Relationship Between Visual Cortical Response Powerlaw and Perceptual Threshold

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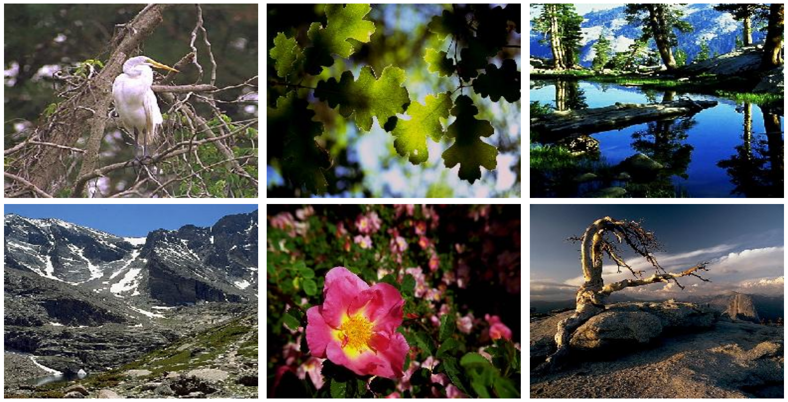
Based on Sarma and Choe (2006) and Lee and Choe (2003)

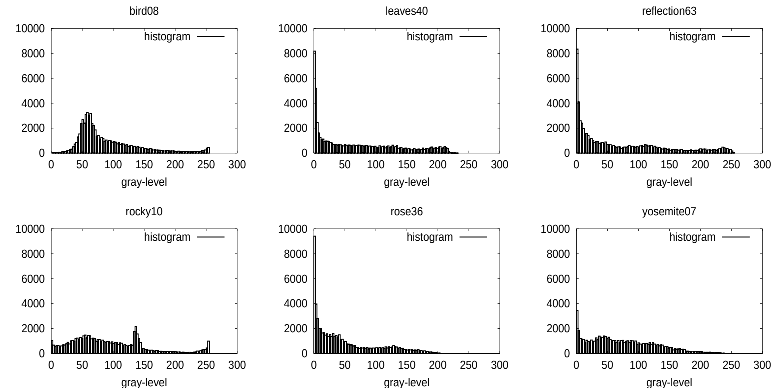
What Is Common in These Images?



What Is Common in These Images?

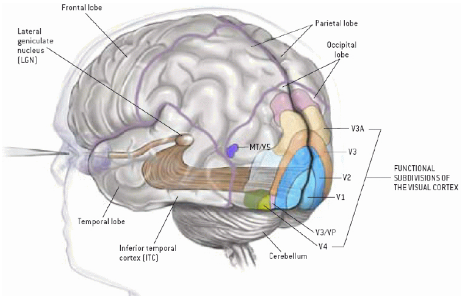
- In color, natural image, from the Kodak data set, ...
  - What about the brightness intensity histogram?

Brightness Intensity Histogram



- They are very different!
- What is similar then?

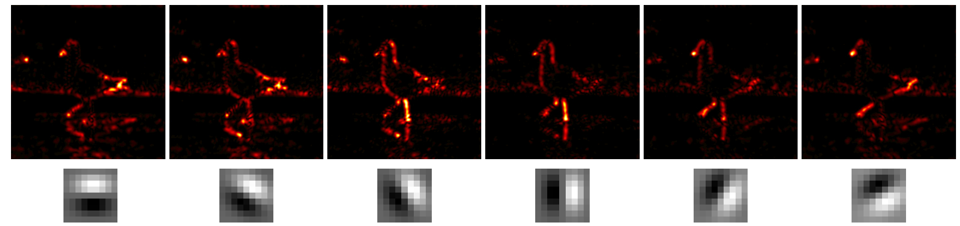
The Visual Cortical Response



(Logothetis 2002)

- Retina: center-surround filter
- LGN (thalamus): center-surround filter
- Visual cortex: oriented Gabor filter

Simulating Visual Cortical Response: Convolution with Oriented Gabor Filters

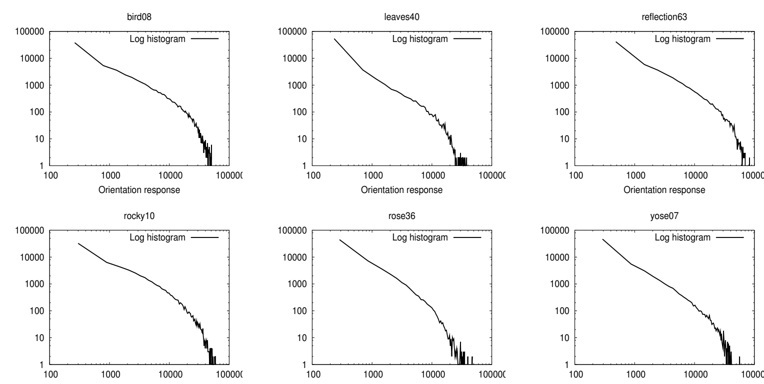


- Oriented Gabor filters simulate visual cortical receptive fields.

Visual Cortical Response (Simulated)

- This is (sort of) how the visual cortex responds to these images (Gabor filtering [next slide]).
  - Oriented edges are most prominently detected.
  - Would the response histogram vary as much as the brightness intensity histogram?

Visual Cortical Response Histogram



- The response (called orientation energy E) distributions are similar across the board!
- **Power law** property is observed (this is already a well-known result; Field 1987): $f(x) = 1/x^a \ \ (a > 0)$.

YAPL:

Yet Another Power Law!

- Power law seems to be ubiquitous in nature and in human-made artifacts:
  - 957,000 documents returned by Google Scholar!
  - Power law phenomena range from www topology, financial market fluctuation, to word frequency and much more (see e.g., Clauset et al. 2009).
- However, it is not often asked:
  - What use is it?
  - What fundamental mechanisms underlie such phenomena?

Overview

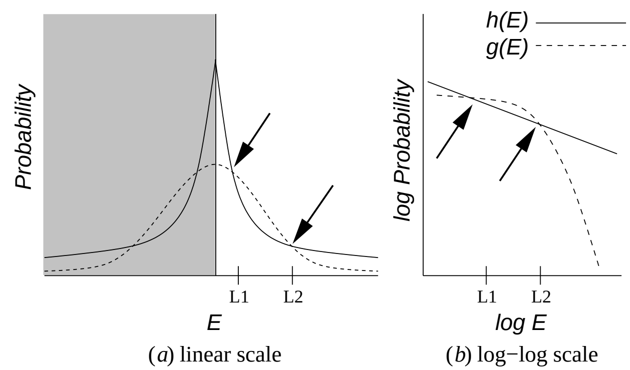
1. Power law + Gaussian baseline = Human perceptual threshold.
2. Why the Gaussian baseline?
3. Deeper questions:
   (a) Neural implementation
   (b) Mathematical/statistical implications

Part I:

Power Law + Gaussian Baseline

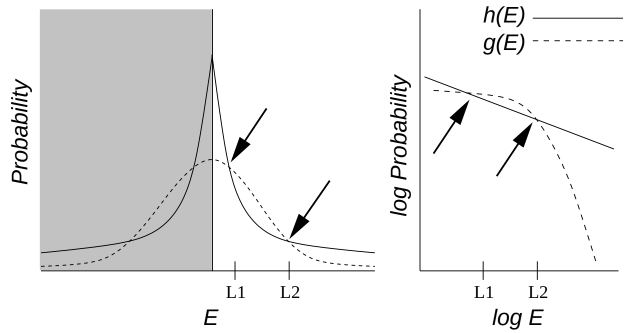
= Human Perceptual Threshold

What to Make of the Power Law?



- **Insight:** Comparing the power law distribution with a normal distribution with the **same variance** can be useful. .
  - **Assumption**: normal distribution can be a suitable baseline.
- The point $L2$ where $h(E)$ becomes greater than $g(E)$ may be important, i.e., orientation energy is **suspiciously high**.

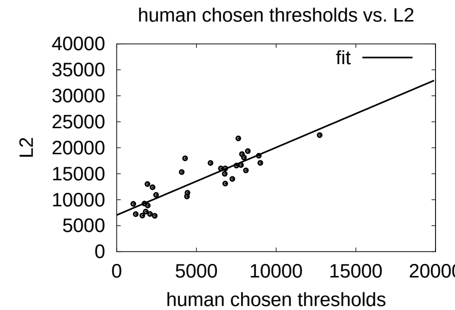
Power Law vs. Human Perceptual Threshold



(a) linear scale (b) log−log scale

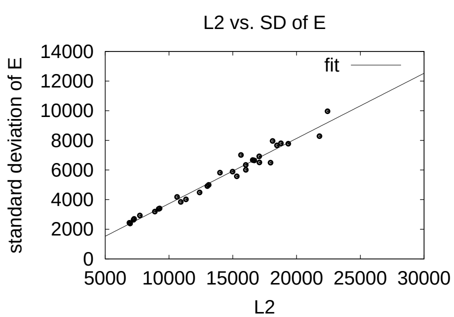
- Can there be a relationship between the threshold of E above which humans see it as **salient** and the point $L2$?
  - **Experiment**: Human participant selected threshold of E so that (1) contours are preserved as much as possible and (2) noise reduced as much as possible.

Checking the Hunch: Comparison to Human Perceptual Threshold



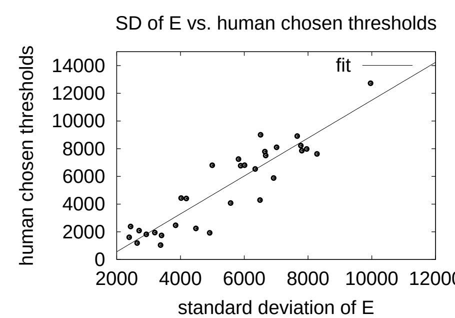
- There is a clear **linear relationship** ($n = 31$) between
  - Human-selected threshold and
  - L2, the intersection of $h(E)$ and $g(E)$.

Further Discoveries: $L2$ and Response Std. Dev.



- Further, the raw standard deviation σ of the response distribution is **linearly related** to $L2$.
  - Question: Is there an analytical solution to $a\frac{1}{x^b} = c \times \exp(-\frac{x^2}{d})$, where the constants a, b, c, and d depend on σ? (more on this later)

Using σ to Estimate Optimal E Threshold



- Relating σ back to the human-chosen E threshold gives again a **linear relation**:

$$T_\sigma = 1.37\sigma - 2176.59.$$

- Thus, instead of calculating the histogram, etc., we can simply calculate the raw standard deviation σ to estimate the appropriate E threshold.

Three Quantities

An unexpected correlation found among:

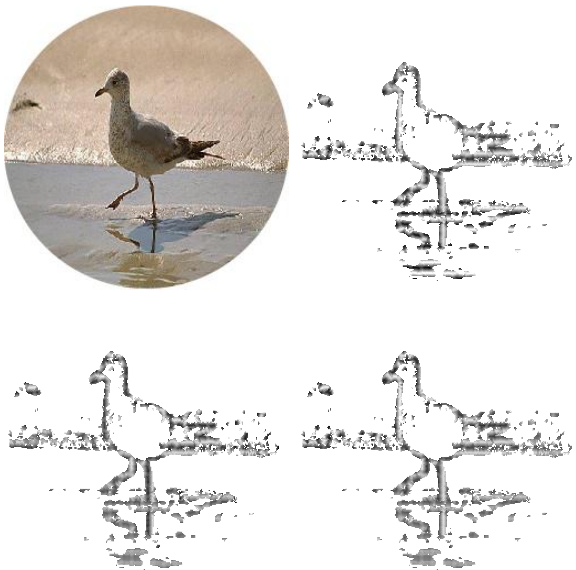
- Human-selected threshold.
- $L2$, point of intersection of response power law and Gaussian baseline.
- σ, standard deviation of response power law.

Application: Thresholding Cortical Response E

(a) Original Image (b) Thresholded Edges (c) Magnified (b)

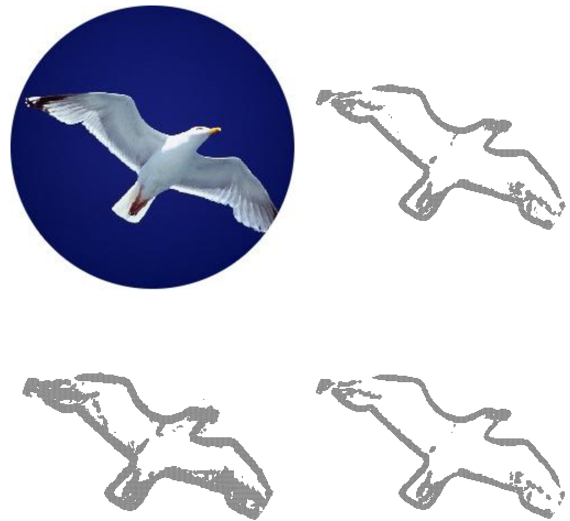
- Using T_σ as a threshold gives good results, comparable to humans' preference.

Thresholding Cortical Response E



- Original, human-selected, 85-percentile, and T_σ.

Thresholding E: Limitations of Fixed Percentile



- Original, human-selected, 85-percentile, and T_σ.

Thresholding E: Limitations of Global Thresholding

- Original, human-selected, 85-percentile, T_σ, and T_σ local.
- Estimating T_σ at a local scale solves the problem.

Part I: Summary

- Visual cortical response exhibits a power law.
- Comparing the power law to a baseline normal distribution results in a quantity ($L2$) that is linearly correlated with human perceptual threshold.
- $L2$ is in turn linearly correlated with the standard deviation of the power law.
- Straight-forward application possible (thresholding, salient edge detection):
  - Simple calculation of response variance is enough!

Part II: Why the Gaussian Baseline?

Why the Gaussian Baseline?

- The results are promising, but why?
- Why is the normal (Gaussian) distribution a reasonable choice as a baseline?
  - Central limit theorem?
  - People commonly use it?

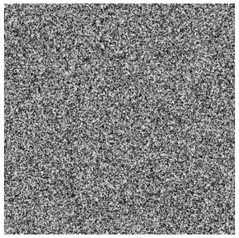
Power Law, Gaussian Dist., vs. Suspicious Coincidence

- What is the relationship between **salience** defined as **super-Gaussian** and the conventional definition of **suspiciousness** (Barlow 1994, 1989)?

$$P(A, B) > P(A)P(B),$$

where A and B are pixels in an image.

White-Noise Analysis

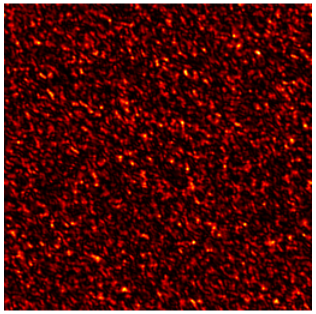


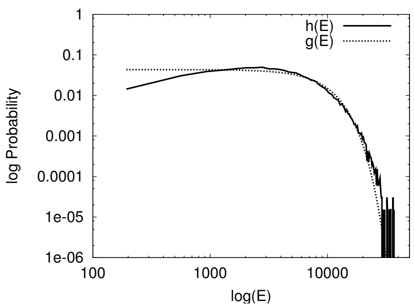
- In white-noise images, each pixel is independent, so, given any pixel pair (A, B):

$$P(A, B) = P(A)P(B).$$

- Would we get a power law response?
  - If the Gaussian baseline assumption was correct, since there is no salient edge, the response distribution should be Gaussian.

Visual Response to White Noise Images



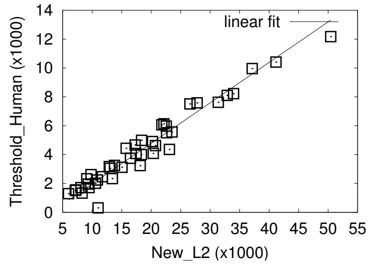


- The orientation energy distribution is very close to a Gaussian, especially near the high E values.
- Thus, the T_σ thresholding will not produce a meaningful threshold.

Use of White Noise Response as a Baseline

- Can we use the white-noise response as a baseline for thresholding E?: Yes!
- Generate white noise response, and scale it by σ_h/σ_r where σ_h and σ_r are the STD in the natural image response and the white noise response.
- Recalculate the response distribution (if necessary).

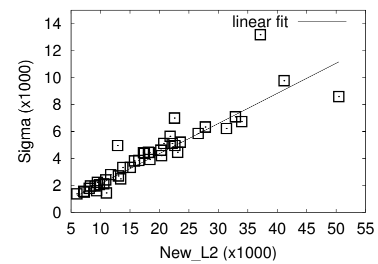
New Baseline for Salience vs. Humans



New L_2 vs. Human Chosen Threshold ($r = 0.98$)*

- Strong linearity is found between the new L_2 and the human selected threshold.
  - * This is much tighter than the Gaussian baseline ($r = 0.91$)!

New Baseline for Salience vs. σ



New L_2 vs. σ ($r = 0.91$)

- The same linearity between L_2 and the σ is maintained.

Part II: Summary

- Gaussian baseline corresponds to response distribution to white noise images.
- In white noise images, each pixel is independent from the others.
- This relates to the idea of suspicious coincidence by Barlow (1994)
- Threshold derived using the white-noise response distribution is even more accurate than earlier results.

Part III: Deeper Questions

Neural Implementation

- The local (or even global) threshold calculation can be easily implemented in a neural circuit:

$$\sigma^2 = \sum_{i,j} w_{ij} g(V_{ij}),$$

where w_{ij} are connection weights serving as normalization constants, $g(x) = x^2$, and V_{ij} is the V1 response at location i, j.

- The resulting value can be passed through another activation function $f(x) = \sqrt{x}$.

$$f(\sigma^2) = \sqrt{\sigma^2} = \sigma$$

- These are all plausible functions that can be implemented in a biological neural network.

Mathematical/Statistical Implications

Is there an analytical solution to $a\frac{1}{x^b} = c \times \exp(-\frac{x^2}{d})$?

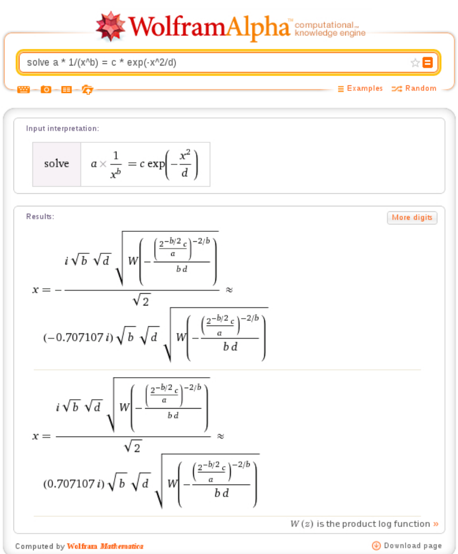
- This leads to another obscure yet surprisingly ubiquitous function called the Lambert W function $W(x)$ which is defined as the inverse of the following function:

$$x = W \exp(W)$$

- The Lambert W function is popping up everywhere: delay differential equations (with applications in population dynamics, economics, control theory), projectile trajectory calculation, voltage/current/resistance in a diode, etc. (see Hayes 2005 for a review)–A *déjà vu?*

- **Speculation**: Power law, Gaussian, and Lambert W function are deeply related.

Power Law, Gaussian, and Lambert W function



Basically, $x = \pm ip\sqrt{W(-q)}$

- How I found out: Wolfram Alpha (Mathematica, prior to that).

Wrap Up

Related Work

- Malik et al. (Malik et al. 1999) used peak values of orientation energy to define boundaries of regions of coherent brightness and texture.
- The non-Gaussian nature of orientation energy (or wavelet response) histograms has also been recognized and utilized, especially in denoising and compression (Simoncelli and Adelson 1996).
- Other kinds of histograms, e.g., spectral histogram by Liu and Wang (2002), or spatial frequency distributions (Field 1987), may be amenable to a similar analysis.

Conclusions

- Visual cortical response shows a power law.
- Power law compared to Gaussian baseline gives accurate predictor for human perceptual threshold.
- Standard deviation of the response is a simple yet powerful approximation.
- Gaussian baseline found to be related to suspicious coincidence.
- Power law, Gaussian baseline, and Lambert W function intricately interrelated.
- **Lesson:** Power law is there for a reason, and it can greatly simplify things downstream.

Acknowledgments

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