CPSC 420-500 Homework #2
Due 11/05/07 (Monday), 3pm

Handwritten or printed hardcopy must be submitted to the TA
Total: 130 pts

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October 29, 2007

1 First-Order Logic

Important: In this section, assume that w, x, y, z are variables; A, B, C, D are constants; and $f(\cdot), g(\cdot), h(\cdot)$ are functions; and $P(\cdot), Q(\cdot), R(\cdot)$ are predicates.

1.1 Standard Forms

To do automatic theorem proving in first-order logic, you need to go through three steps to convert your initial first-order logic expression into a standard form. These are:

1. Prenex normal form,
2. Conjunctive normal form, and
3. Skolemization.

Question 1 (12 pts): Convert to prenex normal form (4 points each):

1. $\forall x, \neg\left(\exists y, \neg P(x, y)\right)$
2. $\neg\forall x\left(P(x) \vee \neg(\exists y, \neg Q(x, y))\right)$
3. $\neg\forall x\left(\exists y, Q(x, y) \rightarrow \neg P(x)\right)$

Question 2 (20 pts): Skolemize the expressions (4 points each):

1. $\exists x P(x)$
2. $\forall x \exists y P(x, y)$
3. $\exists x, \exists y, \forall z P(x, y) \wedge Q(y, z)$
4. $\forall x, \exists y, \exists P(x, y) \wedge Q(y, z)$
5. $\forall x, \forall y, \exists z P(x, y) \wedge Q(y, z)$

Question 3 (9 pts): Convert the following into a standard form:

$$\forall x, [\neg P(x) \rightarrow \neg(\exists y, Q(x, y))]$$

1.2 Substitution and Unification

Question 1 (9 pts): Apply the following substitutions to the expressions (3 point each);

1. Apply $\{x/f(A)\}$ to $P(x, y) \vee Q(x)$.
2. Apply $\{x/A, y/f(z)\}$ to $P(x, y) \vee Q(x)$.
3. Apply $\{y/x\}$ to $P(x, y) \vee Q(x)$.

Question 2 (16 pts): For each of the following, (1) find the unifier, and (2) show the unified expression. For example, given $P(A)$ and $P(x)$, the unifier would be $\{x/A\}$, and the unified expression $P(A)$. If the pair of expressions is not unifiable, indicate so. (4 points each):

1. $P(x, f(B))$ and $P(A, f(y))$
2. $P(x, f(A))$ and $P(y, y)$
3. $P(x, f(y), y)$ and $P(A, f(g(w)), g(A))$
4. $P(A, f(y), y, A)$ and $P(x, f(g(x)), g(B), w)$

Question 3 (20 pts): Show that $R(A)$ is a logical consequence of the following. Use **resolution**. Turn into a normal form as necessary.

1. $\forall x, \forall y, (\neg P(x) \rightarrow (Q(x, y) \vee R(y)))$
2. $\exists x, \neg P(x)$
3. $\forall w, \forall z, (\neg Q(w, z) \vee R(w))$

2 Uncertainty and Probabilistic Reasoning

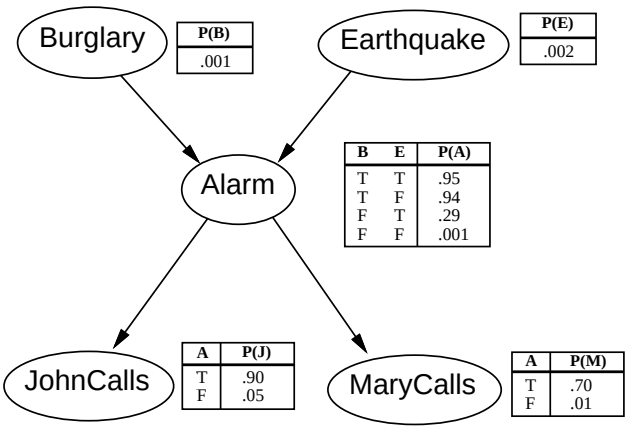

Figure 1: **Belief Network.** See problem 1.

Question 1 (10 pts): Given the Belief network as shown in figure 1, calculate the two joint probability values and answer the question. Note that in this section $P(\cdot)$ denotes the probability of the event. (5 points each):

1. $P(MaryCalls \wedge JohnCalls \wedge Alarm \wedge \neg Earthquake \wedge Burglary)$
2. $P(\neg MaryCalls \wedge \neg JohnCalls \wedge \neg Alarm \wedge \neg Earthquake \wedge Burglary)$

Question 2 (5 pts): Why do belief networks give a much more compact representation of the joint probability distribution, compared to a full joint probability table?

3 Learning

3.1 Decision Tree Learning

Consider the following set of examples where you are trying to make a decision whether to take a course or not.

Example#	Usefulness in life	Toughness	Fun	Decision (Take course?)
1	Stellar	Okay	Medium	Yes
2	Not bad	Very tough	High	Yes
3	Zero	Light	Medium	No
4	Not bad	Light	Low	No
5	Not bad	Light	High	Yes
6	Not bad	Very tough	Low	No
7	Zero	Okay	Medium	No
8	Stellar	Very tough	Low	Yes
9	Stellar	Okay	Medium	Yes
10	Zero	Light	High	No

Question 1 (12 pts): For each of the three attributes above, draw a decision tree rooted at that attribute with a **single depth**. See slide06, page 12, (a) and (b) which show some examples. (4 points each)

Question 2 (12 pts): Calculate the information gain for each of the three attributes. (4 points each)

Question 3 (5 pts): If you are supposed to choose from the three attributes for the first test in a decision tree construction, which one would you choose and why? **Do not** base your answer on your personal preference.