Topics in ALGORITHMIC GAME THEORY *

Spring 2012

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*** Based on slides by Prof. Costis Daskalakis**

Let's play:

game theory society sign

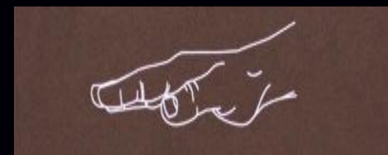
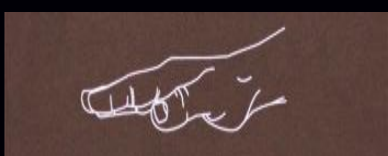
Let's play:
Battle of the Sexes

	Theater	Football
Theater	1, 5	0, 0
Football	0, 0	5, 1

* Numbers denote degree of happiness

WORLD RPS SOCIETY

Serving the needs of decision makers since 1918

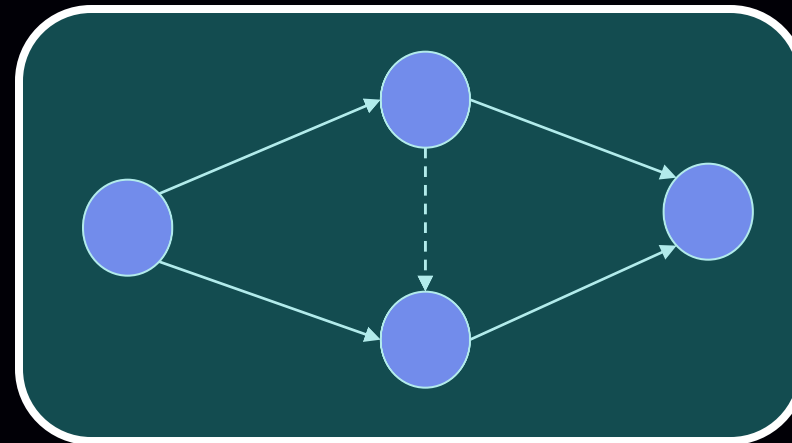
			
	tie	lose, win	win, lose
	win, lose	tie	lose, win
	lose, win	win, lose	tie

What we study in Algorithmic Game Theory…

game theory society sign

Markets

Routing in Networks



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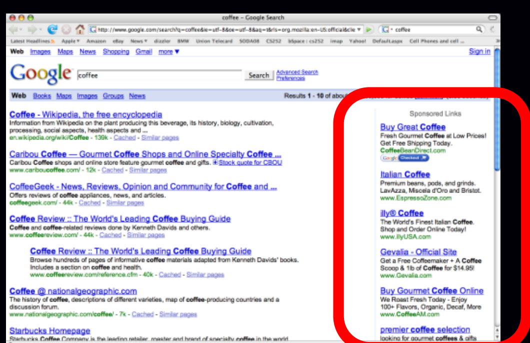
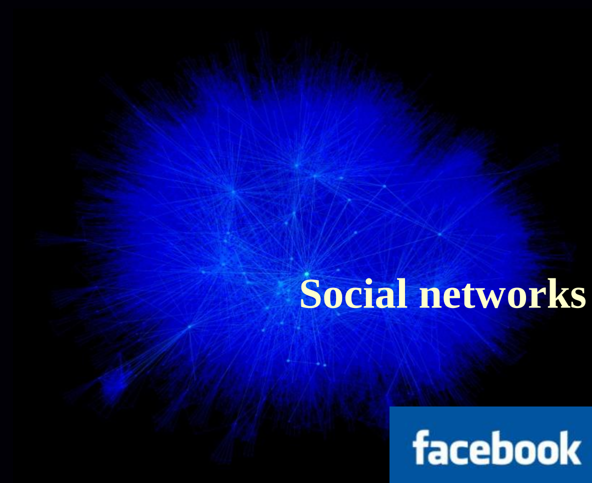
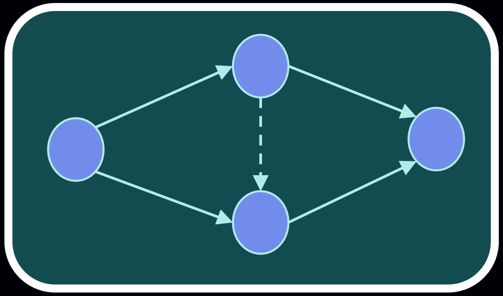
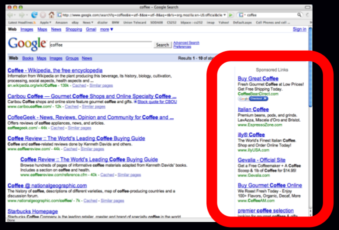


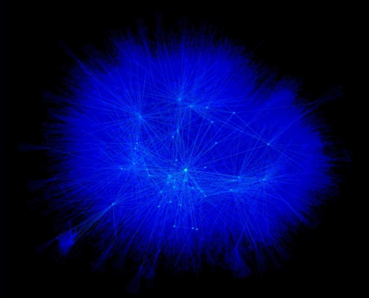
Elections

Social networks









Game Theory

We study (and sometimes question)
the algorithmic foundations of this theory.

Game Theory

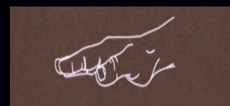
`Games` are thought experiments to help us learn how to ***predict rational behavior*** in ***situations of conflict.***

Situation of conflict: Everybody's actions affect others. This is captured by the tabular game formalism.

Rational Behavior: The players want to maximize their own expected utility. No altruism, envy, masochism, or externalities (if my neighbor gets the money, he will buy louder stereo, so I will hurt a little myself...).

Predict: We want to know what happens in a game. Such predictions are called solution concepts (e.g., Nash equilibrium).

Algorithmic Game Theory

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

Mechanism Design: How can we design a system that will be launched and used by competitive users to optimize our objectives ?

?

Can we predict what will happen in a large system?

Game theory says yes, through its prediction tools (solution concepts).

Can we ***efficiently*** predict what will happen in a large system?

Are the predictions of Game Theory ***plausible*, *i.e. likely to arise***?

Sample topics from Algorithmic Game Theory

- *Solution Concepts*
- *Equilibrium Computation*
- *Price of Anarchy*
- *Mechanism Design*

Battle of the Sexes

	Theater	Football
Theater	1, 5	0, 0
Football	0, 0	5, 1

Think of this game as a **metaphor** of real-life examples, not necessarily in the context of a couple's decision making, not necessarily about football vs theater, and not necessarily with exactly these numerical values associated to different outcomes.

Nash Equilibrium: A pair of strategies (deterministic or randomized) such that the strategy of the row player is a *Best Response* to the strategy of the column player and vice versa.

Above definition is for static (one-shot) games. Other solution concepts for:

- Extensive-form games
- Repeated games
- Games with incomplete/ imperfect information

Back to Rock-Paper-Scissors

0,0	-1,1	1,-1
1,-1	0,0	-1 , 1
-1,1	1 , -1	0,0

The unique Nash Equilibrium is the pair of uniform strategies.

Contrary to the battle of the sexes, in RPS randomization is necessary to construct a Nash equilibrium.

Rock-Paper-Scissors Championship

The behavior observed in the RPS championship is very different from the pair of uniform strategies; indeed, the matrix-form version of RPS did not intend to capture the *repeated* interaction between the same pair of players rather, the intention is to model the behavior of a population of, say, students in a courtyard participating in random occurrences of RPS games

Guess Two-Thirds of the Average

- k players $p_1, p_2, p_3, \ldots, p_k$
- each player submits a number in [0,100]

$$x_1, x_2, \ldots, x_k$$

- compute

$$\bar{x} := \frac{1}{k} \sum_{i=1}^{k} x_i$$

- find x_j, closest to $\frac{2}{3}\bar{x}$
- player p_j wins $100, all other players win nothing

Let's Play!

Guess Two-Thirds of the Average

Is it rational to play above $\frac{2}{3} \cdot 100$?

A: no (why?)

Given that no rational player will play above $\frac{2}{3} \cdot 100$ is it rational to play above $(2/3)^2 \cdot 100$?

A: no (same reasons)

⋮

All rational players should play 0.

The all-zero strategy is the only Nash equilibrium of this game.

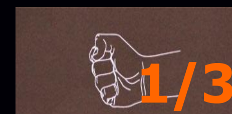
Rationality versus common knowledge of rationality

historical facts: 21.6 was the winning value in a large internet-based competition organized by the Danish newspaper Politiken. This included 19,196 people and with a prize of 5000 Danish kroner.

OK, Nash equilibrium makes sense and is stable, but does it always exist?

2-player Zero-Sum Games

$R + C = 0$



0,0	-1,1	1,-1
1,-1	0,0	-1 , 1
-1,1	1 , -1	0,0

Danzig '47

von Neumann '28:
In two-player zero-sum games, it always exists.

⟺ **LP duality**

[original proof used Brouwer's Fixed Point Theorem]

Poker

von Neuman's predictions are in fact accurate in predicting players' strategies in two-player poker!

Poker

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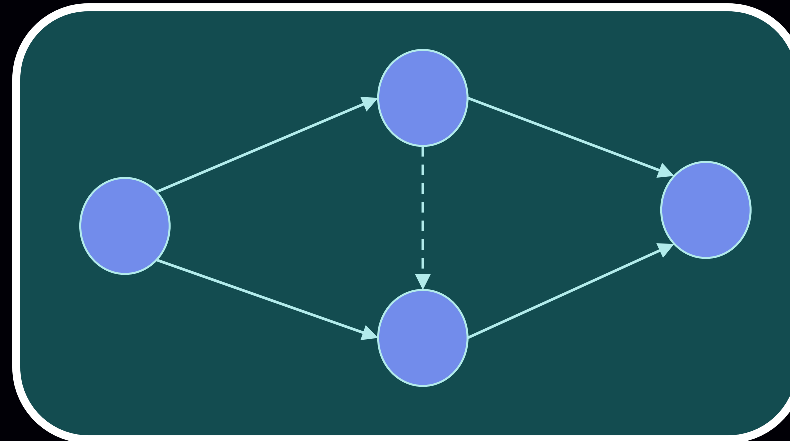
But what about larger systems (more than 2 players) or systems where players do not have directly opposite interests?

Markets

Routing in Networks



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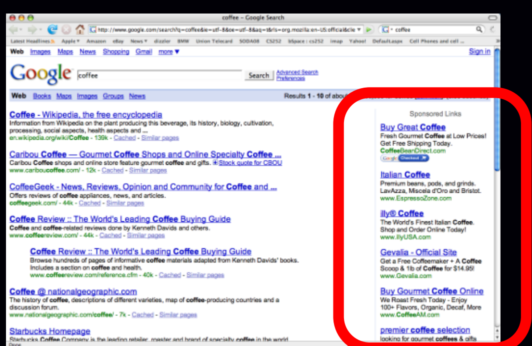
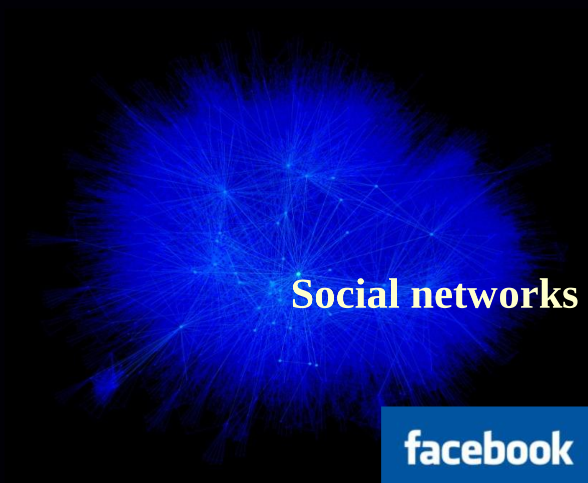


Elections

Social networks



Modified Rock Paper Scissors

Not zero-sum any more

	25%	50%	25%
33%	0,0	-1, 1	2,-1
33%	1,-1	0,0	- 1 , 1
33%	- 2, 1	1 , -1	0,0

Is there still an equilibrium?

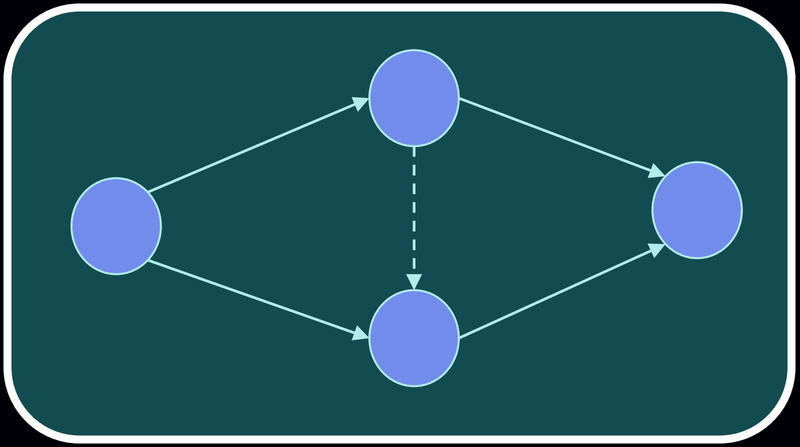
[that is a pair of randomized strategies so that no player has incentive to deviate given the other player's strategy ?]

John Nash '51:

There always exists a Nash equilibrium, regardless of the game's properties.

Markets

Routing in Networks



and every other game!

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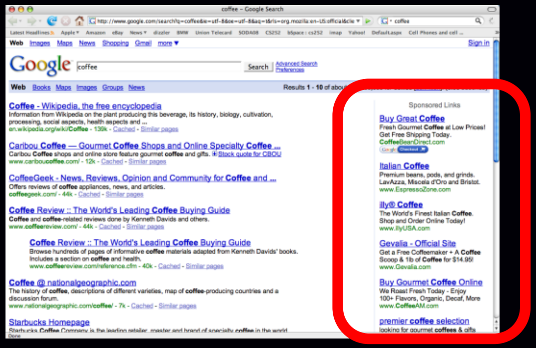

Elections

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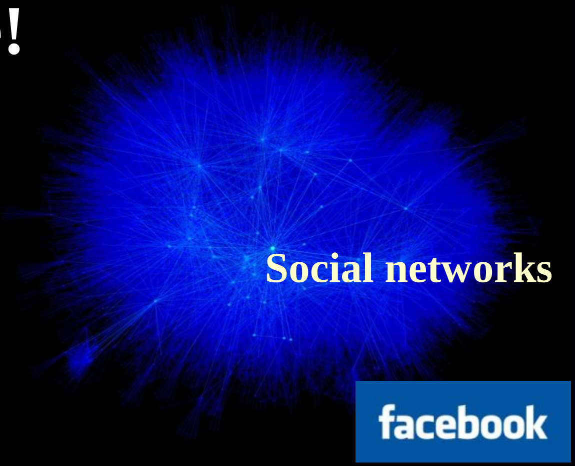

Applications…

game =

market → price equilibrium

Internet → packet routing

roads → traffic pattern

facebook, myspace, … → structure of the social network

Modified Rock Paper Scissors

Not zero-sum any more

	25%	50%	25%
33%	0,0	-1, 1	2,-1
33%	1,-1	0,0	- 1 , 1
33%	- 2, 1	1 , -1	0,0

Highly Non-Constructive

Is there still an equilibrium?

[that is a pair of randomized strategies so that no player has incentive to deviate given the other player's strategy ?]

Brouwer's Fixed Point Theorem

John Nash '51:
There always exists a Nash equilibrium, regardless of the game's properties.

Nobel 1994, due to its large influence in understanding systems of competitors…

Sample topics from Algorithmic Game Theory

- *Solution Concepts*
- *Equilibrium Computation*
- *Price of Anarchy*
- *Mechanism Design*

How can we compute a Nash equilibrium?

- if we had an algorithm for equilibria we could predict what behavior will arise in a system, before the system is launched

- in this case, we can easily compute the equilibrium, thanks to gravity!

Why should we care about computing equilibria?

• First, if we believe our equilibrium theory, efficient algorithms would enable us to make predictions:

Herbert Scarf writes…

"[Due to the non-existence of efficient algorithms for computing equilibria], general equilibrium analysis has remained at a level of abstraction and mathematical theoretizing far removed from its ultimate purpose as a method for the evaluation of economic policy."

The Computation of Economic Equilibria, 1973

• More importantly: If equilibria are supposed to model behavior, computational tractability is an important modeling *prerequisite*.

"If your laptop can't find the equilibrium, then how can the market?"

Kamal Jain, EBay

N.B. computational intractability implies the non-existence of efficient dynamics converging to equilibria; how can equilibria be universal, if such dynamics don't exist?

2-player zero-sum vs General Games

1928 Neumann:

- existence of min-max equilibrium in *2-player*, *zero-sum* games;

- proof uses Brouwer's fixed point theorem;

+ Danzig '47: equivalent to LP duality;

+ Khachiyan'79: poly-time solvable;

+ a multitude of distributed algorithms converge to equilibria.

1950 Nash:

- existence of an equilibrium in *multiplayer*, *general-sum* games;

- proof also uses Brouwer's fixed point theorem;

- intense effort for equilibrium computation algorithms:

  Kuhn '61, Mangasarian '64, Lemke-Howson '64, Rosenmüller '71, Wilson '71, Scarf '67, Eaves '72, Laan-Talman '79, etc.

- Lemke-Howson: simplex-like, works with LCP formulation;

no efficient algorithm is known after 50+ years of research.

hence, also no efficient dynamics …

Sample topics from Algorithmic Game Theory

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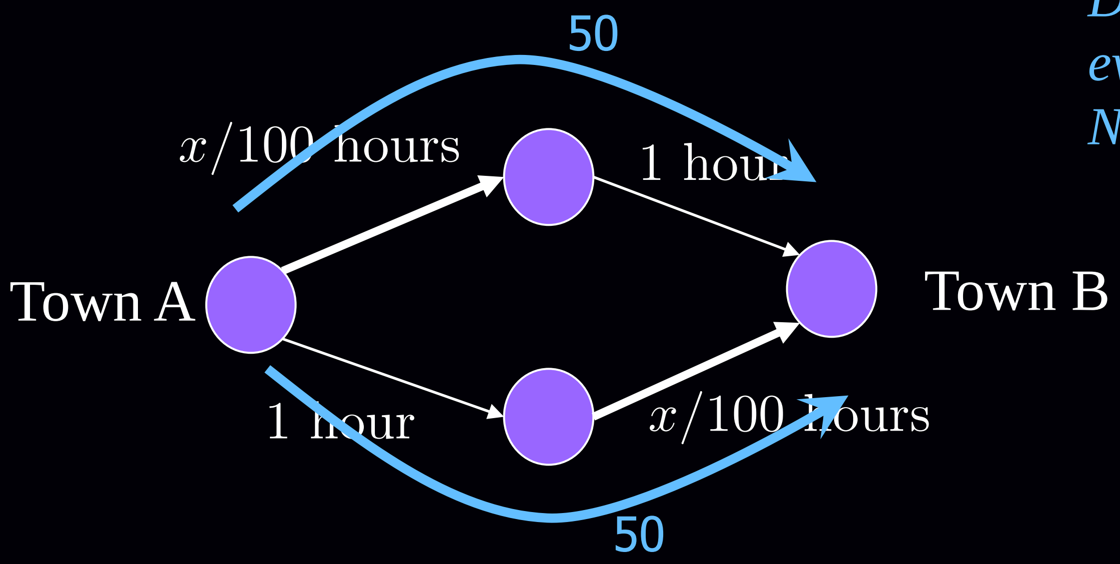
Traffic Routing



Delay is 1.5 hours for everybody at the unique Nash equilibrium

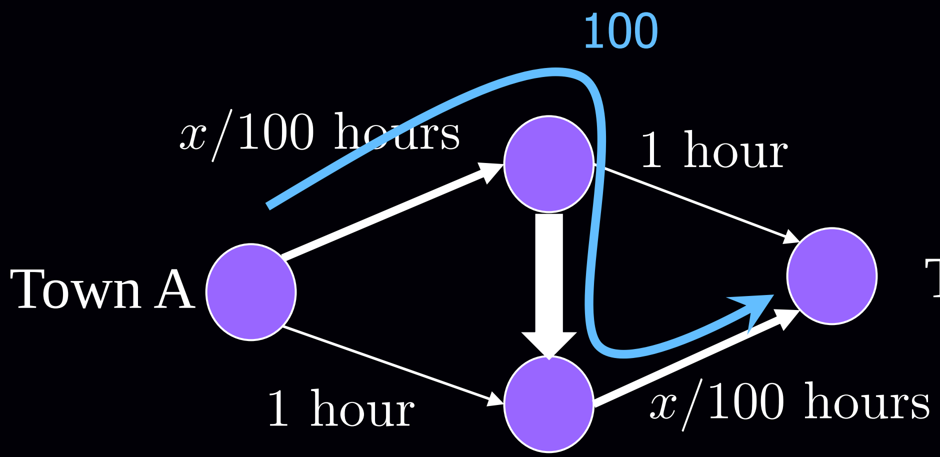
Suppose 100 drivers leave from town A towards town B.

Every driver wants to minimize her own travel time.

What is the traffic on the network?

In any unbalanced traffic pattern, all drivers on the most loaded path have incentive to switch their path.

Traffic Routing



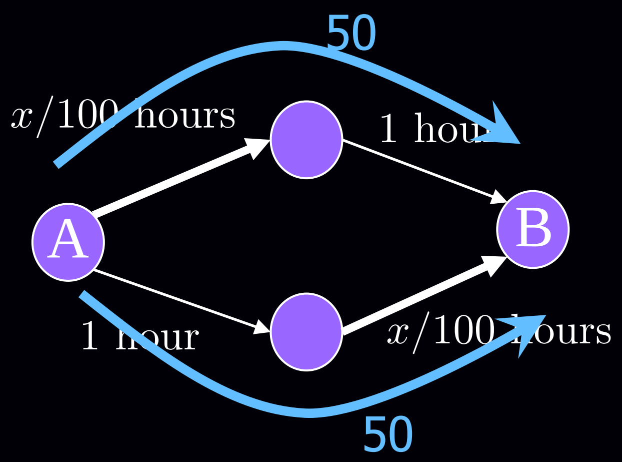
Delay is 2 hours for everybody at the unique Nash equilibrium

A benevolent mayor builds a superhighway connecting the fast highways of the network.

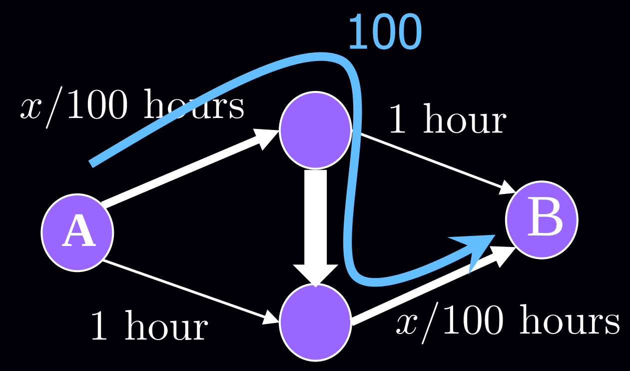
What is now the traffic on the network?

No matter what the other drivers are doing it is always better for me to follow the zig-zag path.

Traffic Routing



vs



Adding a fast road on a road-network is not always a good idea!

Braess's paradox

$4/3$

In the RHS network there exists a traffic pattern where all players have delay 1.5 hours.

$$\text{PoA} = \frac{\text{performance of system in worst Nash equilibrium}}{\text{optimal performance if drivers did not decide on their own}}$$

Price of Anarchy: measures the loss in system performance due to free-will

Traffic Routing

Obvious Questions:

What is the worst-case PoA in a system?

How do we design a system whose PoA is small?

In other words, what incentives can we provide to induce performance that is close to optimal?

E.g. tolls?

Sample topics from Algorithmic Game Theory

- *Solution Concepts*
- *Equilibrium Computation*
- *Price of Anarchy*
- *Mechanism Design*

Auctions

- Suppose we have one item for sale;

- k parties (or *bidders*) are interested in the item;

- party i has value v_i for the item, which is private, and our objective is to give the item to the party with the largest value (alternatively make as much revenue as possible from the sale);

- we ask each party for a bid, and based on their bids $b_1, b_2, \ldots, b_k$ we decide who gets the item and how much they pay;

-if bidder i gets the item and pays price p, her total utility is $v_i - p$ (quasilinear)

Auctions

First Price Auction: Give item to bidder with largest b_i, and charge him b_i

clearly a bad idea to bid above your value (why?)

but you may bid below your value (and you will!)

e.g. two bidders with values $v_1 = \$5$, $v_2 = \$100$

Nash equilibrium = $(b_1, b_2) = (\$5, \$5.01)$ (assume bids are in increments of cents)

non truthful!

- bidders want to place different bids, depending on their opponents' bids, which they don't know a priori; hence cycling may occur while they are trying to learn/guess them, etc.

- it is non-obvious how to play

- in the end, the auctioneer does not learn people's true values

Auctions

Second Price Auction:

Give item to bidder with highest bid and charge him the second largest bid.

e.g. if the bids are (b_1, b_2) = ($5, $100), then second bidder gets the item and pays $5

bidding your value is a *dominant strategy* (i.e. the best you could bid regardless of what others are doing)

truthful!

In conclusion

- *We are going to study and question the algorithmic foundations of Game Theory*

- *Models of strategic behavior*

  dynamics of player interaction:
  e.g. best response, exploration-exploitation,…

- *System Design*

  auctions

  robustness against strategic players, e.g., routing

- *Theory of Networks with incentives*

  information, graph-structure, dynamics…

Find Nash Equilibria: Prisoner's Dilemma

Prisoner 2 / Prisoner 1	Silent	Confess
Silent	1/2, 1/2	10, 0
Confess	0, 10	5, 5

* Numbers denote years serving in prison: smaller is better!

*Definition: A **Nash Equilibrium*** is a pair of strategies (deterministic or randomized) such that the strategy of the row player is a *Best Response* to the strategy of the column player and vice versa.