace from the goal?
2. successors and predecessors: are operations reversible?
3. are goals explicit?: need to know the goal to begin with
4. check overlap in two branches
5. BFS? DFS? which strategy to use? Same or different?

BDS Example: 8-Puzzle

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

→ ... ←

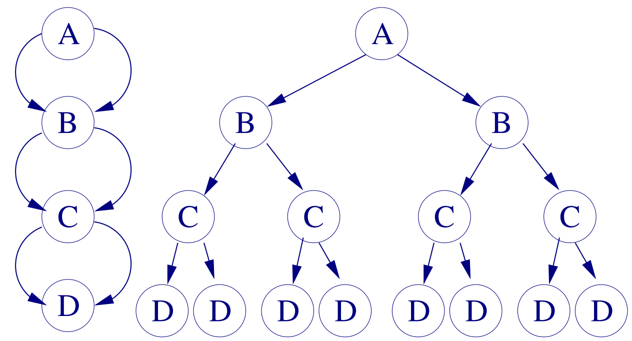
1	2	3
	8	4
7	6	5

←

1	2	3
8		4
7	6	5

- Is it a good strategy?
- What about Chess? Would it be a good strategy?
- What kind of domains may be suitable for BDS?

Avoiding Repeated States



Repeated states can be devastating in search problems.

- Common cases: problems with reversible operators ⟶ search space becomes infinite
- One approach: find a spanning tree of the graph

Avoiding Repeated States: Strategies

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

→

5	4	
6	1	8
7	3	2

→

5	4	8
6	1	
7	3	2

...

- Do not return to the node's parent
- Avoid cycles in the path (this is a huge theoretical problem in its own right)
- Do not generate states that you generated before: use a hash table to make checks efficient

How to avoid storing every state? Would using a short signature (or a checksum) of the full state description help?

Key Points

- DLS, IDS, BDS search order, expansions, and queuing
- DLS, IDS, BDS evaluation
- DLS, IDS, BDS: suitable domains
- Repeated states: why removing them is important

Overview

- Best-first search
- Heuristic function
- Greedy best-first search
- A^*
- Designing good heuristics
- IDA^*
- Iterative improvement algorithms
  1. Hill-climbing
  2. Simulated annealing

Informed Search

From domain knowledge, obtain an **evaluation function**.

- best-first search: order nodes according to the evaluation function value
- greedy search: minimize estimated cost for reaching the goal – fast, but incomplete and non-optimal.
- A^*: minimize $f(n) = g(n) + h(n)$, where $g(n)$ is the current path cost from start to n, and $h(n)$ is the estimated cost from n to goal.

Best First Search

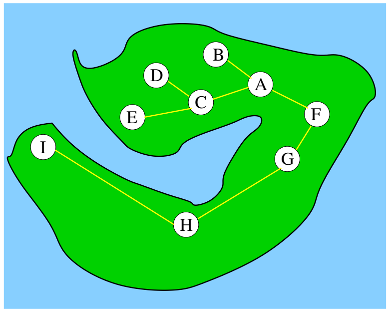
function Best-First-Search (*problem, Eval-Fn*)

Queuing-Fn $\leftarrow$ sorted list by *Eval-Fn*(node)

return General-Search(*problem, Queuing-Fn*)

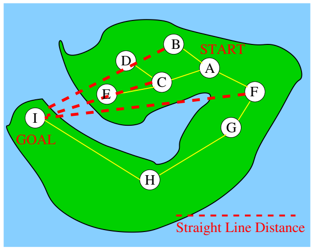
- The queuing function queues the expanded nodes, and sorts it every time by the *Eval-Fn* value of each node.
- One of the simplest Eval-Fn: **estimated cost** to reach the goal.

Heuristic Function



- $h(n)$ = estimated cost of the cheapest path from the state at node n to a goal state.
- The only requirement is the $h(n) = 0$ at the goal.
- **Heuristics** means “to find” or “to discover”, or more technically, “how to solve problems” (Polya, 1957).

Heuristics: Example



- $h_{\mathrm{SLD}}(n)$: straight line distance (SLD) is one example.
- Start from **A** and Goal is **I**: **C** is the most promising next step in terms of $h_{\mathrm{SLD}}(n)$, i.e. $h(C) < h(B) < h(F)$
- Requires some knowledge:
  1. coordinates of each city
  2. generally, cities toward the goal tend to have smaller **SLD**.

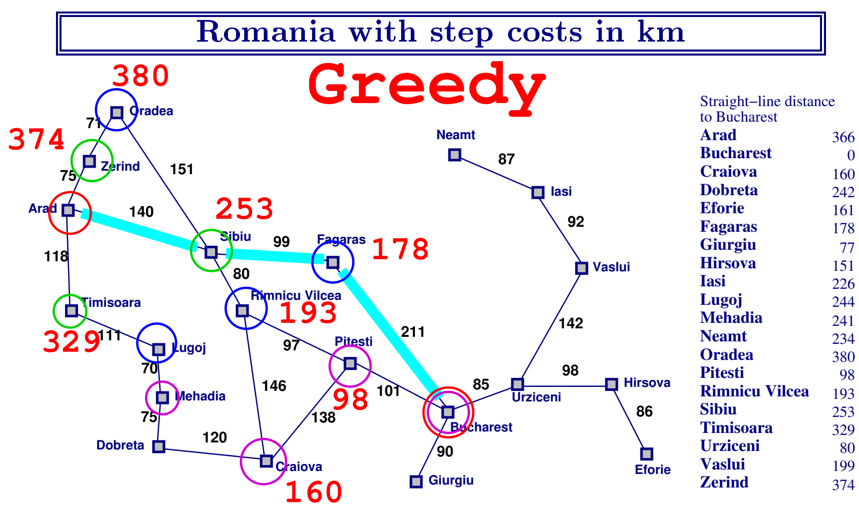
Greedy Best-First Search

function Greedy-Best-First Search (*problem*)

$h(n)$=estimated cost from n to goal

return Best-First-Search(*problem*,h)

- Best-first with heuristic function $h(n)$



Total Path Cost = 450

Greedy Best-First Search: Evaluation

Branching factor b and max depth m:

- Fast, just like Depth-First-Search: single path toward the goal.
- Time: $O(b^m)$
- Space: same as time – all nodes are stored in sorted list(!), **unlike DFS**
- Incomplete, just like DFS
- Non-optimal, just like DFS

A^*: Uniform Cost + Heuristic Search

Avoid expanding paths that are already found to be expensive:

- $f(n) = g(n) + h(n)$
- $f(n)$: estimated cost to goal through node n
- **provably complete and optimal!**
- **restrictions**: $h(n)$ should be an **admissible heuristic**
- admissible heuristic: one that **never overestimate** the actual cost of the best solution through n
- **NOTE:** $f(n)$ can be different depending on the path taken to $f(n)$ if multiple paths exists from root to n!

A^* Search

function A^*-Search (*problem*)

$g(n)$=current cost up till n

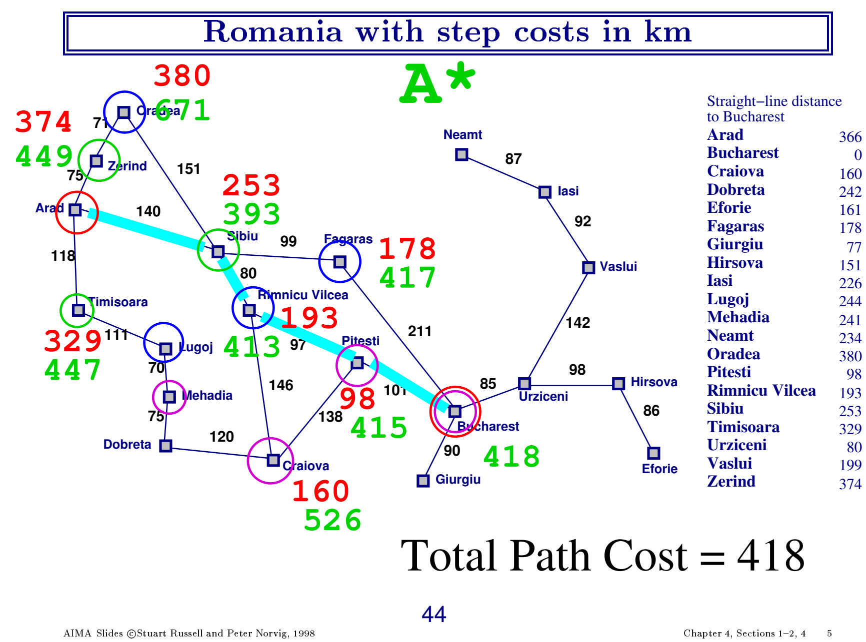
$h(n)$=estimated cost from n to goal

return Best-First-Search(*problem*,$g + h$)

- Condition: $h(n)$ must be an **admissible heuristic function**!
- A^* is **optimal!**

Behavior of A^* Search

- usually, the f value never decreases along a given path: **monotonicity**
- in case it is nonmonotonic, i.e. $f(Child) < f(Parent)$, make this adjustment:
  $f(Child) = max(f(Parent), g(Child) + h(Child))$.
- this is called **pathmax**



Optimality of A^*

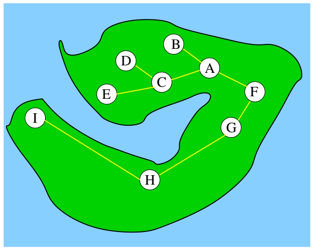
G_2: suboptimal goal in the node-list.

n: unexpanded node on a shortest path to goal G_1

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $> g(G_1)$ since G_2 is suboptimal
- $\geq f(n)$ since h is admissible

Since $f(G_2) > f(n)$, A^* will never select G_2 for expansion.

Optimality of A^*: Example



1. **Expansion of parent allowed**: search fails at nodes **B**, **D**, and **E**.
2. **Expansion of parent disallowed**: paths through nodes **B**, **D**, and **E** with have an inflated path cost $g(n)$, thus will become nonoptimal.

$$\underbrace{A \to C \to E \to C \to}_{\text{inflated path cost}} A \to F \to \ldots$$

Lemma to Optimality of A^*

Lemma: A^* expands nodes in order of increasing $f(n)$ value.

- Gradually adds **f-contours** of nodes (cf. BFS adds layers).
- The goal state may have a f value: let's call it f^*
- This means that all nodes with $f < f^*$ will be expanded!

Complexity of A^*

A^* is complete and optimal, but space complexity can become exponential if the heuristic is not good enough.

- condition for **subexponential** growth:

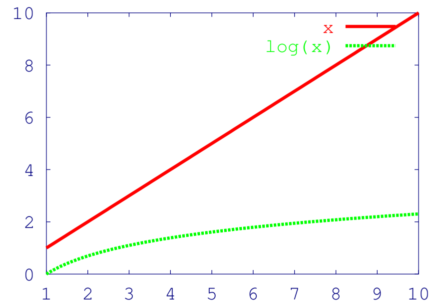
  $|h(n) - h^*(n)| \leq O(\log h^*(n))$,

  where $h^*(n)$ is the **true** cost from n to the goal.

- that is, error in the estimated cost to reach the goal should be less than even linear, i.e. $< O(h^*(n))$.

Unfortunately, with most heuristics, error is at least proportional with the true cost, i.e. $\geq O(h^*(n)) > O(\log h^*(n))$.

Linear vs. Logarithmic Growth Error



- Error in heuristic: $|h(n) - h^*(n)|$.
- For most heuristics, the error is at least linear.
- For A^* to have subexponential growth, the error in the heuristic should be on the order of $O(\log h^*(n))$.

Problem with A^*

Space complexity is usually **exponential**!

- we need a memory bounded version
- one solution is: Iterative Deepening A^*, or IDA^*

A^*: Evaluation

- Complete : unless there are infinitely many nodes with $f(n) \leq f(G)$
- Time complexity: exponential in (relative error in $h \times$ length of solution)
- Space complexity: same as time (keep all nodes immediately outside of current f-contour in memory)
- Optimal

Heuristic Functions: Example

Eight puzzle

5	4	
6	1	8
7	3	2

1	2	3
8		4
7	6	5

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

  $h_1(n)$ = 7 (not counting the blank tile)
  $h_2(n)$ = 2+3+3+2+4+2+0+2 = 18

* Both are admissible heuristic functions.

Dominance

If $h_2(n) \geq h_1(n)$ for all n and both are admissible, then we say that $h_2(n)$ **dominates** $h_1(n)$, and is better for search.

Typical search costs for depth $d = 14$:

- Iterative Deepening : 3,473,941 nodes expanded
- $\mathrm{A}^*(h_1)$: 539 nodes
- $\mathrm{A}^*(h_2)$: 113 nodes

Observe that in A^*, every node with $f < f^*$ is expanded. Since $f = g + h$, nodes with $h(n) < f^* - g(n)$ will be expanded, so larger h will result in less nodes being expanded.

- f^* is the f value for the optimal solution path.

Designing Admissible Heuristics

Relax the problem to obtain an admissible heuristics.

For example, in 8-puzzle:

- allow tiles to move anywhere $\rightarrow h_1(n)$
- allow tiles to move to any adjacent location $\rightarrow h_2(n)$

For traveling:

- allow traveler to travel by air, not just by road: **SLD**

Other Heuristic Design

- Use composite heuristics: $h(n) = max(h_1(n), ..., h_m(n))$
- Use statistical information: random sample h and true cost to reach goal. Find out how often h and true cost is related.

Iterative Deepening A^*: IDA^*

A^* is complete and optimal, but the performance is limited by the available space.

- Basic idea: only search within a certain f bound, and gradually increase the f bound until a solution is found.
- Popular use include path finding in game AI.

IDA^*

function IDA^*(*problem*)

root $\leftarrow$ Make-Node(Initial-State(*problem*))

f-limit $\leftarrow$ f-Cost(*root*)

loop do

solution, f-limit $\leftarrow$ DFS-Contour(*root, f-limit*)

if *solution* != NULL **then return** *solution*

if *f-limit* == ∞ **then return** *failure*

end loop

Basically, iterative deepening depth-first-search with depth defined as the f-cost ($f = g + n$):

DFS-Contour(*root, f-limit*)

Find solution from node **root**, within the f-cost limit of **f-limit**.
DFS-Contour returns **solution sequence** and **new f-cost limit**.

- if f-cost(**root**) $>$ **f-limit**, return fail.
- if **root** is a goal node, return solution and new f-cost limit.
- **recursive call** on all successors and return solution and **minimum f-limit** returned by the calls
- return **null solution** and new f**-limit** by default

Similar to the recursive implementation of DFS.

IDA^*: Evaluation

- complete and optimal (with same restrictions as in A^*)
- space: proportional to longest path that it explores (because it is depth first!)
- time: dependent on the number of different values $h(n)$ can assume.

IDA^*: Time Complexity

Depends on the heuristics:

- small number of possible heuristic function values $\rightarrow$ small number of f-contours to explore $\rightarrow$ becomes similar to A^*
- complex problems: each f-contour only contain one new node

  if A^*expands N nodes,
  IDA^*expands
  $1 + 2 + .. + N = \frac{N(N+1)}{2} = O(N^2)$
- a possible solution is to have a **fixed** increment ϵ for the f-limit $\rightarrow$ solution will be suboptimal for at most ϵ (ϵ-admissible)

Iterative Improvement Algorithms

Start with a complete configuration (all variable values assigned, and **optimal**), and**gradually improve** it.

- Hill-climbing (maximize cost function)
- Gradient descent (minimize cost function)
- Simulated Annealing (probabilistic)

Hill-Climbing

- no queue, keep only the best node
- greedy, no back-tracking
- good for domains where **all nodes are solutions**:
  - goal is to **improve** quality of the solution
  - optimization problems
- note that it is different from greedy search, which keeps a node list

Hill-Climbing Strategies

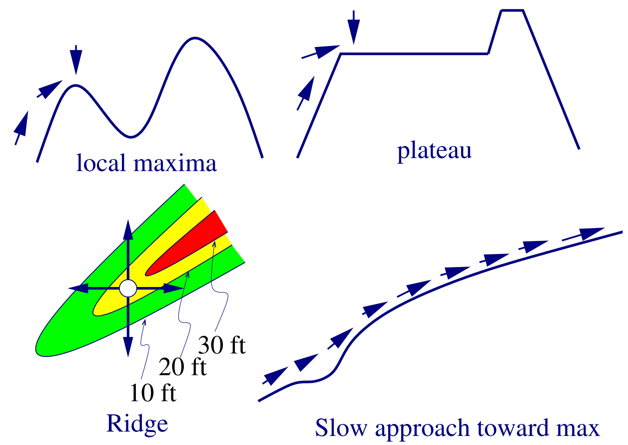
Problems of local maxima, plateau, and ridges:

- try **random-restart**: move to a random location in the landscape and restart search from there
- parallel search
- simulated annealing *

Hardness of problem depends on the **shape of the landscape**.

*: coming up next

Hill-Climbing and Gradient Search: Problems



- Possible solution: **simulated annealing** – gradually decrease randomness of move to attain globally optimal solution (more on this next week).

Simulated Annealing: Overview

Annealing:

- heating metal to a high-temperature (making it a liquid) and then allowing to cool slowly (into a solid); this relieves internal stresses and results in a more stable, lower-energy state in the solid.
- at high temperature, atoms move actively (large distances with greater randomness), but as temperature is lowered, they become more static.

Simulated annealing is similar:

- basically, hill-climbing with randomness that allows going **down** as well as the standard **up**
- randomness (as temperature) is reduced over time

Simulated Annealing (SA)

Goal: **minimize** (not maximize) the energy E, as in statistical thermodynamics.

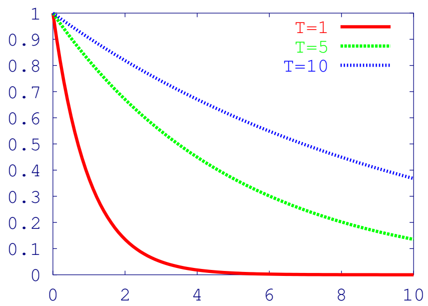
For successors of the current node,

- if $\Delta E \leq 0$, the move is accepted
- if $\Delta E > 0$, the move is accepted with probability $P(\Delta E) = e^{-\frac{\Delta E}{kT}}$, where k is the Boltzmann constant and T is temperature.
- randomness is in the comparison: $P(\Delta E) < \mathrm{rand}(0,1)$

$\Delta E = E_{\mathrm{new}} - E_{\mathrm{old}}$.

The heuristic $h(n)$ or $f(n)$ represents E.

Temperature and $P(\Delta E) < \mathrm{rand}(0,1)$



Downward moves of any size are allowed at high temperature, but at low temperature, only small downward moves are allowed.

- Higher temperature $T \rightarrow$ higher probability of **downward** hill-climbing
- Lower $\Delta E \rightarrow$ higher probability of **downward** hill-climbing

T Reduction Schedule

High to low temperature reduction schedule is important:

- reduction too fast: suboptimal solution
- reduction too slow: wasted time
- question: does the form of the reduction schedule curve matter? linear, quadratic, exponential, etc.?

The proper values are usually found experimentally.

Simulated Annealing Applications

- VLSI wire routing and placement
- Various scheduling optimization tasks
- Traffic control
- Neural network training
- etc.

Key Points

- best-first-search: definition
- heuristic function $h(n)$: what it is
- greedy search: relation to $h(n)$ and evaluation. How it is different from DFS (time complexity, space complexity)
- A^*: definition, evaluation, conditions of optimality
- complexity of A^*: relation to error in heuristics
- designing good heuristics: several rule-of-thumbs
- IDA^*: evaluation, time and space complexity (worst case)
- hill-climbing concept and strategies
- simulated annealing: core algorithm, effect of T and ΔE, source of randomness.

Game Playing

Game Playing

- attractive AI problem because it is **abstract**
- one of the oldest domains in AI
- in most cases, the world state is fully accessible
- computer representation of the situation can be clear and exact
- challenging: uncertainty introduced by the opponent and the complexity of the problem (full search is impossible)
- hard: in chess, branching factor is about 35, and 50 moves by each player $= 35^{100}$ nodes to search
  - compare to 10^{40} possible legal board states
- *game playing is more like real life than mechanical search*

Games vs. Search Problems

"Unpredictable" opponent $\longrightarrow$ solution is a contingency plan

Time limits $\longrightarrow$ unlikely to find goal, must approximate

Plan of attack:

- algorithm for perfect play (Von Neumann, 1944)
- finite horizon, approximate evaluation (Zuse, 1945; Shannon, 1950; Samuel, 1952–57)
- pruning to reduce costs (McCarthy, 1956)

Types of Games

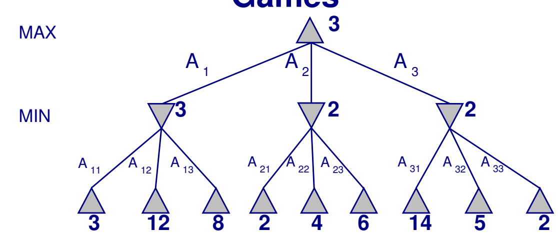
	deterministic	chance
perfect info	chess, checkers, go, othello	backgammon, monopoly
imperfect info	battle ship	bridge, poker, scrabble

Two-Person Perfect Information Game

initial state: initial position and who goes first
operators: legal moves
terminal test: game over?
utility function: outcome (win:+1, lose:-1, draw:0, etc.)

- two players (**MIN** and **MAX**) taking turns to maximize their chances of winning (each turn generates one **ply**)
- one player's victory is another's defeat
- need a **strategy** to win no matter what the opponent does

Minimax: Strategy for Two-Person Perfect Info Games



- generate the whole tree, and apply util function to the leaves
- go back upward assigning utility value to each node
- at MIN node, assign **min(successors' utility)**
- at MAX node, assign **max(successors' utility)**
- **assumption**: the opponent acts optimally

Minimax Decision

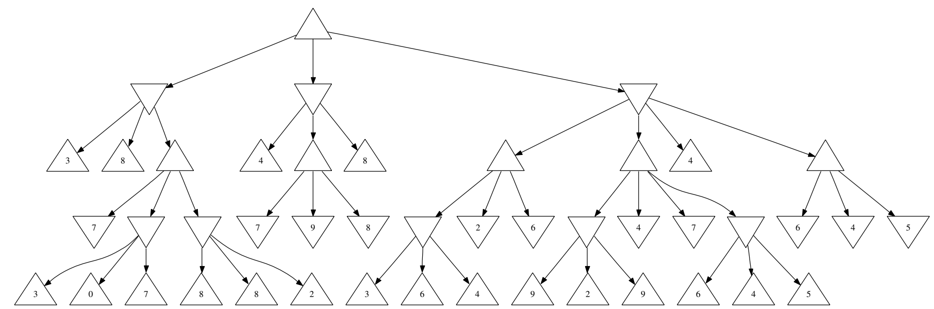
function Minimax-Decision (game) **returns** operator

return operator that leads to a child state with the **max**(Minimax-Value(child state,game))

function Minimax-Value(state,game) **returns** utility value

if Goal(state), **return** Utility(state)
else if Max's move **then**
$\longrightarrow$ **return** max of successors' Minimax-Value
else
$\longrightarrow$ **return** min of successors' Minimax-Value

Minimax Exercise



Minimax: Evaluation

Branching factor b, max depth m:

- **complete**: if the game tree is finite
- **optimal**: if opponent is optimal
- **time**: b^m
- **space**: bm – depth-first (only when utility function values of all nodes are known!)

Resource Limits

- Time limit: as in Chess $\longrightarrow$ can only evaluate a fixed number of paths
- Approaches:
  - **evaluation function** : how desirable is a given state?
  - **cutoff test** : depth limit
  - **pruning**

Depth limit can result in the **horizon effect**: interesting or devastating events can be just over the horizon!

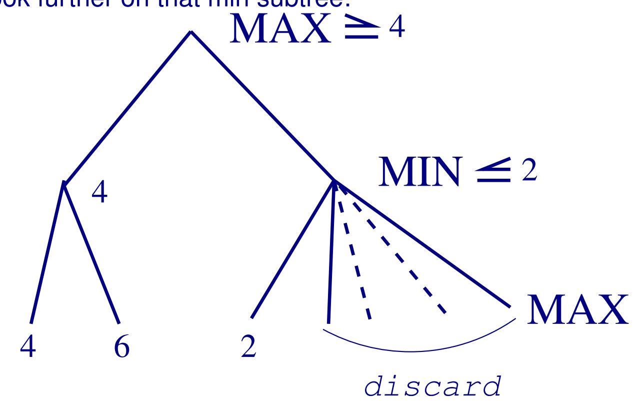
Evaluation Functions

For chess, usually a **linear** weighted sum of feature values:

- $\mathrm{Eval}(s) = \sum_i w_i f_i(s)$
- $f_i(s) =$ (number of white piece X) - (number of black piece X)
- other features: degree of control over the center area
- exact values do not matter: the **order** of Minimax-Value of the successors matter.

α Cuts

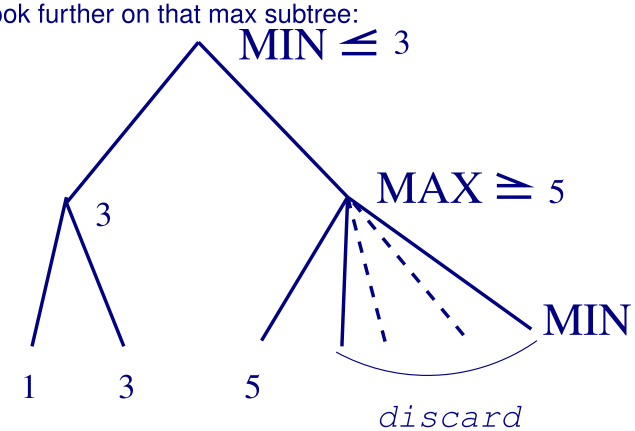
When the current max value is greater than the successor's min value, don't look further on that min subtree:



Right subtree can be **at most** 2, so MAX will always choose the left path regardless of what appears next.

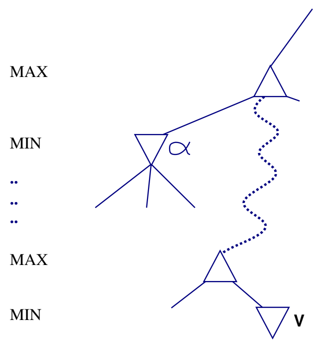
β Cuts

When the current min value is less than the successor's max value, don't look further on that max subtree:



Right subtree can be **at least** 5, so MIN will always choose the left path regardless of what appears next.

$\alpha - \beta$ Pruning



- memory of best MAX value α and best MIN value β
- do not go further on any one that does worse than the remembered α and β

$\alpha - \beta$ Pruning Properties

Cut off nodes that are known to be suboptimal.

Properties:

- pruning **does not** affect final result
- good move ordering improves effectiveness of pruning
- with **perfect ordering**, time complexity = $b^{m/2}$
  $\longrightarrow$ **doubles** depth of search
  $\longrightarrow$ can easily reach 8-ply in chess
- $b^{m/2} = (\sqrt{b})^m$, thus $b = 35$ in chess reduces to $b = \sqrt{35} \approx 6$!!!

Key Points

- Game playing: what are the types of games?
- Minimax: definition, and how to get minmax values
- Minimax: evaluation
- α-β pruning: why it saves time

Overview

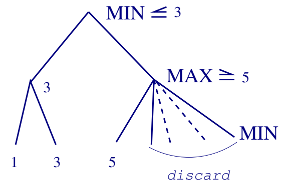
- formal $\alpha - \beta$ pruning algorithm
- $\alpha - \beta$ pruning properties
- games with an element of chance
- state-of-the-art game playing with AI
- more complex games

$\alpha - \beta$ Pruning: Initialization

Along the path from the beginning to the current **state**:

- α: best MAX value
  - initialize to $-\infty$
- β: best MIN value
  - initialize to ∞

$\alpha - \beta$ Pruning Algorithm: Max-Value



function Max-Value (state, game, α, β) **return** utility value

α: best MAX on path to *state* ; β: best MIN on path to *state*

if Cutoff(state) **then return** Utility(state)

$v \leftarrow -\infty$

for each *s* in Successor(state) **do**

· $v \leftarrow$ Max(α, Min-Value(s,game,α,β))

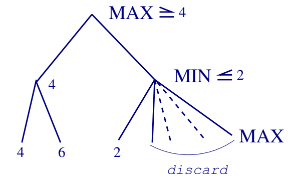
· **if** $v \geq \beta$ **then return** v /* **CUT!!** */

· $\alpha \leftarrow$Max(α, v)

end

return v

$\alpha - \beta$ Pruning Algorithm: Min-Value



function Min-Value (state, game, α, β) **return** utility value

α: best MAX on path to *state* ; β: best MIN on path to *state*

if Cutoff(state) **then return** Utility (state)

$v \leftarrow \infty$

for each *s* in Successor(state) **do**

· $v \leftarrow$ Min(β, Max-Value(s,game,α,β))

· **if** $v \leq \alpha$ **then return** v /* **CUT!!** */

· $\beta \leftarrow$Min(β,v)

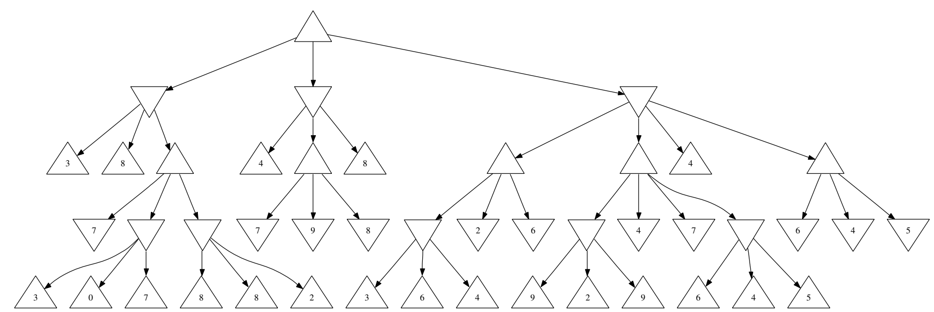
end

return v

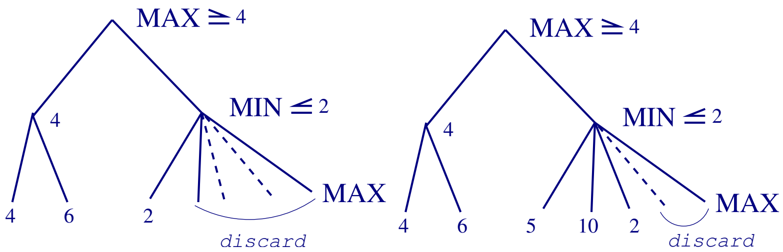
$\alpha - \beta$ Pruning Tips

- At a MAX node:
  - Only α is updated with the MAX of successors.
  - Cut is done by checking if returned $v \geq \beta$.
  - If all fails, MAX(v of succesors) is returned.
- At a MIN node:
  - Only β is updated with the MIN of successors.
  - Cut is done by checking if returned $v \leq \alpha$.
  - If all fails, MIN(v of succesors) is returned.

$\alpha - \beta$ Exercise



Ordering is Important for Good Pruning



- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

Games With an Element of Chance

Rolling the dice, shuffling the deck of card and drawing, etc.

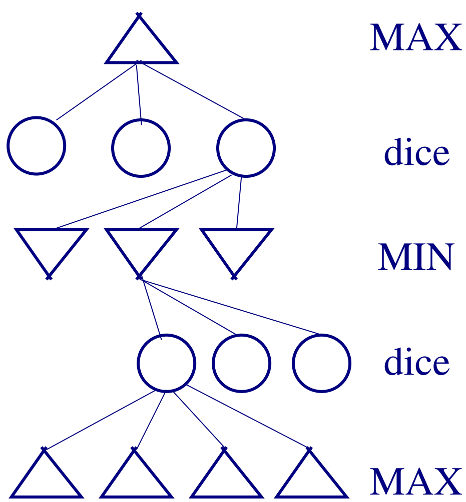
- **chance nodes** need to be included in the minimax tree
- try to make a move that maximizes the **expected value** $\rightarrow$ **expectimax**
- expected value of random variable X:

$$E(X) = \sum_x xP(x)$$

- expectimax

$$\text{expectimax}(C) = \sum_i P(d_i)\max_{s \in S(C,d_i)}(utility(s))$$

Game Tree With Chance Element



- chance element forms a new ply (e.g. dice, shown above)

Design Considerations for Probabilistic Games

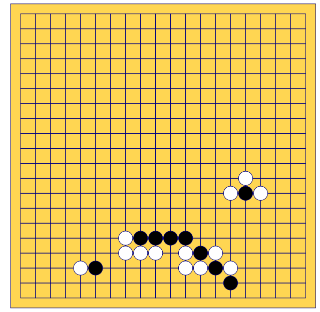
- the **value** of evaluation function, not just the **scale** matters now! (think of what expected value is)
- time complexity: $b^m n^m$, where n is the number of distinct dice rolls
- pruning can be done if we are careful

State of the Art in Gaming With AI

- Chess: IBM's Deep Blue defeated Garry Kasparov (1997)
- Backgammon: Tesauro's Neural Network $\longrightarrow$ top three (1992)
- Othello: smaller search space $\longrightarrow$ superhuman performance
- Checkers: Samuel's Checker Program running on 10Kbyte (1952)

Genetic algorithms can perform very well on select domains.

Hard Games



The game of *Go*, popular in East Asia:

- $19 \times 19 = 361$ grid: branching factor is huge!
- search methods inevitably fail: need more structured rules
- the bet was high: \$1,400,000 prize for the first computer program to beat a select, 12-year old player. The late Mr. Ing Chang Ki (photo above) put up the money from his personal funds.

Photo from http://www.samsloan.com/ing.htm.

Key Points

- formal $\alpha - \beta$ pruning algorithm: know how to apply pruning
- $\alpha - \beta$ pruning properties: evaluation
- games with an element of chance: what are the added elements? how does the minmax tree get augmented?