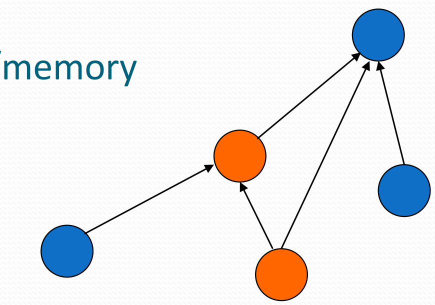
Neural Networks

- Threshold units
- Gradient descent
- Multilayer networks
- Backpropagation
- Hidden layer representations
- Example: Face Recognition
- Advanced topics
- And, more.

Blue slides: from Mitchell. Turquoise slides: from Alpaydin.

Neural Networks

- Networks of processing units (neurons) with connections (synapses) between them
- Large number of neurons: 10^{10}
- Large connectitivity: 10^5
- Parallel processing
- Distributed computation/memory
- Robust to noise, failures



Understanding the Brain

- Levels of analysis (Marr, 1982)
  1. Computational theory
  2. Representation and algorithm
  3. Hardware implementation
- Reverse engineering: From hardware to theory
- Parallel processing: SIMD vs MIMD

  Neural net: SIMD with modifiable local memory

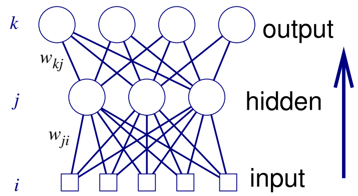
  Learning: Update by training/experience

Biological Neurons and Networks

- Neuron switching time $\sim .001$ second (1 ms)
- Number of neurons $\sim 10^{10}$
- Connections per neuron $\sim 10^{4-5}$
- Scene recognition time $\sim .1$ second (100 ms)
- 100 processing steps doesn't seem like enough

  [$\longrightarrow$] much parallel computation

Artificial Neural Networks



- Many neuron-like threshold switching units (real-valued)
- Many weighted interconnections among units
- Highly parallel, distributed process
- Emphasis on tuning weights automatically: New learning algorithms, new optimization techniques, new learning principles.

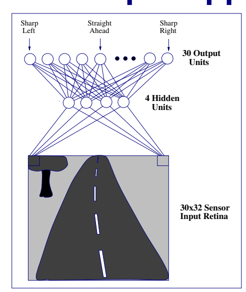
Biologically Motivated (or Accurate) Neural Networks

- Spiking neurons
- Complex morphological models
- Detailed dynamical models
- Connectivity either based on or trained to mimic biology
- Focus on **modeling** network/neural/subneural processes
- Focus on natural **principles** of neural computation
- Different forms of learning: spike-timing-dependent plasticity, covariance learning, short-term and long-term plasticity, etc.

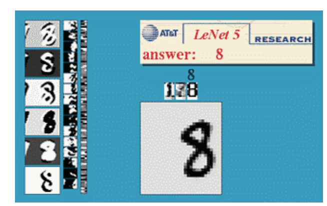
When to Consider Neural Networks

- Input is high-dimensional discrete or real-valued (e.g. raw sensor input)
- Output is discrete or real valued
- Output is a vector of values
- Possibly noisy data
- Long training time (may need occasional, extensive **retraining**)
- Form of target function is unknown
- Fast evaluation of learned target function
- Human readability of result is unimportant

Example Applications (more later)



(a) ALVINN

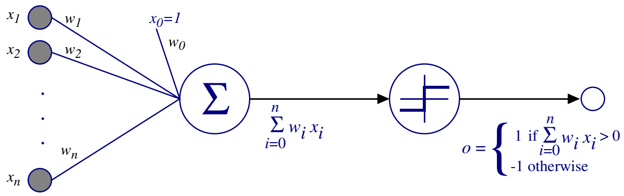


(b) http://yann.lecun.com

Examples:

- Speech synthesis
- Handwritten character recognition (from yann.lecun.com).
- Financial prediction, Transaction fraud detection (Big issue lately)
- Driving a car on the highway

Perceptrons

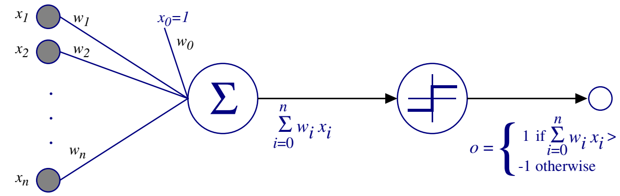


$$o(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Sometimes we'll use simpler vector notation:

$$o(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise.} \end{cases}$$

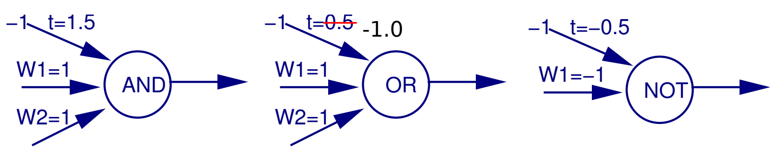
Hypothesis Space of Perceptrons



- The tunable parameters are the weights $w_0, w_1, ..., w_n$, so the space H of candidate hypotheses is the set of **all possible combination of real-valued weight vectors**:

$$H = \{\vec{w} | \vec{w} \in \mathcal{R}^{(n+1)}\}$$

Boolean Logic Gates with Perceptron Units



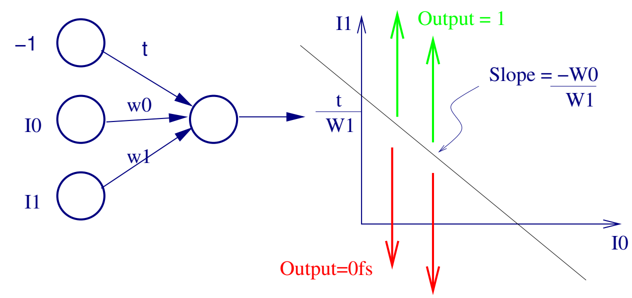
Russel & Norvig

input: $\{-1, 1\}$

- Perceptrons can represent basic boolean functions.
- Thus, a network of perceptron units can compute any Boolean function.

What about `XOR` or `EQUIV`?

What Perceptrons Can Represent



Perceptrons can only represent **linearly separable** functions.

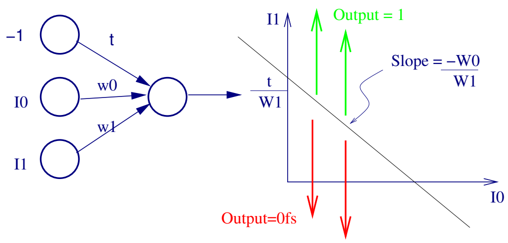
- Output of the perceptron:

$$W_0 \times I_0 + W_1 \times I_1 - t > 0, \text{ then output is } 1$$

$$W_0 \times I_0 + W_1 \times I_1 - t \leq 0, \text{ then output is } -1$$

The hypothesis space is a collection of separating lines.

Geometric Interpretation



- Rearranging

$$W_0 \times I_0 + W_1 \times I_1 - t > 0, \text{ then output is } 1,$$

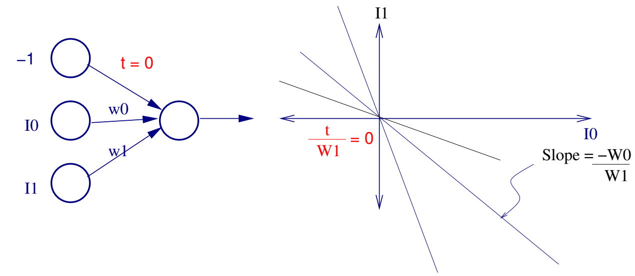
we get (if $W_1 > 0$)

$$I_1 > \frac{-W_0}{W_1} \times I_0 + \frac{t}{W_1},$$

where points above the line, the output is 1, and -1 for those below the line. Compare with

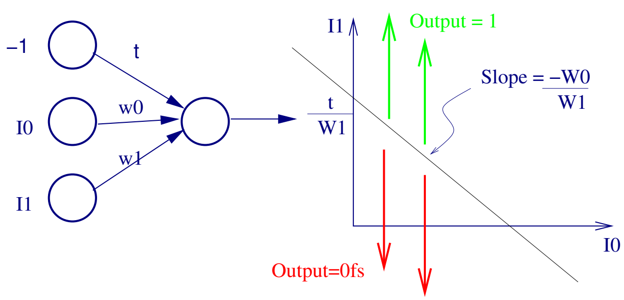
$$y = \frac{-W_0}{W_1} \times x + \frac{t}{W_1}.$$

The Role of the Bias



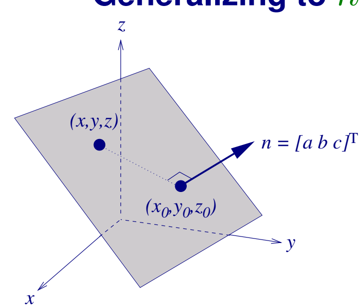
- Without the bias ($t = 0$), learning is limited to adjustment of the slope of the separating line passing through the origin.
- Three example lines with different weights are shown.

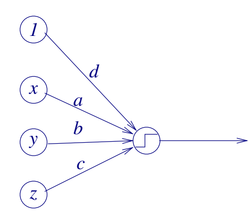
Limitation of Perceptrons



- Only functions where the -1 points and 1 points are clearly separable can be represented by perceptrons.
- The geometric interpretation is generalizable to functions of n arguments, i.e. perceptron with n inputs plus one threshold (or bias) unit.

Generalizing to n-Dimensions

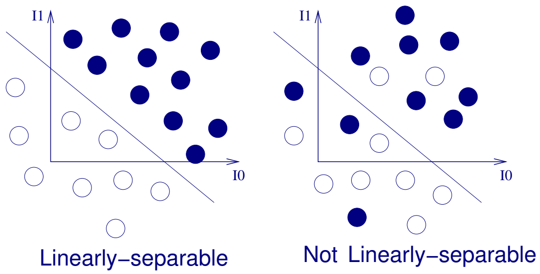




http://mathworld.wolfram.com/Plane.html

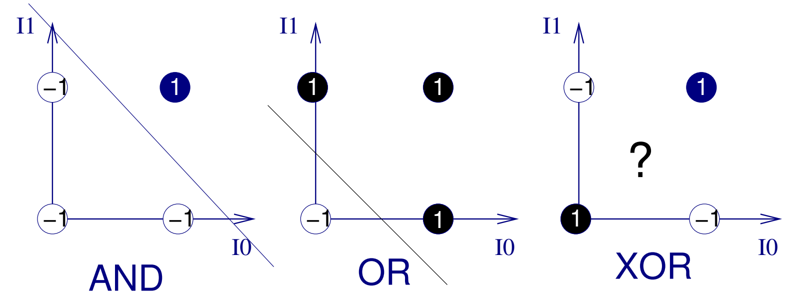
- $\vec{n} = (a, b, c), \vec{x} = (x, y, z), \vec{x_0} = (x_0, y_0, z_0)$.
- Equation of a plane: $\vec{n} \cdot (\vec{x} - \vec{x_0}) = 0$
- In short, $ax + by + cz + d = 0$, where a, b, c can serve as the weight, and $d = -\vec{n} \cdot \vec{x_0}$ as the bias.
- For n-D input space, the decision boundary becomes a $(n-1)$-D hyperplane (1-D less than the input space).

Linear Separability



- For functions that take integer or real values as arguments and output either -1 or 1.
- Left: linearly separable (i.e., can draw a straight line between the classes).
- Right: not linearly separable (i.e., perceptrons cannot represent such a function)

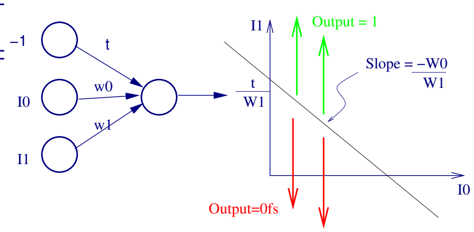
Linear Separability (cont'd)



- Perceptrons cannot represent `XOR`!
- Minsky and Papert (1969)

XOR in Detail

#	I_0	I_1	XOR
1	0	0	-1
2	0	1	1
3	1	0	1
4	1	1	-1



$W_0 \times I_0 + W_1 \times I_1 - t > 0$, then output is 1:

1 $\quad -t \leq 0 \quad \rightarrow \quad t \geq 0$

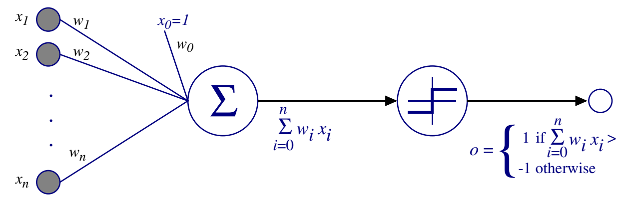
2 $\quad W_1 - t > 0 \quad \rightarrow \quad W_1 > t$

3 $\quad W_0 - t > 0 \quad \rightarrow \quad W_0 > t$

4 $\quad W_0 + W_1 - t \leq 0 \quad \rightarrow \quad W_0 + W_1 \leq t$

$2t < W_0 + W_1 < t$ (from 2, 3, and 4), but $t \geq 0$ (from 1), a contradiction.

Learning: Perceptron Rule



- The weights do not have to be calculated manually.
- We can train the network with (input,output) pair according to the following weight update rule:

$$w_i \leftarrow w_i + \eta(t - o)x_i$$

where η is the learning rate parameter.

- Proven to converge **if input set is linearly separable** and η is small.

Learning in Perceptrons (Cont'd)

$$w_i \leftarrow w_i + \eta(t - o)x_i$$

- When $t = o$, weight stays.
- When $t = 1$ and $o = -1$, change in weight is:

$$\eta(1 - (-1))x_i > 0$$

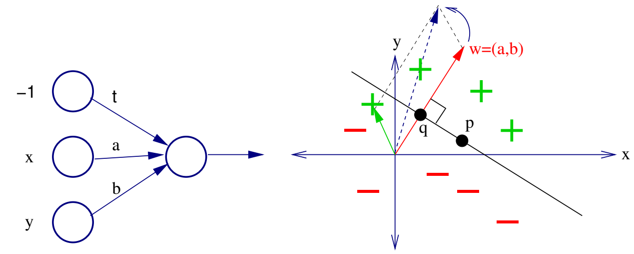
  if x_i are all positive. Thus $\vec{w} \cdot \vec{x}$ will increase, thus eventually, output o will turn to 1.

- When $t = -1$ and $o = 1$, change in weight is:

$$\eta(-1 - 1)x_i < 0$$

  if x_i are all positive. Thus $\vec{w} \cdot \vec{x}$ will decrease, thus eventually, output o will turn to -1.

Learning in Perceptron: Another Look



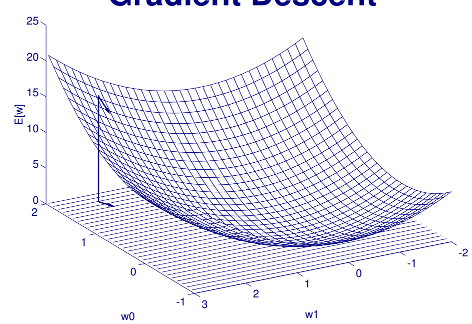
- The perceptron on the left can be represented as a line shown on the right (why? see page 14).
- Learning can be thought of as adjustment of $\vec{w}$ turning toward the input vector $\vec{x}$: $\quad \vec{w} \leftarrow \vec{w} + \eta(t - o)\vec{x}$.
- Adjustment of the bias t moves the line closer or away from the origin.

Another Learning Rule: Delta Rule

- The perceptron rule cannot deal with noisy data.
- The delta rule will find an approximate solution even when input set is not linearly separable.
- Use **linear unit** without the step function: $o(\vec{x}) = \vec{w} \cdot \vec{x}$.
- Want to reduce the error by adjusting $\vec{w}$:

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

Gradient Descent



- Want to minimize by adjusting $\vec{w}$: $\quad E(\vec{w}) \equiv \frac{1}{2}\sum_{d \in D}(t_d - o_d)^2$
- Note: the error surface is defined by the training data D. A different data set will give a different surface.
- $E(w_0, w_1)$ is the error function above, and we want to change (w_0, w_1) to position under a low E.

Gradient Descent (Cont'd)

Gradient

$$\nabla E[\vec{w}] \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots \frac{\partial E}{\partial w_n} \right]$$

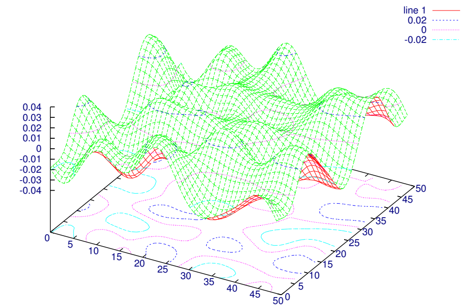
Training rule:

$$\Delta \vec{w} = -\eta \nabla E[\vec{w}]$$

i.e.,

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

Gradient Descent (Example)



- Gradient points in the **maximum increasing direction**.
- Gradient is prependicular to the level curve (uphill direction).
- $E(w_0, w_1)$ is the error function above, so $\nabla E = (\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1})$, a vector on a 2D plane.

Gradient Descent (Cont'd)

$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_d (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\
&= \sum_d (t_d - o_d) \frac{\partial}{\partial w_i} (t_d - \vec{w} \cdot \vec{x_d}) \\
\frac{\partial E}{\partial w_i} &= \sum_d (t_d - o_d)(-x_{i,d})
\end{aligned}$$

Since we want $\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$, $\Delta w_i = \eta \sum_d (t_d - o_d) x_{i,d}$.

Gradient Descent: Summary

Gradient-Descent ($training\_examples, \eta$)

Each training example is a pair of the form $\langle \vec{x}, t \rangle$, where $\vec{x}$ is the vector of input values, and t is the target output value. η is the learning rate (e.g., .05).

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - Initialize each Δw_i to zero.
  - For each $\langle \vec{x}, t \rangle$ in $training\_examples$, Do
    - Input the instance $\vec{x}$ to the unit and compute the output o
    - For each linear unit weight w_i, Do

$$\Delta w_i \leftarrow \Delta w_i + \eta (t - o) x_i$$

  - For each linear unit weight w_i, Do

$$w_i \leftarrow w_i + \Delta w_i$$

Gradient Descent Properties

Gradient descent is effective in searching through a large or infinite H:

- H contains continuously parameterized hypotheses, and
- the error can be **differentiated** wrt the parameters.

Limitations:

- convergence can be slow, and
- finds local minima (global minumum not guaranteed).

Stochastic Approximation to Grad. Desc.

Avoiding local minima: Incremental gradient descent, or stochastic gradient descent.

- Instead of weight update based on **all** input in D, immediately update weights after each input example:

$$\Delta w_i = \eta(t - o)x_i,$$

instead of

$$\Delta w_i = \eta \sum_{d \in D} (t_d - o_d)x_i,$$

- Can be seen as minimizing error function

$$E_d(\vec{w}) = \frac{1}{2}(t_d - o_d)^2.$$

Standard and Stochastic Grad. Desc.: Differences

- In the standard version, error is defined over entire D.
- In the standard version, more computation is needed per weight update, but η can be larger.
- Stochastic version can **sometimes** avoid local minima.

Summary

Perceptron training rule guaranteed to succeed if

- Training examples are linearly separable
- Sufficiently small learning rate η

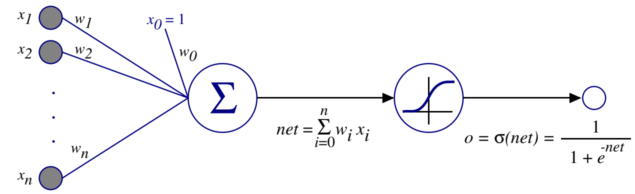
Linear unit training rule using gradient descent

- Asymptotic convergence to hypothesis with minimum squared error
- Given sufficiently small learning rate η
- Even when training data contains noise
- Even when training data not separable by H

Exercise: Implementing the Perceptron

- It is fairly easy to implement a perceptron.
- You can implement it in any programming language: C/C++, etc.
- Look for examples on the web, and JAVA applet demos.

Multilayer Networks



- Differentiable threshold unit: **sigmoid**

$$\sigma(y) = \frac{1}{1 + \exp(-y)}.$$

Interesting property: $\frac{d\sigma(y)}{dy} = \sigma(y)(1 - \sigma(y))$.

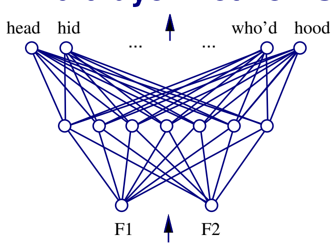
- Output:

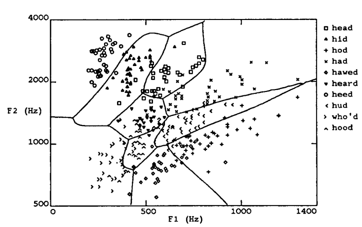
$$o = \sigma(\vec{w} \cdot \vec{x})$$

- Other functions:

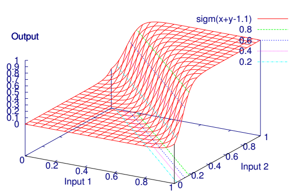
$$\tanh(y) = \frac{\exp(-2y) - 1}{\exp(-2y) + 1}$$

Multilayer Networks and Backpropagation

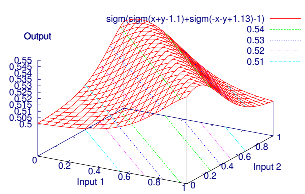




- Nonlinear decision surfaces.



(a) One output



(b) Two hidden, one output

- Another example: XOR

Error Gradient for a Sigmoid Unit

$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\
&= \sum_d (t_d - o_d) \left(- \frac{\partial o_d}{\partial w_i} \right) \\
&= - \sum_d (t_d - o_d) \frac{\partial o_d}{\partial net_d} \frac{\partial net_d}{\partial w_i}
\end{aligned}$$

Error Gradient for a Sigmoid Unit

From the previous page:

$$\frac{\partial E}{\partial w_i} = -\sum_d (t_d - o_d) \frac{\partial o_d}{\partial net_d} \frac{\partial net_d}{\partial w_i}$$

But we know:

$$\frac{\partial o_d}{\partial net_d} = \frac{\partial \sigma(net_d)}{\partial net_d} = o_d(1 - o_d)$$

$$\frac{\partial net_d}{\partial w_i} = \frac{\partial (\vec{w} \cdot \vec{x}_d)}{\partial w_i} = x_{i,d}$$

So:

$$\frac{\partial E}{\partial w_i} = -\sum_{d \in D} (t_d - o_d) o_d (1 - o_d) x_{i,d}$$

Backpropagation Algorithm

Initialize all weights to small random numbers.

Until satisfied, Do

- For each training example, Do
  1. Input the training example to the network and compute the network outputs
  2. For each output unit k
     $\delta_k \leftarrow o_k(1 - o_k)(t_k - o_k)$
  3. For each hidden unit h
     $\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in outputs} w_{kh} \delta_k$
  4. Update each network weight $w_{i,j}$
     $w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$ where
     $\Delta w_{ji} = \eta \delta_j x_i$.

Note: w_{ji} is the weight from i to j (i.e., $w_{j \leftarrow i}$).

The δ Term

- For output unit:

$$\delta_k \leftarrow \underbrace{o_k(1 - o_k)}_{\sigma'(net_k)} \underbrace{(t_k - o_k)}_{\text{Error}}$$

- For hidden unit:

$$\delta_h \leftarrow \underbrace{o_h(1 - o_h)}_{\sigma'(net_h)} \underbrace{\sum_{k \in outputs} w_{kh} \delta_k}_{\text{Backpropagated error}}$$

- In sum, δ is the derivative times the error.
- Derivation to be presented later.

Derivation of Δw

- Want to update weight as:

$$\Delta w_{ji} = -\eta \frac{\partial E_d}{\partial w_{ji}},$$

where error is defined as:

$$E_d(\vec{w}) \equiv \frac{1}{2} \sum_{k \in outputs} (t_k - o_k)^2$$

- Given $net_j = \sum_j w_{ji} x_i$,

$$\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} \frac{\partial net_j}{\partial w_{ji}}$$

- Different formula for output and hidden.

Derivation of Δw: Output Unit Weights

From the previous page, $\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} \frac{\partial net_j}{\partial w_{ji}}$

- First, calculate $\frac{\partial E_d}{\partial net_j}$:

$$\frac{\partial E_d}{\partial net_j} = \frac{\partial E_d}{\partial o_j} \frac{\partial o_j}{\partial net_j}$$

$$\begin{aligned}
\frac{\partial E_d}{\partial o_j} &= \frac{\partial}{\partial o_j} \frac{1}{2} \sum_{k \in outputs} (t_k - o_k)^2 \\
&= \frac{\partial}{\partial o_j} \frac{1}{2} (t_j - o_j)^2 \\
&= 2 \frac{1}{2} (t_j - o_j) \frac{\partial (t_j - o_j)}{\partial o_j} \\
&= -(t_j - o_j)
\end{aligned}$$

Derivation of Δw: Output Unit Weights

From the previous page,
$\frac{\partial E_d}{\partial net_j} = \frac{\partial E_d}{\partial o_j} \frac{\partial o_j}{\partial net_j} = -(t_j - o_j) \frac{\partial o_j}{\partial net_j}$:

- Next, calculate $\frac{\partial o_j}{\partial net_j}$: Since $o_j = \sigma(net_j)$, and $\sigma'(net_j) = o_j(1 - o_j)$,

$$\frac{\partial o_j}{\partial net_j} = o_j(1 - o_j).$$

Putting everything together,

$$\frac{\partial E_d}{\partial net_j} = \frac{\partial E_d}{\partial o_j} \frac{\partial o_j}{\partial net_j} = -(t_j - o_j) o_j (1 - o_j).$$

Derivation of Δw: Output Unit Weights

From the previous page:

$$\frac{\partial E_d}{\partial net_j} = \frac{\partial E_d}{\partial o_j} \frac{\partial o_j}{\partial net_j} = -(t_j - o_j) o_j (1 - o_j).$$

Since $\frac{\partial net_j}{\partial w_{ji}} = \frac{\partial \sum_k w_{jk} x_k}{\partial w_{ji}} = x_i$,

$$\begin{aligned}
\frac{\partial E_d}{\partial w_{ji}} &= \frac{\partial E_d}{\partial net_j} \frac{\partial net_j}{\partial w_{ji}} \\
&= -\underbrace{(t_j - o_j) o_j (1 - o_j)}_{\delta_j = error \times \sigma'(net)} \underbrace{x_i}_{input}
\end{aligned}$$

Derivation of Δw: Hidden Unit Weights

Start with $\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} \frac{\partial net_j}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} x_i$:

$$\begin{aligned}
\frac{\partial E_d}{\partial net_j} &= \sum_{k \in Downstream(j)} \frac{\partial E_d}{\partial net_k} \frac{\partial net_k}{\partial net_j} \\
&= \sum_{k \in Downstream(j)} -\delta_k \frac{\partial net_k}{\partial net_j} \\
&= \sum_{k \in Downstream(j)} -\delta_k \frac{\partial net_k}{\partial o_j} \frac{\partial o_j}{\partial net_j} \\
&= \sum_{k \in Downstream(j)} -\delta_k w_{kj} \frac{\partial o_j}{\partial net_j} \\
&= \sum_{k \in Downstream(j)} -\delta_k w_{kj} \underbrace{o_j (1 - o_j)}_{\sigma'(net)} \qquad (1)
\end{aligned}$$

Derivation of Δw: Hidden Unit Weights

Finally, given

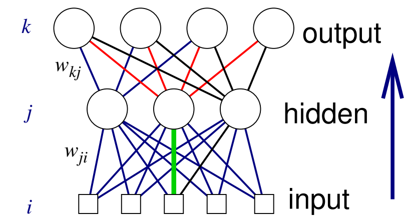
$$\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j}\frac{\partial net_j}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} x_i,$$

and

$$\frac{\partial E_d}{\partial net_j} = \sum_{k \in Downstream(j)} -\delta_k w_{kj} \underbrace{o_j(1-o_j)}_{\sigma'(net)},$$

$$\Delta w_{ji} = -\eta \frac{\partial E_d}{\partial w_{ji}} = \underbrace{\eta\, [\underbrace{o_j(1-o_j)}_{\sigma'(net)} \underbrace{\sum_{k \in Downstream(j)} \delta_k w_{kj}}_{error}]}_{\delta_j} x_i$$

Extension to Different Network Topologies



- Arbitrary number of layers: for neurons in layer m:

$$\delta_r = o_r(1-o_r) \sum_{s \in layer\ m+1} w_{sr}\delta s.$$

- Arbitrary acyclic graph:

$$\delta_r = o_r(1-o_r) \sum_{s \in Downstream(r)} w_{sr}\delta s.$$

Backpropagation: Properties

- Gradient descent over entire *network* weight vector.
- Easily generalized to arbitrary directed graphs.
- Will find a local, not necessarily global error minimum:
  - In practice, often works well (can run multiple times with different initial weights).
- Often include weight *momentum* α

$$\Delta w_{i,j}(n) = \eta \delta_j x_{i,j} + \alpha \Delta w_{i,j}(n-1).$$

- Minimizes error over *training* examples:
  - Will it generalize well to subsequent examples?
- Training can take thousands of iterations $\longrightarrow$ slow!
- Using the network after training is very fast.

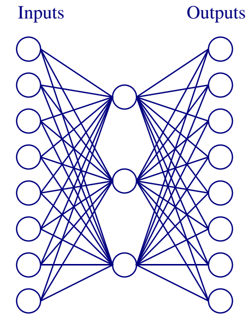
Representational Power of Feedforward Networks

- Boolean functions: every boolean function representable with two layers (hidden unit size can grow exponentially in the worst case: one hidden unit per input example, and “OR” them).
- Continous functions: Every **bounded** continuous function can be approximated with an arbitrarily small error (output units are linear).
- Arbitrary functions: with three layers (output units are linear).

H-Space Search and Inductive Bias

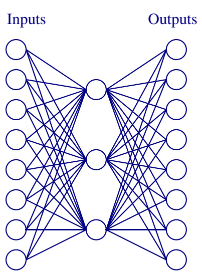
- H-space = n-D weight space (when there are n weights).
- The space is **continuous**, unlike decision tree or general-to-specific concept learning algorithms.
- Inductive bias:
  - Smooth interpolation between data points.

Learning Hidden Layer Representations



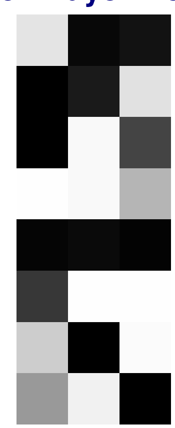
Input		Output
10000000	$\longrightarrow$	10000000
01000000	$\longrightarrow$	01000000
00100000	$\longrightarrow$	00100000
00010000	$\longrightarrow$	00010000
00001000	$\longrightarrow$	00001000
00000100	$\longrightarrow$	00000100
00000010	$\longrightarrow$	00000010
00000001	$\longrightarrow$	00000001

Learned Hidden Layer Representations



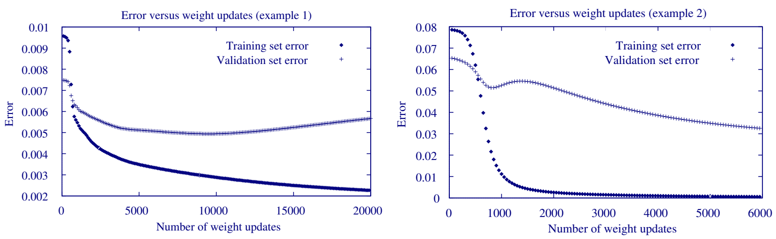
Input		Hidden Values				Output
10000000	$\longrightarrow$	.89	.04	.08	$\longrightarrow$	10000000
01000000	$\longrightarrow$	.01	.11	.88	$\longrightarrow$	01000000
00100000	$\longrightarrow$	.01	.97	.27	$\longrightarrow$	00100000
00010000	$\longrightarrow$	.99	.97	.71	$\longrightarrow$	00010000
00001000	$\longrightarrow$	.03	.05	.02	$\longrightarrow$	00001000
00000100	$\longrightarrow$	.22	.99	.99	$\longrightarrow$	00000100
00000010	$\longrightarrow$	.80	.01	.98	$\longrightarrow$	00000010
00000001	$\longrightarrow$	.60	.94	.01	$\longrightarrow$	00000001

Learned Hidden Layer Representations



- Learned encoding is similar to standard 3-bit binary code.
- Automatic discovery of **useful hidden layer representations** is a key feature of ANN.
- Note: The hidden layer representation is **compressed**.

Overfitting



- Error in two different robot perception tasks.
- Training set and validation set error.
- Early stopping ensures good performance on unobserved samples, but must be careful.
- Weight decay, use of validation sets, use of k-fold cross-validation, etc. to overcome the problem.

Alternative Error Functions

Penalize large weights:

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2 + \gamma \sum_{i,j} w_{ji}^2$$

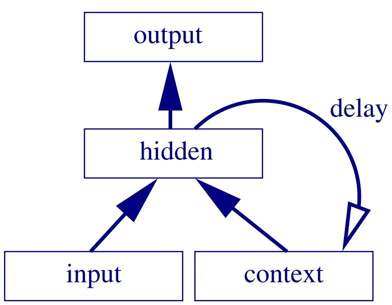
Train on target slopes as well as values (when the slope is available):

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in outputs} \left[(t_{kd} - o_{kd})^2 + \mu \sum_{j \in inputs} \left(\frac{\partial t_{kd}}{\partial x_d^j} - \frac{\partial o_{kd}}{\partial x_d^j} \right)^2 \right]$$

Tie together weights:

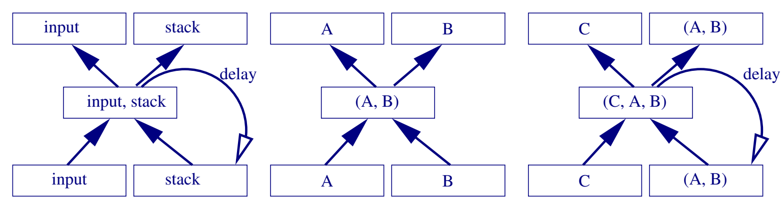
- e.g., in phoneme recognition network, or
- handwritten character recognition (weight sharing).

Recurrent Networks



- Sequence recognition.
- Store tree structure (next slide).
- Can be trained with plain backpropagation.
- Generalization may not be perfect.

Recurrent Networks (Cont'd)

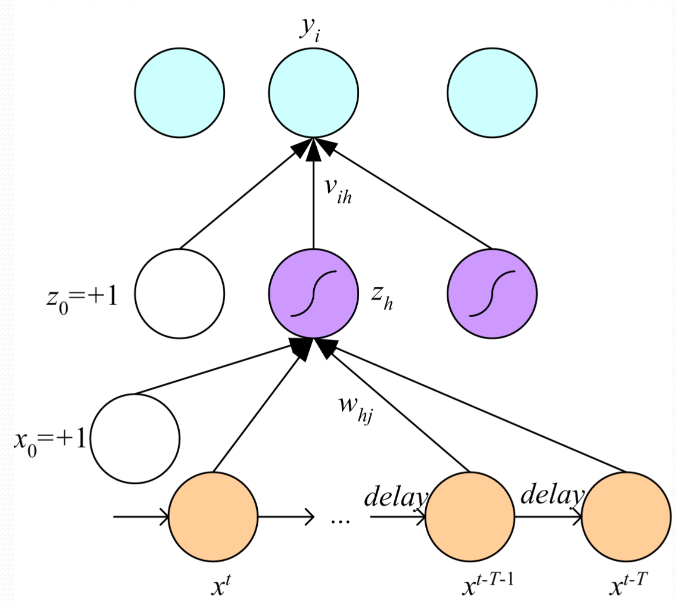


- Autoassociation (intput = output)
- Represent a stack using the hidden layer representation.
- Accuracy depends on numerical precision.

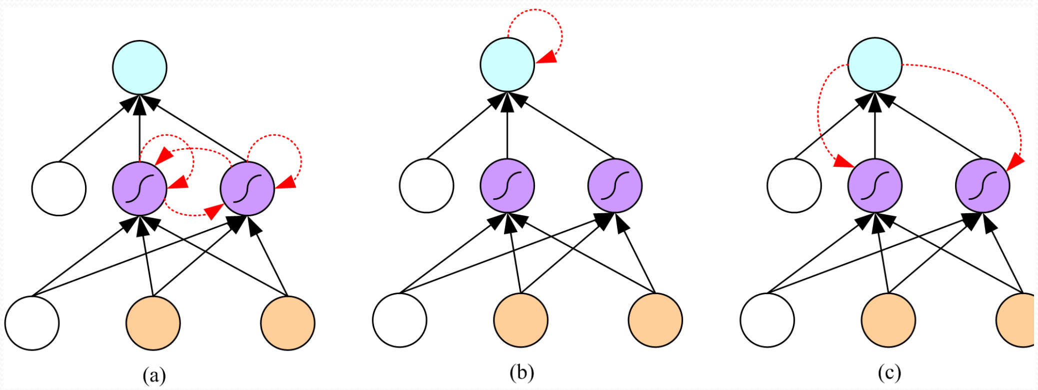
Learning Time

- Applications:
  - Sequence recognition: Speech recognition
  - Sequence reproduction: Time-series prediction
  - Sequence association
- Network architectures
  - Time-delay networks (Waibel et al., 1989)
  - Recurrent networks (Rumelhart et al., 1986)

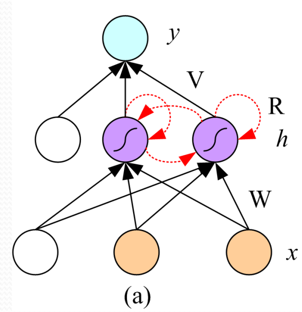
Time-Delay Neural Networks

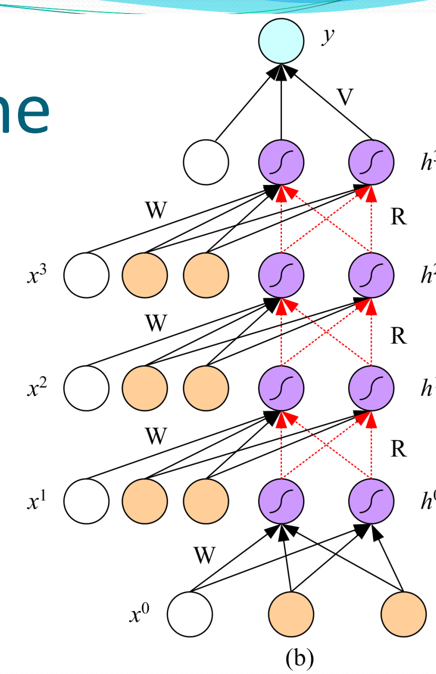


Recurrent Networks

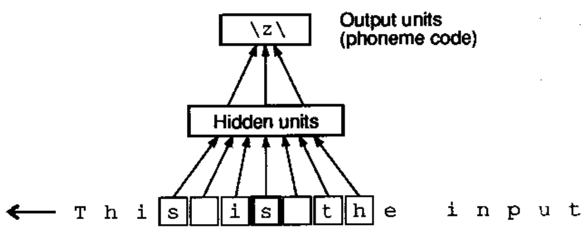


Unfolding in Time





Some Applications: NETtalk



- NETtalk: Sejnowski and Rosenberg (1987).
- Learn to pronounce English text.
- Demo
- Data available in UCI ML repository

NETtalk data

```
aardvark a-rdvark 1<<<>2<<0
aback xb@k-0>1<<0
abacus @bxkxs 1<0>0<0
abaft xb@ft 0>1<<0
abalone @bxloni 2<0>1>0 0
abandon xb@ndxn 0>1<>0<0
abase xbes-0>1<<0
abash xb@S-0>1<<0
abate xbet-0>1<<0
abatis @bxti-1<0>2<2
...
```

- Word – Pronunciation – Stress/Syllable
- about 20,000 words

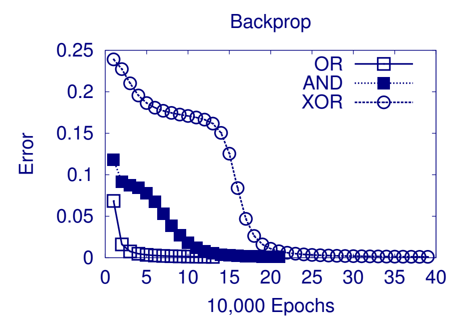
Backpropagation Exercise

- **URL:** http://www.cs.tamu.edu/faculty/choe/src/backprop-1.6.tar.gz
- Untar and read the `README` file:

  ```
  gzip -dc backprop-1.6.tar.gz | tar
  xvf -
  ```

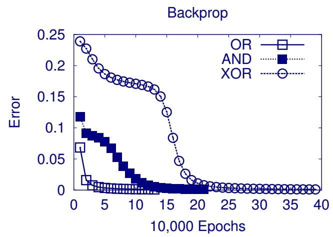
- Run `make` to build (on departmental unix machines).
- Run `./bp conf/xor.conf` etc.

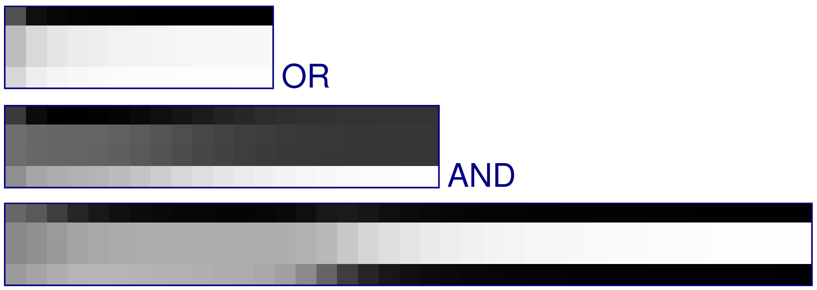
Backpropagation: Example Results



- Epoch: one full cycle of training through all training input patterns.
- `OR` was easiest, `AND` the next, and `XOR` was the most difficult to learn.
- Network had 2 input, 2 hidden and 1 output unit. Learning rate was 0.001.

Backpropagation: Example Results (cont'd)



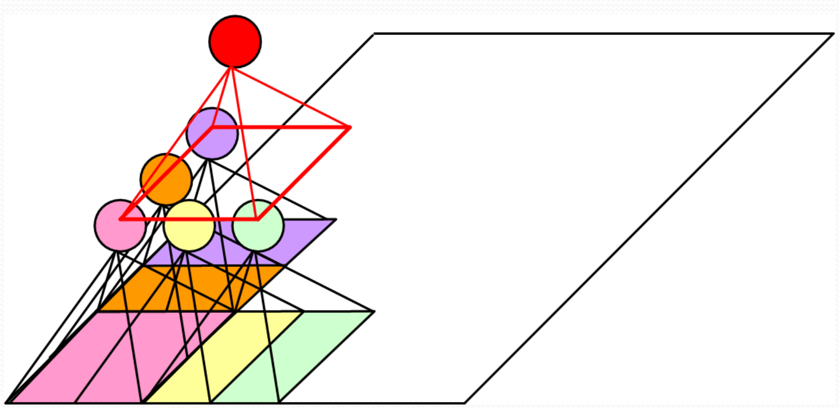


Output to (0,0), (0,1), (1,0), and (1,1) form each row.

Backpropagation: Things to Try

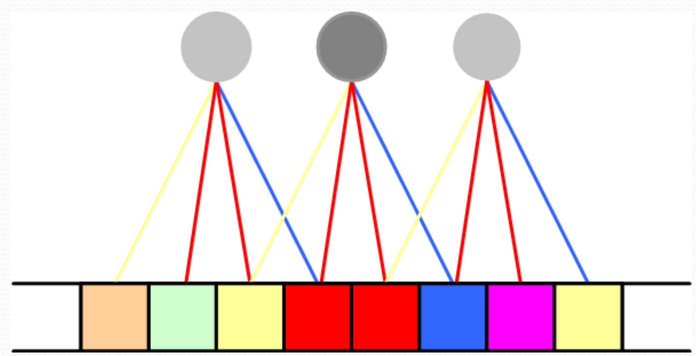
- How does increasing the number of hidden layer units affect the (1) time and the (2) number of epochs of training?
- How does increasing or decreasing the learning rate affect the rate of convergence?
- How does changing the slope of the sigmoid affect the rate of convergence?
- Different problem domains: handwriting recognition, etc.

Structured MLP



(Le Cun et al, 1989)

Weight Sharing



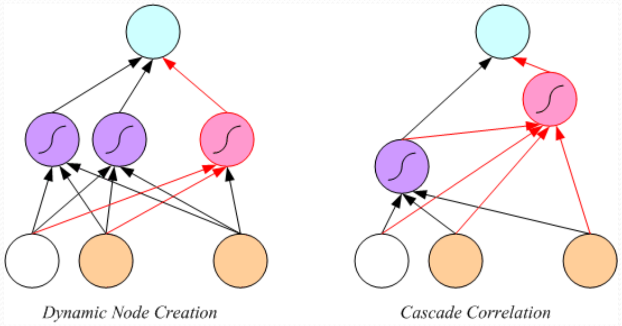
Tuning the Network Size

- Destructive
- Weight decay:

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i} - \lambda w_i$$

$$E' = E + \frac{\lambda}{2}\sum_i w_i^2$$

- Constructive
- Growing networks



(Ash, 1989) (Fahlman and Lebiere, 1989)

Bayesian Learning

- Consider weights w_i as random vars, prior $p(w_i)$

$$p(\mathbf{w}|\mathcal{X}) = \frac{p(\mathcal{X}|\mathbf{w})p(\mathbf{w})}{p(\mathcal{X})} \quad \hat{\mathbf{w}}_{MAP} = \arg\max_{\mathbf{w}} \log p(\mathbf{w}|\mathcal{X})$$

$$\log p(\mathbf{w}|\mathcal{X}) = \log p(\mathcal{X}|\mathbf{w}) + \log p(\mathbf{w}) + C$$

$$p(\mathbf{w}) = \prod_i p(w_i) \text{ where } p(w_i) = c \cdot \exp\left[-\frac{w_i^2}{2(1/2\lambda)}\right]$$

$$E' = E + \lambda \|\mathbf{w}\|^2$$

- Weight decay, ridge regression, regularization

  cost=data-misfit + λ complexity

  More about Bayesian methods in chapter 14

Summary

- ANN learning provides general method for learning real-valued functions over continuous or discrete-valued attributed.
- ANNs are robust to noise.
- H is the space of all functions parameterized by the weights.
- H space search is through gradient descent: convergence to local minima.
- Backpropagation gives novel hidden layer representations.
- Overfitting is an issue.
- More advanced algorithms exist.