Complex Dynamics Is Abolished in Delayed Recurrent Systems with Distributed Feedback Times

by Thiel et al. (2003)

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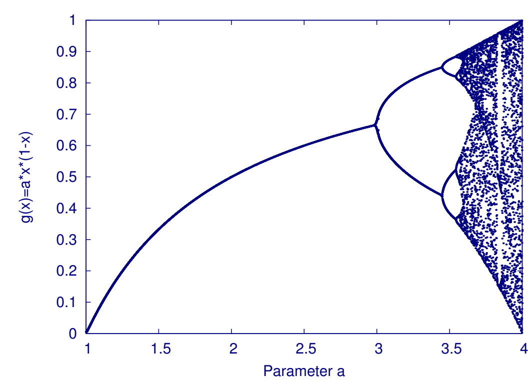
Introduction

- Feedback systems with a single delay time: known to exhibit various dynamical behaviors including complex oscillations and chaos.
- With broad distribution of delays, yields a larger set of parameter values that results in fixed point behavior or simple osciallatory behavior.

Background

- In time-lagged recurrent feedback systems, feedback gain and delay may serve as a bifurcation parameter whose increase yields a sequence of bifurcations leading from fixed point behavior to periodic orbits, and finally chaos.
- In most studies, recurrent signals are assumed to come from a singular instant in the past.
- However, in biological systems, there may be a wider range of delay in the feedback.

Basic Concept: Bifurcation



- Logistic map: $x = a \times x \times (1 - x)$.
- With random initial values x_0, calculate sequence of x's, and find the steady-state.
- Plot the steady states for different parameter values: Bifurcation diagram.

Approach

- Build up from existing models (with singular delay) showing complex dynamic.
  - Inhibitory feedback in hippocampus.
  - Mackey-Glass equation (regulation process of white blood cells).
  - Logistic growth of an ecological population under resource limits.
- Introduce distributed delay and observe resulting change in behavior.

Neural Feedback in the Hippocampus

- Mossy fibers (exc) $\longrightarrow$ CA3 pyramidal cell (exc) $\longrightarrow$ interneuronal basket cells (inh) $\longrightarrow$ CA3 pyramidal cell
- Delay in the feedback inhibitory loop can vary.
- Amount of feedback may also affect dynamic behavior: penicillin can modulate this (GABA antagonist).
- Model:

$$\frac{dv(t)}{dt} = -\Gamma v(t) + \Gamma e - \beta \frac{F_\xi(v(t))}{1 + F_\xi(v(t))^n},$$

$$F_\xi(v(t)) = f_0 \int_0^\infty [v(t-\tau) - \theta]_+ \xi(\tau) d\tau.$$

$v(t)$: membrane potential; Γ: inverse time const.; e: external input; β: feedback gain factor; $F(\cdot)$: basket cell firing rate; θ: threshold; $[x]_+ = xH(x)$; ξ: distribution function.

Distribution of Delay

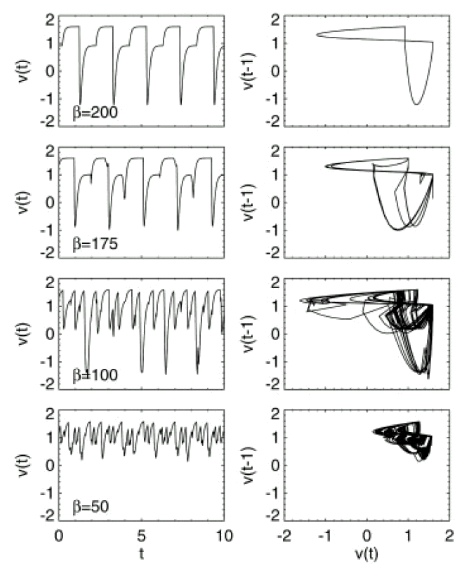
- Some assumptions:

$$\int_0^\infty \xi(\tau) d\tau = 1$$

- Simplest form:

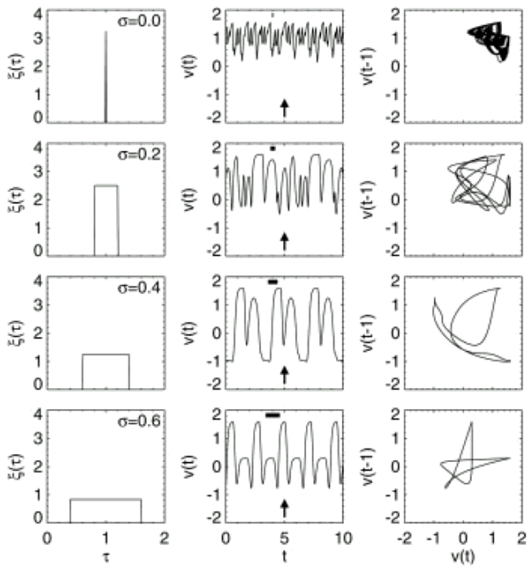
$$\xi(\tau) = \begin{cases} 1/2\sigma & \text{if } \tau_m - \sigma \le \tau \le \tau_m + \sigma \\ 0 & \text{otherwise} \end{cases}$$

Results: Hippocampus Model with Singular Delay



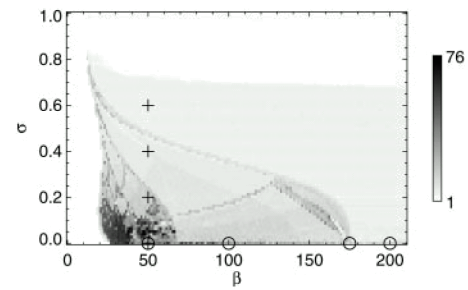
- Singular delay.
- Bifurcation parameter β increased from top to bottom.
- Complex dynamic results.

Results: Hippocampus Model with Distributed Delay



- Various delay distributions.
- Dynamic becomes simpler.

Results: Period Numbers against Parameters



- β: bifurcation parameter (inhibitory gain).
- σ: temporal dispersion.
- Higher σ gives wider region with low period number as β varies.

Mackey-Glass System: White Blood Cell Regulation

- Production of neutrophil granulocytes (a type of white blood cell).
- Production depends on present amount, but new production matures with a delay.
- Altered feedback gain or delay causes period-doubling bifurcations leading to chaos: Suspected cause of chronic granulocytic leukemia.

Mackay-Glass System

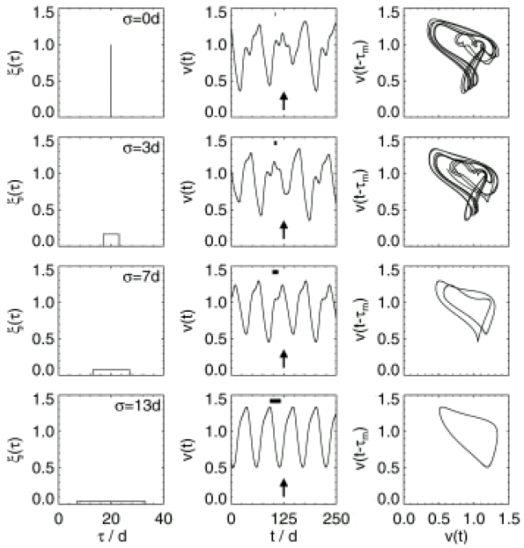
- Normalized concentration of cells:

$$\frac{dv(t)}{dt} = -\gamma v(t) + \beta \frac{V_\xi(v(t))}{1 + V_\xi(v(t))^n},$$

$$V_\xi(v(t)) = \int_0^\infty v(t-\tau)\xi(\tau)d\tau,$$

γ: cell loss rate; $beta$: gain in regulation; Same ξ as before.

Results: Mackay-Glass Model with Distributed Delay



- More distributed delay gives simpler dynamic.
- Bar indicates the integration interval.

Population Dynamics

- Typical model is the logistic equation (introduced earlier).
- Individual maturation time may differ, causeing a spread in the delay distribution.

Population Density

- Delay-differential equation for population density N:

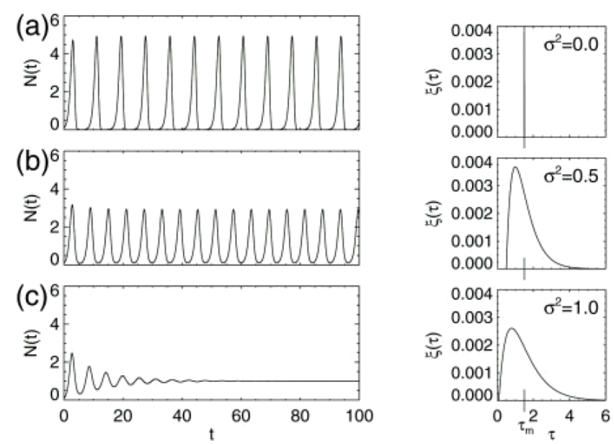
$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{1}{K}N_\xi(N(t))\right),$$

$$N_\xi(N(t)) = \int_0^\infty N(t-\tau)\xi(\tau)d\tau,$$

$$\xi(\tau) = \begin{cases} 0 & \text{if } 0 \leq \tau \leq \tau_{\min} \\ (\tau - \tau_{\min})\exp(-(\tau-\tau_{\min})/\theta)/\theta^2 & \text{if } \tau > \tau_{\min} \end{cases}$$

Mean delay: $\tau_m = \tau_{\min} + 2\theta$; Variance in delay: $\sigma^2 = 2\theta^2$.

Population Density for Different Delay Distributions



- High-amplitude oscillation is not good due to the risk of extinction during low-population periods.
- Distributed delay causes population to stabilize into a stable equilibrium.

Summary and Discussions

- Increasing the spread of delay distribution has a profound effect of dynamics in biological systems.
- Why does the dynamic become simpler in this case?: smoothing, reduced variance.
- Integration interval is shorter than period of oscillation, so there's no over-smoothing.
- The observed effects are robust.

References

Thiel, A., Schwegler, H., and Eurich, C. W. (2003). Complex dynamics is abolished in delayed recurrent systems with distributed feedback times. *Complexity*, 8:102–108.