

Classes of Network Connectivity and Dynamics

by Sporns and Tononi (2002)

CPSC 644

Presented by Yoonsuck Choe

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Connectivity

- Anatomical
 - Area-to-area connectivity known.
 - Some detailed connectivity known.
- Functional
 - Temporal correlation or deviation from statistical independence.
 - Segregation (functional units) and integration needed.
 - Covariance matrix (second-order effects) can express these aspects.
 - “Complexity” measure.

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Introduction

- Complex systems with patterns of temporal correlation: Arise from functional interactions within a structured network.
- The brain is an example.
- Activation and coactivation patterns (or functional connectivity) underlies perceptual and cognitive functions.
- Tools to investigate structure-function relation.

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Computational Methods for Analysis

- Graph theory: vertices and edges, connection matrix, graph space, path, path length, cluster index, etc.
- Functional dynamics: sigmoidal activation units (with noise), covariance matrix derived from connectivity matrix and noise
 - Entropy: overall statistical independence

$$H(X) = 0.5 \cdot \ln((2\pi e)^n |\mathbf{COV}(X)|).$$

- Integration: deviation for statistical independence

$$I(X) = \sum_i H(x_i) - H(X).$$

- Complexity: degree of segregation and integration (next page)

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Complexity

- Complexity: size n and connectivity k

$$C_N(X) = \sum (k/n)I(X) - \langle I(X_j^k) \rangle$$

$$= \sum_k \langle MI(X_j^k; X - X_j^k) \rangle$$

$$C(X) = H(X) - \sum_i H((x)_i | X - \mathbf{x}_i)$$

$$= MI(\mathbf{x}_i; X - \mathbf{x}_i) - I(X)$$

$$= (n - 1)I(X) - n \langle I(X - \mathbf{x}_i) \rangle$$

- $C(X)$ high if single elements are highly informative about the system while not being overly alike.

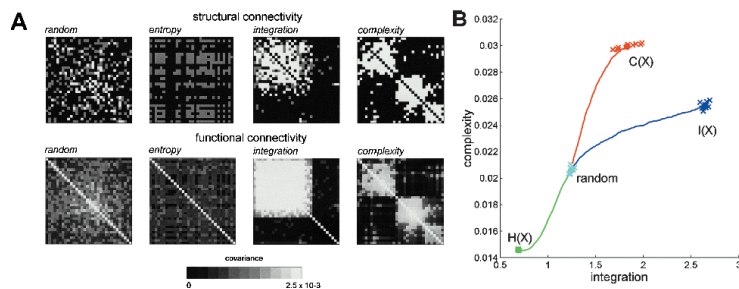
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Graph Selection

- Randomly generate and simulate graphs.
- Try maximizing different measures.
- Perform various measures on connectivity and dynamic properties.

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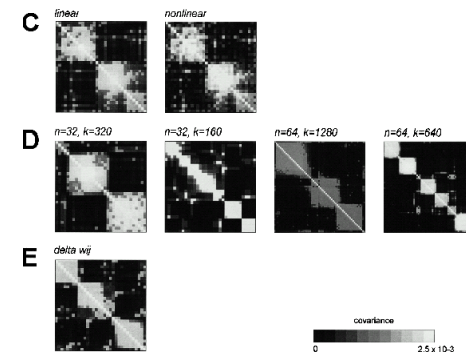
Networks Optimized for Different Criteria



- A(top): Connection matrix (static): unidirectional (gray), bidirectional (white)
- A(bottom): Covariance matrix (dynamic)
- B: $I(X)$ vs. $C(X)$ in networks optimized for three quantities.

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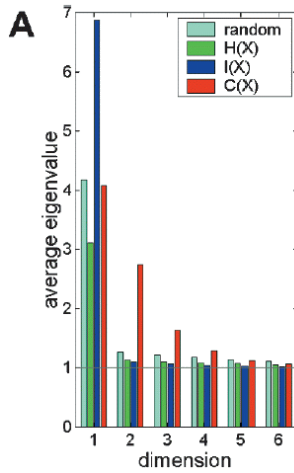
Covariance Matrix Analysis: Dynamics



- C: Analytic (linear) vs. simulation (nonlinear).
- D: Varying network size n and connectivity k .
- E: Redistributing synaptic weight rather than connections themselves.

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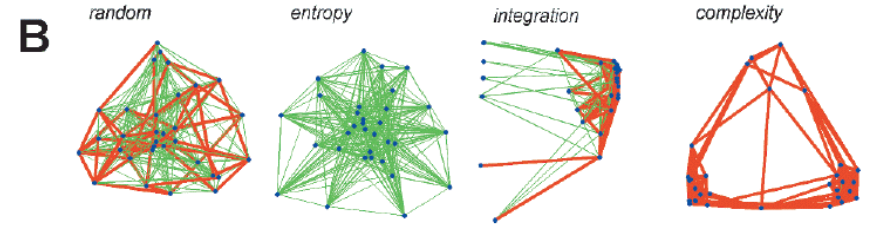
Eigenvalue Spectra for Correlation Matrix



- For complex networks, two to three terms dominate.

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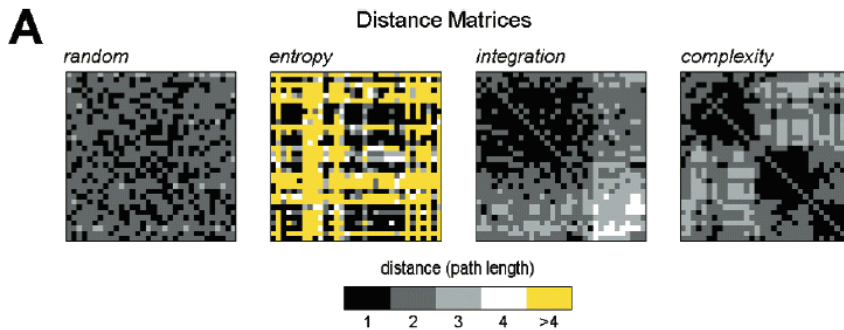
Multidimensional Scaling for Measuring Distance



- Bidirectional connections (red); unidirectional connections (green)
- Some have spatial ordering, while some don't.
- Complex networks show clustering and long-range interconnections: functionally segregated subsets emerge (reduced dimensionality).

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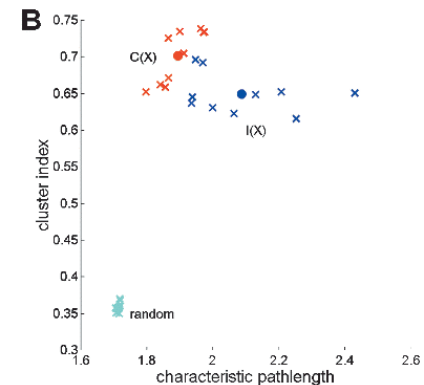
Distance Matrices: Structural Feature



- Path length in different networks.
- Entropy: long distance.
- Integration: short distance between core, long distance to outliers.
- Complexity: short distance within cluster, long distance across cluster.

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Characteristic Path Length: Structural Feature

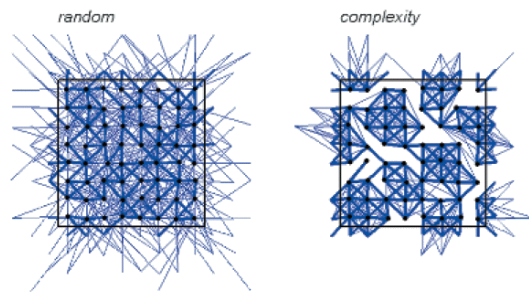


- CPL: global average of the distance matrix.
- Cluster index: number of connections among immediate neighbors / all possible connections among immediate neighbors.
- High complexity: High cluster index and low path length.

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Wire-Length Minimization

2-D Minimized Wiring



- Map graph onto 2D space: Vertices are given (x, y) locations.
- How to map to minimize wiring length?: Run optimization program.
- Complex networks give short total wire length compared to random networks.

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Organization, development, and function of complex brain networks

by Sporns et al. (2004)

CPSC 644, Spring 2007

Presented by Yoonsuck Choe

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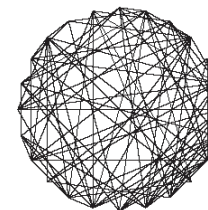
Conclusion

- It is important to understand functional connectivity based on anatomical connectivity.
- Complexity is an interesting measure of simultaneous segregation and integration: Not too ordered, not too random.
- Cortical networks: found to be complex.
- Role of complexity in evolution: wiring length reduction may be a side effect of maximizing complexity.

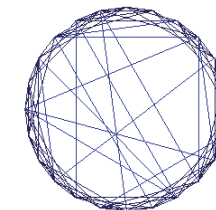
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Complex Networks: Small World vs. Scale-Free

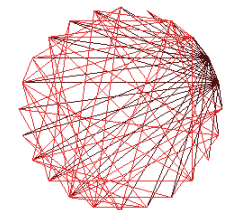
(a) $L=1.68$ (0.01)
 $C=0.35$ (0.03)



(b) $L=1.79$ (0.04)
 $C=0.52$ (0.04)



(c) $L=1.73$ (0.06)
 $C=0.52$ (0.05)



- Random
- Small-world: dense local connections, occasional long-distance connections. Often found in nature.
- Scale-free: degree distribution follows a power-law. Different degree values exist at every scale (scale-free). Often found in artifacts.
- L: characteristic path length; C: clustering coefficient

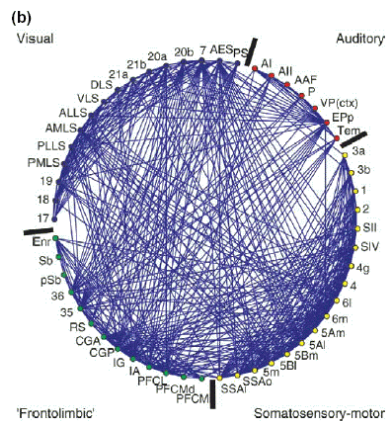
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Brain Connectivity

- Anatomical connectivity: actual physical connections
- Functional connectivity: activation/coactivation patterns.
- Effective connectivity: causal effects of one element over another.
Not model-free: requires causal model with parameters. Use perturbation to infer connectivity.

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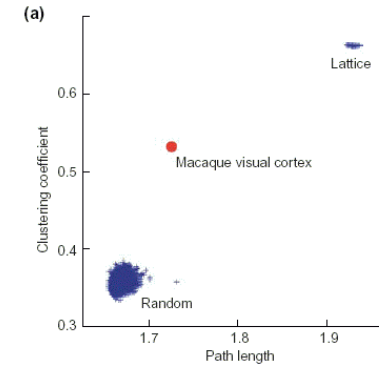
Cat Corticocortical Connectivity



- Closer nodes represent higher inter-linked areas.
- Clusters (separated by bars) correspond to visual, auditory, somatosensory-motor, and frontolimbic cortices.

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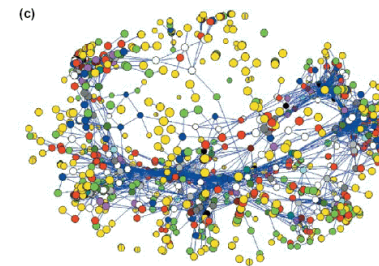
Characterization of Networks



- Area-wise connectivity in Macaque visual cortex: High cluster coefficient, low path length.

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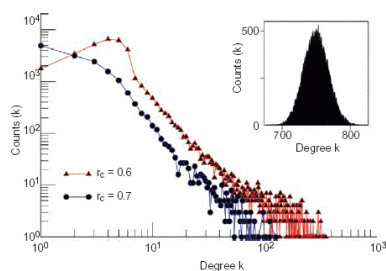
Functional Connectivity in Human fMRI Data



- Areas treated as connected when correlation is above a certain threshold.
- Degree: yellow=1, green=2, red=3, blue=4, other > 4 .

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Degree Distribution in Human fMRI Data



- Power-law is observed regardless of the correlation threshold: It seems to be scale-free?
- Random graphs show a unimodal degree distribution.

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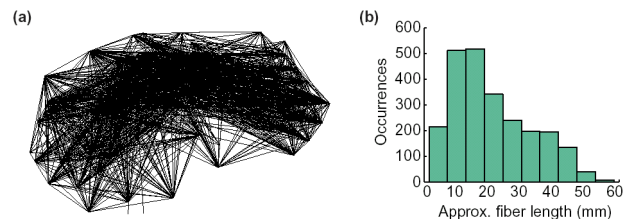
Questions for Future Research

Box 4. Questions for future research

- What are the best experimental approaches to generate large and comprehensive connective datasets for neural systems, especially for the human brain?
- What is the time scale for changes in functional and effective connectivity that underlie perceptual and cognitive processes?
- Are all cognitive processes carried out in distributed networks? Are some cognitive processes carried out in more restricted networks, whereas others recruit larger subsets?
- Does small-world connectivity reflect developmental and evolutionary processes designed to conserve or minimize physical wiring, or does it confer other unique advantages for information processing?
- What is the relationship between criticality, complexity and information transfer?
- Is the brain optimized for robustness towards lesions, or is such robustness the by-product of an efficient processing architecture?
- What is the role of hubs within scale-free functional brain networks?
- How can scale-free functional networks arise from the structural organization of cortical networks?

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Growth: Local Rules vs. Global Design



- Structure largely determined by growth and development.
- Growth mechanisms governed by constraints gives rise to different kinds of networks.
- Role of experience-dependent plasticity?
- Local growth rules: preferential attachment vs. distance-modulated spatial growth mechanism.
- Global network design

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Conclusion

- Systematic and global regularities in brain networks.
- Segregation and integration.
- Small-world attributes: Why? – for signal transformation?
- Relation to cognitive functions?

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References

Sporns, O., Chialvo, D. R., Kaiser, M., and Hilgetag, C. C. (2004). Organization, development and function of complex brain networks. *Trends in Cognitive Sciences*, 8:418–425.

Sporns, O., and Tononi, G. (2002). Classes of network connectivity and dynamics. *Complexity*, 7:28–38.