CSCE 644 Homework 1

Fall 2012

Octave tutorial and exercise (no need to submit the results from this page)

Install and use GNU Octave for this assignment (`http://octave.org`). Octave is a Matlab clone that runs on Linux, Windows (Cygwin), MacOS, etc. Here are some octave plotting and programming tips:

```
# comment starts with '#'
# ';' at the end of each statement suppresses default output

#################### Plotting
x = (0:0.1:10);
plot(x,sin(x));
plot(x,sin(x),"x");         # mark points
plot(x,sin(x),"x");         # mark points and draw line
plot(x,sin(x),"2-x");   # mark points and draw line using color 2
plot(x,sin(x),"2-x",x,cos(x),"1");   # multiple plots

#################### Defining series of numbers
1:5                       # range of values
1:0.1:5                   # range of values at interval of 0.1
size((1:0.1:5))           # size of vector/matrix (row x column)
[r,c] = size((1:0.1:5))  # redirect func return values into variables

#################### For loop
dat = zeros(10,1);
for i=1:10                # simple for loop
        dat(i) = i*i;
end
dat                       # show content of a variable

#################### If statement
a = 10;
if (a>5)
        b=a*a;
end

#################### Function definition
function [ret1,ret2] = func_name(arg1, arg2, arg3)
  # some stuff
  # whatever is assigned to ret1 or ret2 throughout the code will be
  # returned
  ret1 = arg1;
  ret2 = arg2*arg3;
end

func_name(10,20)               # call function
[r1, r2] = runc_name(10,20,30) # call function and retrieve all return values

#################### Vector and matrix operations
a = [1, 2, 3]; b = [1, 10, 100];  # define row vectors
a, b                      # show content (',' to show multiple items)
a', b'                    # transpose
a * b'                    # dot product
a' * b                    # outer product
a(1:2)                    # take sub vector
```

Problem 1 (Program: 25 pts): To solve the following ordinary differential equation (for an arbitrary function $f(x)$ with the initial condition $f(0)$), implement a numerical integration algorithm using the finite difference method based on Taylor series expansion (called forward Euler method). See slide04.pdf (page 15) and diffeq.pdf (page 6).

$$\frac{dV}{dt} = f(V), \text{given } f(0)$$

A quick summary is as follows (Δt is the step size, typically a small value like 0.01):

$$V(t + \Delta t) = V(t) + V'(t)\Delta t = V(t) + f(V)\Delta t$$

So, the actual calculation, starting from $V(0)$, goes like this:

$$\begin{aligned} V(0) &= \text{given} && (1)\\ V(1) &= V(0) + f(V(0))\Delta t && (2)\\ V(2) &= V(1) + f(V(1))\Delta t && (3)\\ &\ldots && (4) \end{aligned}$$

1. Numerically solve $\frac{dV}{dt} = -V$ and compare with the analytic solution $V(t) = \exp(-t) + c$ (note $V(0) = \exp(0) + c = 1 + c$). Try $\Delta t = 0.1, 0.05, 0.01$ for the interval $t = 0..10$.
   (1) Plot the numerical solution against the analytic solution for all step sizes.
   (2) Calculate the errors and plot them for all step sizes.
2. Repeat the above problem for $\frac{dV}{dt} = V$.
3. Discuss the role of Δt on the accuracy of the results.

Problem 2 (Program: 25 pts): Implement an integrate-and-fire neuron using the numerical integration technique from problem 1.

$$\begin{aligned} \tau\frac{dV}{dt} &= (E_{\text{rest}} - V) + RI(t)\\ V &= E_{\text{rest}}, \quad \text{when } V \geq V_{\text{thresh}} \end{aligned}$$

1. Simulate and plot the following:
   (1) Simulate with these parameters: $R = 10.0, C = 1.0, \tau = RC, V_{\text{thresh}} = -50, E_{\text{rest}} = -65$ for $t = 0..150$ with an integration time step of ($\Delta t = 0.05$). The input I should be a constant input of 2.0 ($I(t) = 2.0$). Experiment with various initial values $V(0)$.
   (2) **Written:** Calculate by hand the steady state V_∞ using the parameters in (1). Show all your work.
   (3) Set $R = 20.0$ (all other conditions same as (1)). Predict the behavior, explain why you think so, and observe the behavior. Does the firing rate increase or decrease?

(4) Set $R = 5.0$ (all other conditions same as (1)). Predict the behavior, explain why you think so, and observe the behavior. Does the firing rate increase or decrease?

(5) Set $C = 2.0$ (all other conditions same as (1)). Predict the behavior, explain why you think so, and observe the behavior. Does the firing rate increase or decrease?

(6) Set $C = 0.5$ (all other conditions same as (1)). Predict the behavior, explain why you think so, and observe the behavior. Does the firing rate increase or decrease?

2. Use a step input $I(t) = 2.0,\ \text{ where }\ 0 \leq t \leq 50$ and repeat Problem 2.1-(1) with all other parameters being equal. What happens to V when the input is turned off?

3. Use a sinusoidal input $I(t) = \sin(\frac{t}{2}) + 2.0$ and repeat Problem 2.1-(1) with all other parameters being equal.

Problem 3 (Program: 25 pts): Add spike-rate adaptation to the integrate-and-fire equation in problem 2 and implement it using Octave. Note: you have to change the integrate-and-fire equation (equation 5.13) and implement equation 5.14 as well.

1. Use parameters in Figure 5.6 (Dayan and Abbott) and run a similar experiment. Plot the voltage trace.

   (1) Parameters not explicitly stated in Figure 5.6 are: $C = 0.3, R_m = 90.0, r_m = 45.0, \Delta g_{\text{sra}} = 0.06$. Try a constant input $I(t) = 2.0$.

   (2) Set $E_K = -90$ (other conditions same as (1)). Predict what will happen, explain why, run the simulation, nd plot the results.

   (3) Set $E_K = -65$ (other conditions same as (1)). Predict what will happen, explain why, run the simulation, and plot the results.

   (4) Set $E_K = -70$, $\tau_g = 50$ (other conditions same as (1)). Predict what will happen, explain why, run the simulation, and plot the results.

   (5) Set $E_K = -70$, $\tau_g = 100$ (other conditions same as (1)). Predict what will happen, explain why, run the simulation, and plot the results.

2. Repeat experiment in Figure 5.6A by gradually increasing the input current $I(t) = 0.2, 0.4, 0.8, ...$ and measuring the firing rate $r_{\text{isi}} = \frac{1}{t_{\text{isi}}}$. You must read the figure caption carefully to understand how the data for the black discs and white discs were calculated from the spike train. Use the same parameters as Problem 3.1-(1).

Problem 4 (Program: 25 pts): Implement coupled integrate-and-fire neurons as shown in Figure 5.20. You will have to use equation 5.43 and equation 5.35 (turn this into ODE form: see Ermentrout (2002), chapter 3 `http://courses.cs.tamu.edu/choe/12fall/644/docs/ermentrout.pdf`).

1. Repeat the experiment in Figure 5.20 and show the plots.

2. Does changing the initial voltage lead to synchronization or desynchronization to break?

3. Does changing τ_s lead to synchronization or desynchronization to break? Try halving or doubling the time constant.

Submission: Submit your results as a single PDF file to `http://elearning.tamu.edu`.