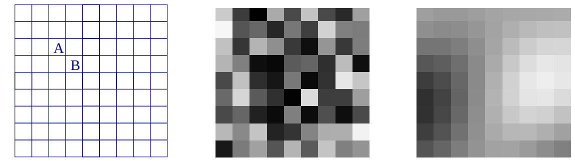
Unsupervised Learning

- No teacher signal (i.e. no feedback from the environment).
- The network must discover patterns, features, regularities, correlations, or categories in the input data and code them in the output.
- The units and connections must display some degree of **self-organization**.
- Unsupervised learning can be useful when there is **redundancy** in the input data.
- A data channel where the input data content is less than the channel capacity, there is redundancy.

Structure, Redundancy, Statistical Dependence



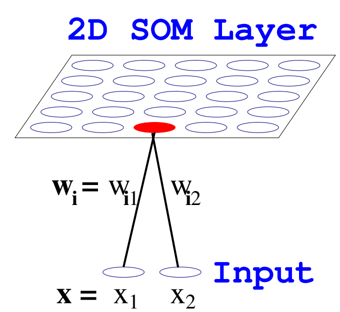
(a) (b) (c)

- Each pixel can be seen as a random variable.
- When pixel A can be predicted from looking at pixel B:
  - They are dependent.
  - They are redundant.
  - There is structure.
- Unsupervised learning needs such structure in the input.

What Can Unsupervised Learning Do?

- **Familiarity**: how similar is the current input to past inputs?
- **Principal Component Analysis**: find orthogonal basis vectors (or axes) against which to project high dimensional data.
- **Clustering**: n output class, each representing a distinct category. Each cluster of similar or nearby patterns will be classified as a single class.
- **Prototyping**: For a given input, the most similar output class (or **exemplar**) is determined.
- **Encoding**: application of clustering/prototyping.
- **Feature Mapping**: topographic mapping of input space onto output network configuration.

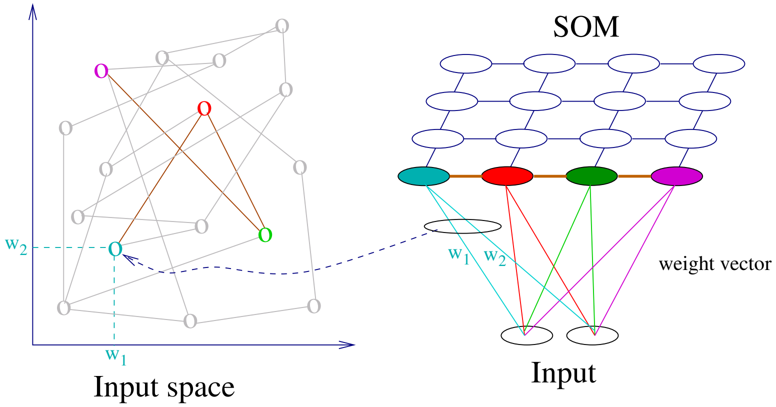
Self-Organizing Map (SOM)



Kohonen (1982)

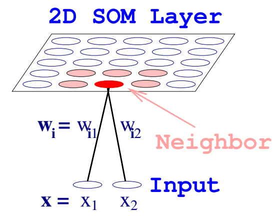
- 1-D or 2-D layout of units.
- One reference vector for each unit.
- Unsupervised learning (no target output).

SOM: Map vs. Input Space



- Each weight vector can be plotted in the input space.
- They can then be linked together based on their proximity in the map.

SOM Algorithm



1. Randomly initialize reference vectors $\mathbf{w_i}$
2. Randomly sample input vector $\mathbf{x}$
3. Find Best Matching Unit (BMU):

$$\mathbf{i}(\mathbf{x}) = \text{argmin}_j \parallel \mathbf{x} - \mathbf{w_j} \parallel$$

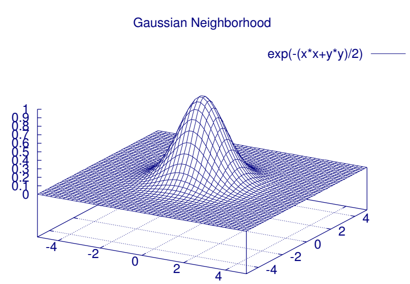
4. Update reference vectors:

$$\mathbf{w_j} \leftarrow \mathbf{w_j} + \alpha\Lambda(j, i(x))(\mathbf{x} - \mathbf{w_j})$$

α : learning rate
$\mathbf{\Lambda}(\mathbf{j}, \mathbf{i}(\mathbf{x}))$: neighborhood function of BMU.

5. Repeat steps 2 – 4.

Typical Neighborhood Functions

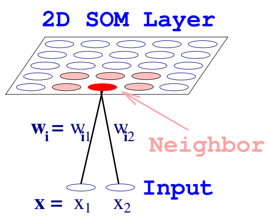


- Gaussian: $\Lambda(j, i(x)) = exp(-|\mathbf{r_j} - \mathbf{r_{i(x)}}|^2/2\sigma^2)$
- Flat: $\Lambda(j, i(x)) = 1$ if $|\mathbf{r_j} - \mathbf{r_{i(x)}}| \leq \sigma,$ and 0 otherwise.
- σ is called the **neighborhood radius.**

Training Tips

- Start with large neighborhood radius.
  Gradually decrease radius to a small value.
- Start with high learning rate α.
  Gradually decrease α to a small value.

Properties of SOM



- **Approximation of input space.**
  Maps continuous input space to discrete output space.
- **Topology preservation.**
  Nearby units represent nearby points in input space.
- **Density mapping.**
  More units represent input space that are more frequently sampled.

Performance Measures

- **Quantization Error**
  Average distance between each data vector and its BMU.

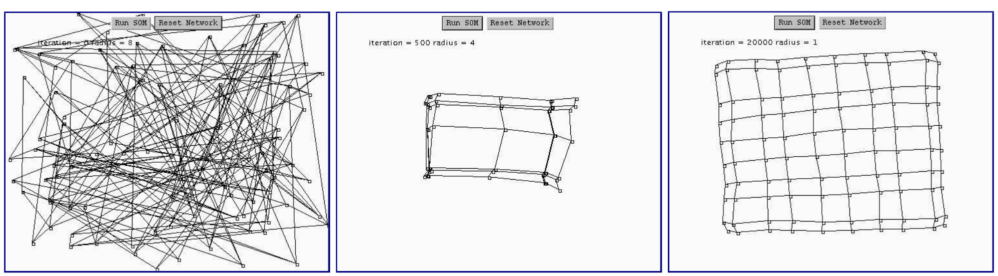
$$\epsilon_Q = \frac{1}{N} \sum_{j=1}^{N} \| \mathbf{x_j} - \mathbf{w_{i(x_j)}} \|$$

- **Topographic Error**
  The proportion of all data vectors for which first and second BMUs are not adjacent units.

$$\epsilon_T = \frac{1}{N} \sum_{j=1}^{N} u(\mathbf{x_j}),$$

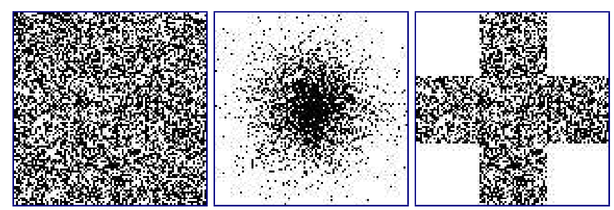
$u(\mathbf{x})$ = 1 if the 1st and 2nd BMUs are not adjacent
$u(\mathbf{x})$ = 0 otherwise.

Example: 2D Input / 2D Output



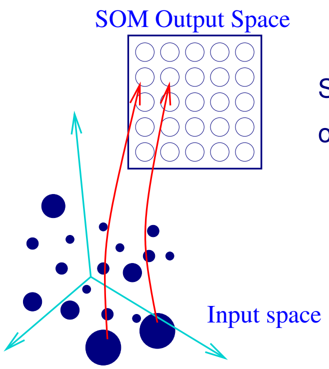
- Train with uniformly random 2D inputs.
  Each input is a point in Cartesian plane.
- Nodes: reference vectors (x and y coordinate).
- Edges: connect immediate neighbors on the map.

Different 2D Input Distributions



- What would the resulting SOM map look like?
- Why would it look like that?

High-Dimensional Inputs



SOM can be trained with inputs of arbitrary dimension.

- Dimensionality reduction: **N-D** to **2-D**.
- Extracts topological features.
- Used for visualization of data.

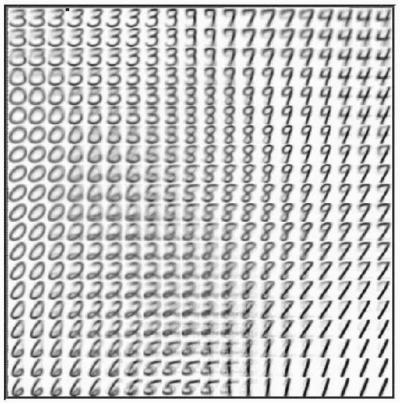
Applications

- **Data clustering and visualization.**
- **Optimization problems:**
  Traveling salesman problem.
- **Semantic maps:**
  Natural language processing.
- **Preprocessing for signal and image-processing.**
  1. Hand-written character recognition.
  2. Phonetic map for speech recognition.

Exercise

1. What happens when $\mathbf{N}_{\mathbf{i}(\mathbf{x})}$ and α was reduced quickly vs. slowly?
2. How would the map organize if different input distributions are given?
3. For a fixed number of input vectors from real-world data, a different visualization scheme is required. How would you use the number of input vectors that best match each unit to visualize the property of the map?

SOM Example: Handwritten Digit Recognition



- Preprocessing for feedforward networks (supervised learning).
- Better representation for training.
- Better generalization.

SOM Demo

Jochen Fröhlich's *Neural Networks with JAVA* page:

http://fbim.fh-regensburg.de/~saj39122/jfroehl/diplom/e-index.html

Check out the `Sample Applet` link.

SOM Demo: Traveling Salesman Problem

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 8.
- Stop when radius $<$ 0.001.
- Try 50 nodes, 20 input points.

Click on `[Parameters]` to bring up the config panel. After the parameters are set, click on `[Reset]` in the main applet, and then `[Start learning]`.

SOM Demo: Space Filling in 2D

Using Fröhlich's SOM applet:

- 1D SOM map ($1 \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 100.
- Stop when radius $<$ 0.001.
- Try 1000 nodes, and 1000 input points.

SOM Demo: Space Filling in 3D

Using Fröhlich's SOM applet:

- 2D SOM map ($n \times n$, where n is the number of nodes).
- 2D input space.
- Initial neighborhood radius of 10.
- Stop when radius $<$ 0.001.
- Try 30×30 nodes, and 500 input points. Limit the y range to 15.

Also try 50×50, 1000 input points, and 16 initial radius.

Other Unsupervised Learning Algorithms

- Hebbian learning: activity-dependent plasticity
- Principal component analysis
- Independent component analysis
- Competetive learning
- Vector quantization
- Various clustering algorithms

Course Wrap Up

- A thought: In ML, learning task is defined by humans. Can machines define their own leanrning tasks?
- Learning vs. understanding.
- Related courses: Pattern Recognition (689), Neural Networks (636), Cortical Networks (644), Information Retrieval, Sketch Recognition, Robotics, ...
- Conferences: ICML, NIPS, COLT, AAAI, IJCAI, GECCO, IJCNN.

Books

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