6
Mechanical Reliability

6.1 INTRODUCTION

The concept of constant failure rate is used to evaluate electronic component reliability. This concept is derived from the bathtub hazard rate belief that the failure rate remains constant during the useful life of electronic components. However, this is not normally the case when evaluating mechanical component reliability. It is an established fact in many cases that the mechanical components follow an increasing failure rate pattern that is generally represented by the exponential hazard function.

The field of mechanical reliability is relatively new as compared to the electronic reliability. The in-depth effort in this field appears to have been started since the early 1960s and may be credited to the U.S. space program. During those years, the failure of mechanical and electromechanical components was one of NASA's (National Aeronautics and Space Administration) prime concerns. For example, due to a mechanical failure caused by a busting high pressure tank, the SYNCOM I is believed to have been lost in space in 1963. Another typical example is the failure of Mariner III in 1964. It is also believed to have been lost due to a mechanical failure. There are several other instances where systems had mechanical failures. The researchers in the field felt that the design improvements were needed to improve reliability and longevity of mechanical and electromechanical components. Therefore, the space agency spent millions of dollars to test, replace, and redesign components such as filters, pressure switches, pressure gauges, mechanical valves, and actuators.

In 1965 NASA [80] initiated some major research projects entitled:

1. Reliability demonstration using overstress testing.
2. Reliability of structures and components subjected to random dynamic loading.
3. Designing specified reliability levels into mechanical components with time-dependent stress and strength distributions.

Ever since many publications on the subject have appeared. An up to date but selective literature on the subject is listed at the end of this chapter. In addition, a comprehensive literature survey up to 1974 on structural reliability is presented in reference 63.

At present, the most acceptable way of predicting mechanical component reliability may be by applying the interference theory. This approach is well documented in references 49 and 50. The topics presented in this chapter are as follows:

1. Statistical distributions in mechanical reliability.
2. Fundamentals of mechanical reliability.
3. Mechanical equipment basic failure modes.
4. Theory of mechanical failures.
5. Safety indices.
6. Load factors.
7. Design by reliability methodology.
8. Interference theory models.
9. Reliability optimization.

6.2 STATISTICAL DISTRIBUTIONS IN MECHANICAL RELIABILITY

This section presents failure distributions useful for representing the failure behavior of mechanical components. As compared to other distributions the extreme value distribution is the most likely candidate for the failure behavior of mechanical components. Its examples are presented in references 28 and 44.

The distributions discussed in the following sections are closely related to the reliability evaluation of mechanical components:

6.2.1 The Exponential Distribution

The probability density function is represented by the equation:

$$f=\lambda\exp(-\lambda t) \qquad t\geqslant 0 \qquad \lambda>0 \tag{6.1}$$

where t is time and λ is the constant failure rate.

The reliability function R and hazard rate z of the exponential distribution are:

$$R=\exp(-\lambda t) \tag{6.2}$$

and

$$z=\lambda \tag{6.3}$$

This distribution is widely used in reliability engineering. One of the reasons for its widespread use is its simplicity in performing reliability analysis. Its validity to represent a real-life failure data was first presented in reference 19.

6.2.2 The Extreme Value Distribution

The density function f of this distribution is defined by

$$f=\exp(t)\exp\{-\exp(t)\} \qquad -\infty<t<\infty \tag{6.4}$$

where t is time. The extreme value reliability and hazard rate functions, respectively, are

$$R=\exp\{-\exp(t)\} \tag{6.5}$$

and

$$z=\exp(t) \tag{6.6}$$

This distribution was first used to analyze flood data by Gumbel [28]. Therefore, it is sometimes known as the Gumbel's distribution. The failure behavior of many mechanical components may be represented by this distribution. From more fundamental considerations, this distribution can be developed by considering a corrosion process [66].

6.2.3 The Weibull Distribution

The Weibull density function is given by

$$f=\beta\lambda t^{\beta-1}e^{-\lambda t^{\beta}} \qquad \text{for} \quad \beta>0 \quad \lambda>0 \quad t\geqslant 0 \tag{6.7}$$

where λ = the scale parameter
β = the shape parameter
t = time

Weibull reliability and hazard functions are

$$R=e^{-\lambda t^{\beta}} \tag{6.8}$$

and

$$z=\beta\lambda t^{\beta-1} \tag{6.9}$$

This distribution was developed by Weibull [99], who described some of its applications. Ball bearing failures applications are given in reference [64].

The exponential ($\beta=1$) and Raleigh ($\beta=2$) are the special cases of this distribution.

6.2.4 The Mixed Weibull Distribution

This distribution was first presented by Kao [43]. He applied it to measure reliability of electron tubes. The probability density function is defined as

$$f=\frac{k\alpha_1}{\beta_1}\left(\frac{t}{\beta_1}\right)^{\alpha_1-1}\exp\left(-\frac{t}{\beta_1}\right)^{\alpha_1}$$

$$+\frac{(1-k)}{\beta_2}\alpha_2\left(\frac{t-\theta}{\beta_2}\right)^{\alpha_2-1}\exp\left\{-\left(\frac{t-\theta}{\beta_2}\right)^{\alpha_2-1}\right\} \tag{6.10}$$

for $\beta_1, \beta_2>0, 0<\alpha_1<1, \alpha_2>1, \theta>0, 0\leqslant k\leqslant 1$

The reliability expression for the above density function is

$$R=1-k\left[1-\exp\left(\frac{t}{\beta_1}\right)^{\alpha_1}\right]-(1-k)\left[1-\exp\left\{-\left(\frac{t-\theta}{\beta_2}\right)^{\alpha_2}\right\}\right] \tag{6.11}$$

6.2.5 The Gamma Distribution

The gamma probability density function is defined as

$$f=\frac{\lambda^{\beta}t^{\beta-1}\exp(-\lambda t)}{\Gamma(\beta)} \qquad \text{for} \quad \lambda>0 \quad \beta>0 \quad t\geqslant 0 \tag{6.12}$$

where $\Gamma(\beta)=\int_0^{\infty} t^{\beta-1}e^{-t}\,dt$

β = the shape parameter

λ = the scale parameter

The reliability and hazard rate expressions are

$$R=\left[\int_t^{\infty} x^{\beta-1}\exp(-\lambda x)\,dx\right]\lambda^{\beta}/\Gamma(\beta) \tag{6.13}$$

and

$$z=\frac{t^{\beta-1}\exp(-\lambda t)}{\int_t^{\infty} x^{\beta-1}\exp(-\lambda x)\,dx} \tag{6.14}$$

This distribution is an extended version of the exponential distribution. It was applied to the life test problems by Gupta and Groll [26].

The gamma distribution is related to the exponential and Chi-squared distributions. For its applications one should consult reference [57].

6.2.6 The Log-Normal Distribution

The probability density function is

$$f=\frac{1}{(t-\theta)\sqrt{2\pi}\,\sigma}\exp-\frac{\{\ln(t-\theta)-\mu\}^2}{2\sigma^2} \tag{6.15}$$

for $t>\theta>0$

where μ = is the mean

σ = the standard deviation

The reliability and hazard rate expressions for the above function are given by

$$R=\frac{1}{\sqrt{2\pi}\,\sigma}\int_t^{\infty}\frac{1}{(t-\theta)}e^{-\{\ln(t-\theta)-\mu\}^2/2\sigma^2}\,dt$$

$$\text{for} \quad t>\theta \tag{6.16}$$

and

$$z=\frac{[1/(t-\theta)]e^{-\{\ln(t-\theta)-\mu\}^2/2\sigma^2}}{\int_t^{\infty}[1/(t-\theta)]e^{-\{\ln(t-\theta)-\mu\}^2/2\sigma^2}\,dx} \tag{6.17}$$

Normally, the hazard rate of this distribution is an increasing function of time followed by a decreasing function. The hazard rate approaches zero for initial and infinite times. A representative example of this distribution is the failures due to fatigue cracks.

6.2.7 The Fatigue Life Distribution Models

These distribution models were presented by Birnbaum and Saunders [9], who proposed two-parameter distributions. The main applications of a family of distributions are to characterize failures due to fatigue.

The probability density function is defined as

$$f=\frac{(t^2-\lambda^2)}{2\sqrt{2\pi}\,\alpha^2\lambda t^2(t/\lambda)^{1/2}-(\lambda/t)^{1/2}}\exp\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\lambda}+\frac{\lambda}{t}-2\right)\right]$$

for $t>0 \quad \alpha, \lambda>0$

where α and λ are the shape and scale parameters, respectively. Readers requiring in depth material on these distributions should consult references 27, 44, and 91. Other hazard rate models are presented in references [67, 91].

6.3 FUNDAMENTALS OF MECHANICAL RELIABILITY

Like any other field of reliability engineering, mechanical reliability is also a joint responsibility of design and reliability engineers. A reliability engineer augments the designer's knowledge with design review procedures and statistical analysis; however, the designer still remains the key person to ensure component or system reliability.

The old concept of merely good design practices is not satisfactory to ensure reliability of a complex system. Reference 72 lists several reasons for the discipline of mechanical reliability.

1. *Lack of design experience.* Changes in technology are quite rapid and the mechanical designers no longer have the time to master the design especially when a complex equipment is designed for use in aerospace or military applications.
2. *Cost and time constraints.* Because of the cost and time involved, the designer cannot learn from past mistakes. In other words the cut-and-try approach cannot be used.
3. *Optimization of resources.* The workable design is no longer considered sufficient. The design must be optimized subject to constraints such as reliability, cost, weight, performance, and size.
4. *Stringent requirements and severe environments.* Because of large-scale investments in developing systems to be used under severe environments (military and space) the reliability problem becomes important.
5. *Influence from electronic reliability.* The vastly improved techniques for predicting electronic component reliability also stimulated similar developments in mechanical engineering.

6.4 MECHANICAL EQUIPMENT BASIC FAILURE MODES

Unlike electronic components, the mechanical components have numerous failure modes. Some of the basic failure modes pertaining to mechanical equipment are fatigue, leakage, wear, thermal shock, creep, impact, corrosion, erosion, lubrication failure, elastic deformation, surface fatigue, radiation damage, spalling, corrosion wear, delamination, and buckling. These basic failure modes are described in detail in reference 72. Some of these

failure modes may be associated with the following:

1. Leakage and distorted flow failure modes are associated with the fluid flow equipment.
2. The principal failure modes associated with a structural system are fracture, and excessive deflection.
3. The overheating and reduction of efficiency may be categorized as the thermodynamic system failure modes.
4. Bearing seizure and reduced accuracy of relative movement pertain to the kinemetic systems.
5. Incorrect material properties and incorrect component geometry are called the material conversion failure modes.

6.5 THEORY OF FAILURES ✓

When the strength of a material, a component, or a device is less than the stress imposed on it, the failure occurs. Stress and strength are defined as follows:

Stress. A stress (load) tends to produce a failure of a component, a device, or a material. The term "load" may be defined as mechanical load, environment, temperature, electrical current and so on.

Strength. Strength is defined as the ability of a component, a device, or a material to accomplish its required mission satisfactorily without a failure when subject to the external loading and environment.

Both stress and strength may be described by probability distributions. All types of stresses and strengths cannot, however, be represented by the existing distributions.

Because of the variation in material properties (e.g., production processes, geometric dimensions) the strength of nominally identical components subject to the same conditions may vary from component to component. The variability may be described by a distribution function. All the important variabilities (and their distributions) of a component must be considered and known (or assumed) when estimating the expected strength distribution function of a component. The methods to predict the expected strength distribution from the variability distributions are presented in references 72, 11, 15, 96, 59, 100, and 76.

It is always desirable to have a narrow spread of the strength distributions because a narrow distribution yields a higher reliability than its counterpart, which is widely spread out and is of the same mean value. Therefore, efforts should always be directed toward obtaining a narrow

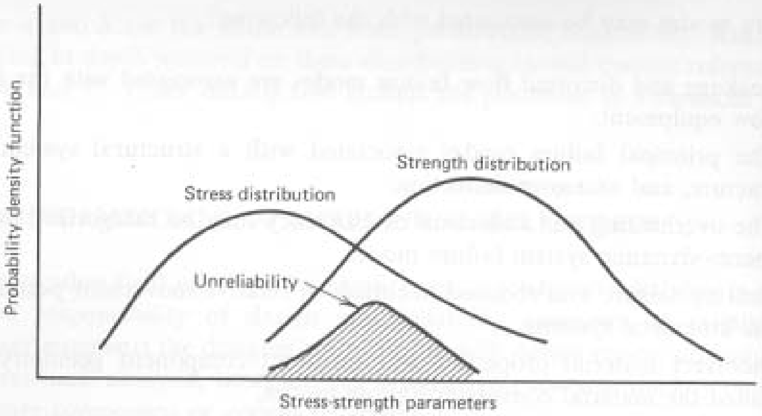

Figure 6.1 Interference theory of stress-strength distribution concept.

strength distribution; however, there should always be some degree of limitation to obtain such narrowness because the strength of a mechanical component or a material is generally reduced by fatigue, corrosion, wear, which are factors that increase the spread of the strength distribution. One should note that these factors take time to become effective. Therefore, it must be understood that the strength distribution is a function of time. Similarly, the stress distribution also changes under different conditions like use, maintenance, environment, and so on. The duty or the stress distribution for a component under controlled laboratory environments or conditions remains constant.

If the expected distributions of stress and strength can be estimated for a mechanical part, then by employing interference theory, the probability of failure of a mechanical part can be obtained. This concept is presented in detail in references 45 and 48–56.

The concept of interference is illustrated in Figure 6.1. The unreliability or the probability of failure is represented by the shaded area in Figure 6.1.

The interference theory is applicable only to those cases in which no significant changes occur in the item over the specified time interval. Furthermore, it is assumed that the failure is dependent on the instantaneous stress and not on the history of the stress.

As mentioned earlier, the stress and strength distributions may change with time. To illustrate this point Figure 6.2*a*, *b* display stress-strength distributions for two different times t_1 and t_2. For the sake of simplicity the stress distribution is assumed to be constant but the strength distribution varies with time. Furthermore, the stress-strength distributions need not be symmetrical and may be skewed or irregular. Once the stress-strength probability density functions are known, reliability can be computed by applying the interference theory.

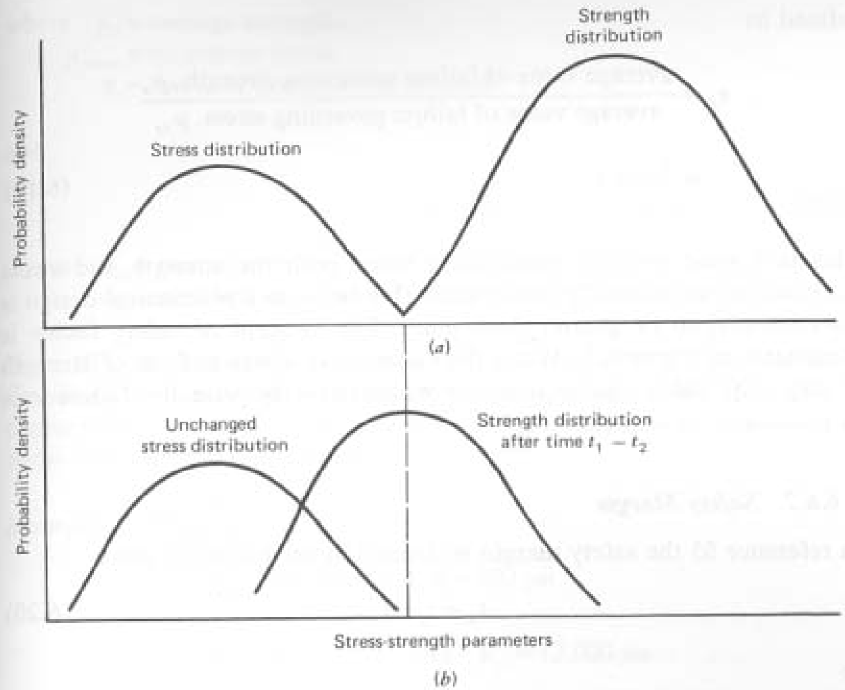

Figure 6.2 (*a*) Stress-strength distribution at time t_1; (*b*) stress-strength distribution at time t_2.

6.6 SAFETY INDICES

The safety factor approach is a conventional design technique. This method uses safety margins and safety factors that are simply arbitrary multipliers. In some cases, these factors provide satisfactory design, if they are established from the past experience. In the days of modern technology, however, the new design involves new applications and new materials and more consistent methods are needed. The mechanical component design based entirely upon safety factors, could be misleading, and may be costly due to overdesign or could end up in a catastrophic failure due to underdesign. It is emphasized that whenever a designer makes use of safety factors these must be based upon considerable experience on similar items.

6.6.1 *Safety Factor*

There are several different ways of defining a safety factor as outlined in reference 55. In reference 10, the theoretical definition of a safety factor, is defined as

$$s_f = \frac{\text{average value of failure governing strength, } \mu_s}{\text{average value of failure governing stress, } \mu_{ss}}$$

$$= \frac{\mu_s}{\mu_{ss}} \geqslant 1 \tag{6.19}$$

This is a good measure particularly when both the strength and stress distributions are normally distributed. This factor in a mechanical design is always equal to or greater than unity. The concept of safety factor is illustrated in Figure 6.3. When the variation of stress and/or of strength is large, the safety factor becomes meaningless because the failure rate is positive.

6.6.2 *Safety Margin*

In reference 55 the safety margin is defined in the following ways:

$$s_m = s_f - 1 \tag{6.20}$$

or

$$s_m = \frac{\mu_s - \mu_{max}}{\sigma_s} \tag{6.21}$$

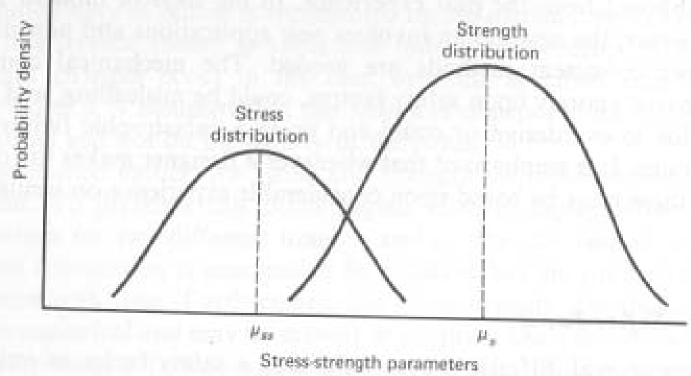

Figure 6.3 Safety factor for stress-strength distribution.

where μ_s = average strength
μ_{max} = maximum stress
σ_s = standard deviation of strength

and

$$\mu_{max} = \mu_{ss} + k\sigma_{ss} \tag{6.22}$$

where μ_{ss} = mean stress
σ_{ss} = standard deviation of stress

Normally, the value of k is between 3 and 6. It can be observed from discussion on safety margins that (a) it is a random variable just like its counterpart, the safety factor, and (b) it presents the idea of separation of stress and strength mean values.

Example 1. Suppose

$$\sigma_{ss} = 200 \text{ psi}, \; k = 4, \; \sigma_s = 900 \text{ psi}$$

$$\mu_s = 25{,}000 \text{ psi} \quad \text{and} \quad \mu_{ss} = 12{,}000 \text{ psi}$$

Find the safety margin for given data. By substituting the above information in (6.21), we get

$$s_m = \frac{25{,}000 - (12{,}000 + 4 \times 200)}{900}$$

$$= \frac{25{,}000 - 12{,}800}{900} = \frac{122}{9}$$

$$\simeq 13.6$$

6.7 LOAD FACTORS

In the last decade or so it has been realized that the loads as well as the capacities of structures are not necessarily deterministic but are probabilistic, because of the random variation in magnitude and the random occurrence of loads. In this section we discuss the determination of load factors in the structural design. In references 78 and 85 this subject is discussed in detail. In this section we mainly deal with the dead and live loads. Earlier analysis on the topic were initiated by the authors of references 78 and 85.

6.7.1 Deterministic Resistance with Normally Distributed Loads

For the deterministic resistance the design load, D_1, may be formulated as follows:

$$D_1 = L_d \mu_d + L_1 \mu_1 \tag{6.23}$$

where L_d = the dead load factor
L_1 = the live load factor
μ_d = nominal mean dead load
μ_1 = nominal mean live load

Suppose the live and dead loads are normally distributed random variables with mean values of μ_1 and μ_d, respectively. The design load then also follows the normal law. In the case of independent dead and live loads, the design load, D_1, may be described by (6.24).

$$D_1 = (\mu_d + \mu_1) + c\sqrt{\sigma_d^2 + \sigma_1^2} \tag{6.24}$$

where σ_d, σ_l are the standard deviations of the dead and live load and c is the reliability coefficient for the combined dead and live load. Also, the design load in terms of component loads may be written as

$$D_1 = m_d + m_1 = (\mu_d + c'\sigma_d) + (\mu_1 + c'\sigma_1) \tag{6.25}$$

where c' is the reliability coefficient for each load component, that is,

$$c' = \frac{c}{\sigma_1 + \sigma_d}\sqrt{\sigma_1^2 + \sigma_d^2}$$

m_d = magnitude of component dead load

m_1 = magnitude of component live load

By manipulating (6.23), (6.24), (6.25), we obtain the following load factor equations:

$$L_d = \frac{m_d}{\mu_d} = 1 + scV_d \tag{6.26}$$

$$L_1 = \frac{m_1}{\mu_1} = 1 + scV_1 \tag{6.27}$$

where $s = \dfrac{(\sigma_d^2 + \sigma_1^2)^{1/2}}{\sigma_d + \sigma_1}$

$$V_d = \frac{\mu_d}{\sigma_d} \qquad V_1 = \frac{\mu_1}{\sigma_1}$$

where V_d is the coefficient of variation of dead load and V_1 is the coefficient of variation of live load.

6.7.2 Normally Distributed Loads and Resistance

When the resistance follows a normal distribution, (6.26) and (6.27) are modified to the following form:

$$L_d = \frac{1 + scV_d}{1 - cV_R} \tag{6.28}$$

$$L_1 = \frac{1 + scV_1}{1 - cV_R} \tag{6.29}$$

where V_R is the resistance coefficient of variation. When V_R is equal to zero, the resistance follows the deterministic law. The value of c can be determined from (6.30) when loads and resistance follow the normal distribution:

$$c = \frac{\left[(V_R \mu_R)^2 + \sigma_1^2 + \sigma_d^2\right]^{1/2}}{V_R \mu_R + (\sigma_1^2 + \sigma_d^2)^{1/2}} \cdot c^* \tag{6.30}$$

where μ_R is the mean resistance and c^* is the reliability coefficient of the system.

By substituting (6.30) into (6.28) and (6.29), we can determine the load factors for any desired level of reliability. Therefore the value of the c^* can be obtained from the table of the error function. For example at the desired level of reliability, say $R = 0.9901$, $c^* = 2.33$. For a solved numerical example see reference 86.

6.8 "DESIGN BY RELIABILITY" METHODOLOGY

The "design by reliability" methodology is described in considerable detail in references 50 and 49. To design an equipment or a component by taking reliability into consideration, the following steps are needed:

1. Define the design problem in question.
2. List and identify all the associated design variables and parameters in the problem.
3. Perform failure modes, effect, and criticality analysis (FMECA).
4. Determine the failure governing stress and strength functions and distributions of a failure mode.
5. Use failure governing stress and strength distribution to evaluate each critical failure mode reliability.

6. Iterate the design until the assigned reliability goals are met.
7. Optimize design under specified constraints such as cost, weight, volume, reliability, maintainability, safety performance, and so on.
8. Repeat the above steps for each vital component or device of a system.
9. Calculate the system reliability by applying the classical reliability theory.
10. Iterate the design until the specified system reliability goal is fulfilled.

Step 4 is probed in depth in the following section:

6.8.1 Determination of Failure Governing Stress Distribution

The following steps are to be followed to determine the failure governing stress distribution:

1. List and identify all the important failure modes.
2. In the case of a fracture failure mode, if any, determine the most likely locations where the combination of stresses are likely to act which may result in component failure.
3. At each location calculate the nominal stress of components.
4. Evaluate maximum value of each component stress with the use of necessary stress modifying factors.
5. At each location combine all the stresses into the failure governing stress in accordance with particular failure mode being considered.
6. In the failure governing stress equation determine each nominal stress, modifying factor and parameter distribution.
7. Determine a failure governing stress distribution from the step 6 distributions.
8. Repeat steps 2–7 for each significant failure mode listed in step 1.

Readers who require more information should consult references 51 and 54.

6.8.2 Determination of the Failure Governing Strength Distribution

To determine failure governing strength distribution, the following steps are outlined:

1. Set up the failure governing strength procedure by taking the failure modes into consideration. This criterion should be based upon the one used to determine failure governing stress.
2. Evaluate the nominal strength.

3. Use appropriate strength factors to modify nominal strength. This is to convert the nominal strength obtained under the standardized and idealized test conditions.
4. Determine the nominal strength distribution, of each modifying factor and parameter associated with the failure governing strength equation.
5. Establish the failure governing strength distribution by utilizing the normal distributions of step 4.

For more detailed information regarding the determination of the failure governing strength distribution, the interested reader should consult references 49 and 50.

6.9 RELIABILITY DETERMINATION—CONSTANT STRESS-STRENGTH INTERFERENCE THEORY MODELS

This section deals with situations in which the stress and strength are represented by well-defined probability density functions. Furthermore, the stress-strength distributions are not time dependent.

When the probability density functions of both stress and strength are known, the component reliability may be determined analytically. Reliability is defined as the probability that the failure governing stress will not exceed the failure governing strength. In a mathematical equation it can be written as

$$R=P(s<S)=P(S>s) \tag{6.31}$$

where R = the reliability of a component or a device
P = the probability
S = the strength
s = the stress

Equation (6.31) can be rewritten in the following form:

$$R=\int_{-\infty}^{\infty} f_{st}(s)\left[\int_{s}^{\infty} f_{Sth}(S)\,dS\right] ds \tag{6.32}$$

where $f_{st}(s)$ = the probability density function of the stress, s
$f_{Sth}(S)$ = the probability density function of the strength S

Reliability for a single failure mode can also be computed from (6.33) on the basis that the stress will be less than the strength:

$$R=\int_{-\infty}^{\infty} f_{Sth}(S)\left[\int_{-\infty}^{S} f_{st}(s)\,ds\right] dS \tag{6.33}$$

The above equation may be used to obtain numerical solutions if the analytical solution is difficult to obtain. In addition, when the empirical data is sufficient but the stress or strength distribution cannot be identified, the graphical approach can be applied to obtain component reliability.

6.9.1 Reliability Calculation by Graphical Approach

This technique makes use of the Mellin transforms, which can be applied to any distribution. The Mellin transforms of the reliability equation (6.32) are defined as

$$M=\int_s^{\infty} f_{Sth}(S)\,dS$$
$$=1-F_{Sth}(s) \tag{6.34}$$

and

$$L=\int_0^s f_{st}(s)\,ds=F_{st}(s) \tag{6.35}$$

Equation 6.35 may be rewritten as

$$dL=f_{st}(s)\,ds \tag{6.36}$$

By substituting (6.36) and (6.34) into (6.32) we get

$$R=\int_0^1 M\,dL \tag{6.37}$$

Obviously, L takes values from 0 to 1. Therefore, if we plot (6.37), that is, M versus L, the area under the curve will represent the single failure mode component reliability. A typical plot of (6.37) is shown in Figure 6.4. Simpson's rule can be used to calculate area under the M versus L curve.

Example 2. Suppose the strength of a component follows the Rayleigh distribution with known scale parameter value of 15,000 psi. Similarly, the stress follows a Weibull distribution with the shape parameter equal to 3 and the scale parameter value of 12,000 psi.

Therefore, the stress and strength density functions become

$$f_{Sth}(S)=\frac{2}{15{,}000}\left(\frac{S}{15{,}000}\right)\exp\left[-\left(\frac{S}{15{,}000}\right)^2\right] \tag{6.38}$$

and

$$f_{st}(s)=\frac{3}{12{,}000}\left(\frac{s}{12{,}000}\right)^2\exp\left[-\left(\frac{s}{12{,}000}\right)^3\right] \tag{6.39}$$

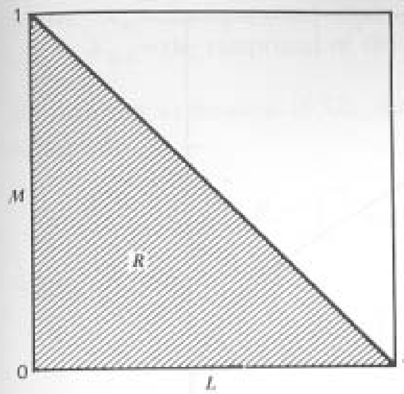

Figure 6.4 A hypothetical plot of L versus M, where the shaded area represents the component reliability, R.

By substituting (6.38) and (6.39) into (6.34) and (6.35), respectively, we get:

$$M=1-F_{Sth}(s)=\exp\left[-\left(\frac{s}{15{,}000}\right)^2\right] \tag{6.40}$$

$$L=F_{st}(s)=1-\exp\left[-\left(\frac{s}{12{,}000}\right)^3\right] \tag{6.41}$$

Table 6.1 presents tabulation for M and L for the various values of s. Figure 6.5 shows a plot of values for M and L from Table 6.1. Using

Table 6.1

s	M	L
0	1	0
2,000	0.98	0.005
4,000	0.93	0.04
6,000	0.85	0.12
8,000	0.75	0.26
10,000	0.64	0.44
12,000	0.53	0.63
14,000	0.42	0.8
16,000	0.32	0.91
18,000	0.24	0.97
20,000	0.17	0.99
22,000	0.12	0.997≃1

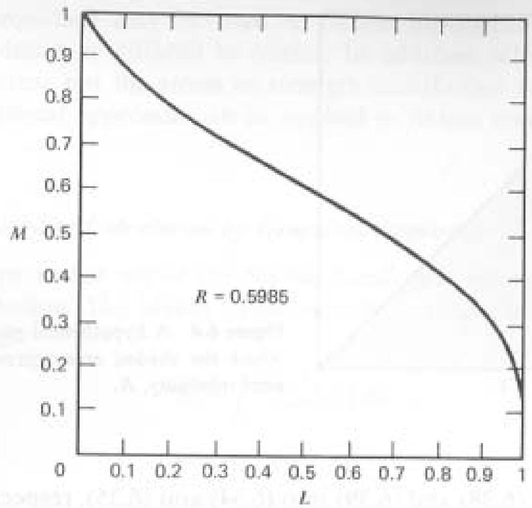

Figure 6.5 *M* versus *L* plot.

Simpson's rule, the component reliability R is estimated from Figure 6.5

$$R=\frac{0.25}{3}\{y_0+4y_1+2y_2+4y_3+y_4\}$$

$$R=\frac{0.25}{3}\{1+4\times0.75+2\times0.61+4\times0.46+0.12\}$$

$$R=0.5985$$

Reliability calculation when stress and strength data can not be represented by any existing distribution is discussed in reference 79.

6.9.2 *Analytical: Constant Stress-Strength Interference Theory Models**

This section presents three interference theory models when probability density functions are defined.

Component Reliability Determination for Exponential Stress and Strength Distributions. Both stress and strength probability functions are defined as

$$f_{st}(s)=\lambda_{st}e^{-\lambda_{st}s} \qquad 0\leqslant s<\infty \tag{6.42}$$

and

$$f_{Sth}(S)=\lambda_{Sth}e^{-\lambda_{Sth}S} \qquad 0\leqslant S<\infty \tag{6.43}$$

*For these models it is assumed that the component has only one significant failure mode.

where λ_{st} = the reciprocal of the mean value of stress, $\bar{s}$
λ_{Sth} = the reciprocal of the mean value of strength, $\bar{S}$

By utilizing expression (6.33), the component reliability, R_c, can be determined:

$$R_c=\int_0^\infty f_{Sth}(S)\left[\int_0^S f_{st}(s)\,ds\right]dS \tag{6.44}$$

where

$$\int_0^S f_{st}(s)\,ds=\int_0^S \lambda_{st}e^{-\lambda_{st}s}\,ds=1-e^{-\lambda_{st}S} \tag{6.45}$$

Therefore by substituting (6.45) into (6.44) we get

$$R_c=\int_0^\infty \lambda_{Sth}e^{-\lambda_{Sth}S}\left[1-e^{-\lambda_{st}S}\right]dS$$

$$=1-\int_0^\infty \lambda_{Sth}e^{-(\lambda_{st}+\lambda_{Sth})S}\,dS$$

$$=1-\frac{\lambda_{Sth}}{\lambda_{Sth}+\lambda_{st}}\int_0^\infty (\lambda_{st}+\lambda_{Sth})e^{-(\lambda_{st}+\lambda_{Sth})S}\,dS$$

$$R_c=\frac{\lambda_{st}}{\lambda_{st}+\lambda_{Sth}} \tag{6.46}$$

Dividing numerator and denominator of expression (6.46) by λ_{st} we get:

$$R_c=\frac{1}{1+\lambda_{Sth}/\lambda_{st}} \qquad \text{for } \bar{S}\neq 0$$

$$=\frac{1}{1+\rho} \tag{6.47}$$

where $\rho=\bar{s}/\bar{S}$ for $\bar{S}\geqslant\bar{s}$, $\rho\leqslant 1$. Values of R_c are presented in Table 6.2 for the various values of ρ. A plot of (6.47) is shown in Figure 6.6.

6.9.3 *Component Reliability Determination when Stress and Strength Follow Rayleigh Distribution*

Both Rayleigh stress and strength density functions are defined as follows:

$$f_{st}(s)=2k_{st}se^{-k_{st}s^2} \qquad 0\leqslant s<\infty \tag{6.48}$$

and

$$f_{Sth}(S)=2k_{Sth}Se^{-k_{Sth}S^2} \qquad 0\leqslant S<\infty \tag{6.49}$$

Table 6.2

ρ	$1+\rho$	$R_c=\frac{1}{1+\rho}$
1	2	0.5
0.9	1.9	0.53
0.8	1.8	0.56
0.7	1.7	0.59
0.6	1.6	0.63
0.5	1.5	0.67
0.4	1.4	0.71
0.3	1.3	0.77
0.2	1.2	0.83
0.1	1.1	0.91
0	1	1

where k_{st} = the stress parameter
k_{Sth} = the strength parameter

Component reliability is determined by substituting (6.48) and (6.49) into (6.32):

$$R_c=\int_0^{\infty} 2k_{st}se^{-k_{st}s^2}2\left[\int_s^{\infty} k_{Sth}Se^{-k_{Sth}S^2}\,dS\right]ds$$

$$=\int_0^{\infty} 2k_{st}se^{-k_{st}s^2}2\left[e^{-k_{Sth}s^2}\right]ds=2\int_0^{\infty} 2k_{st}se^{-(k_{st}+k_{Sth})s^2}\,ds$$

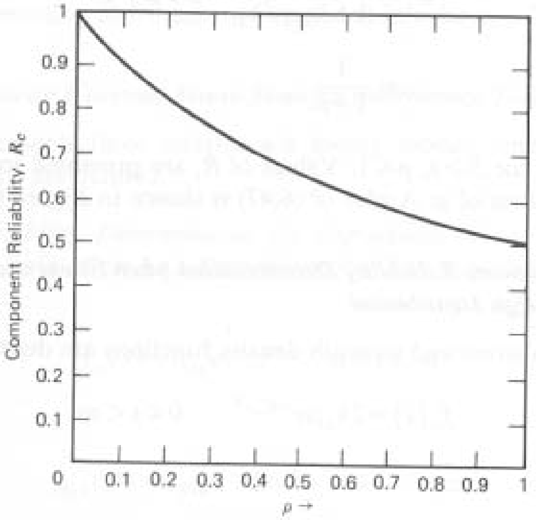

Figure 6.6 Component reliability versus mean stress-strength ratio.

Let

$$a=(k_{st}+k_{Sth})$$

$$=4k_{st}\int_0^{\infty} se^{-as^2}\,ds$$

$$\therefore R_c=4k_{st}\frac{\Gamma(1/2)}{2a}=\frac{2k_{st}\sqrt{\pi}}{k_{st}+k_{Sth}} \tag{6.50}$$

6.9.4 Component Reliability Calculation with Normally Distributed Stress and Gamma Distributed Strength

Stress and strength probability density functions are defined as

$$f_{st}(s)=\frac{1}{\sigma_{st}\sqrt{2\pi}}e^{-(s-\mu_{st})^2/2\sigma_{st}} \qquad 0\leqslant s\leqslant\infty \tag{6.51}$$

and

$$f_{Sth}(S)=\frac{1}{\Gamma(\beta)}\lambda^{\beta}S^{\beta-1}e^{-\lambda S} \qquad 0\leqslant S\leqslant\infty \tag{6.52}$$

where β and λ are the shape and scale parameters, respectively, and μ_{st} and σ_{st} are the mean and the standard deviation, respectively. By substituting the probability density functions (6.51) and (6.58) into (6.32) and integrating, the following reliability expression is obtained,

$$R_c=\sum_{\theta=0}^{\theta=\beta-1}\sum_{s=0}^{s=\theta}\frac{(\lambda\sigma_{st})^{\theta}}{\theta!}e^{-\frac{\lambda}{2}(2\mu_{st}-\sigma_{st}^2\lambda)}XYZ \tag{6.53}$$

where

$$X=\left[2^{\theta/2-1}\binom{\theta}{s}a^{\theta-c/\sqrt{\pi}}\right]$$

$$Y=\Gamma\left(\frac{s+1}{2}\right)$$

$$Z=\left[1-I\left(r,\frac{\theta-1}{2}\right)\right]$$

where I is the incomplete gamma function.

$$r=\frac{1}{\sqrt{(s+1)}}\frac{1}{\sqrt{2}}\left(\frac{\mu_{st}-\sigma_{st}^2\lambda}{\sigma_{st}}\right)^2$$

For the detailed derivation of (6.53) see reference 102. Many other interference theory models to calculate component reliability are developed in references 45, 83, 101, and 102. These models are developed for the following:

1. Normally distributed stress and strength.
2. Log-normally distributed stress and strength.
3. Exponentially (normally) distributed strength and normally (exponentially) distributed stress.
4. Gamma distributed stress and strength.
5. Weibull distributed strength and normally distributed stress.
6. Weibull distributed stress and strength.
7. Weibull distributed strength and extreme value distributed stress.
8. Maxwellian distributed stress and Weibull distributed strength.

6.9.5 *Component Reliability with Multiple Failure Modes*

Reliability of a component with many independent failure modes is given by

$$R=\prod_{i=1}^{n} R_i \tag{6.54}$$

where R = the overall component reliability
n = the number of significant failure modes
R_i = the reliability of a significant failure mode i

Similarly, the system reliability can be computed for a series configuration, the component reliability being obtained by applying (6.54) or directly from the stress-strength models (i.e., if the component under study has only one significant failure mode).

6.9.6 *Chain Model*

This model represents a situation in which a chain is composed of n number of identical series links subject to the same environmental stress [83]. The probability of any link having strength S_0 or greater is given by

$$P(S_{Sth}\geq S_0)=\int_{S_0}^{\infty} f_{Sth}(S)\,dS \tag{6.55}$$

In the case of n number of identical and independent links, the probability

that the chain has strength S_0 or greater is given by

$$P(S_{Sth}\geq S_0)=\left[\int_{S_0}^{\infty} f_{Sth}(S)\,dS\right]^n \tag{6.56}$$

To obtain the probability density function of the chain strength, $f_{cSth}(S)$, differentiate expression (6.56) with respect to S:

$$f_{cSth}(S)=n\left[\int_{S_0}^{\infty} f_{Sth}(S)\,dS\right]^{n-1} f_{Sth}(S) \tag{6.57}$$

When all the chain links are under the same environmental stress, the chain reliability R_{ch} can be obtained by substituting (6.57) into (6.33):

$$R_{ch}=\int_0^{\infty}\left\{\left[\int_0^{S} f_{st}(s)\,ds\right]n\left[\int_S^{\infty} f_{Sth}(S)\,dS\right]^{n-1} f_{Sth}(S)\right\}dS \tag{6.58}$$

Reliability of the above equation may be determined by graphical, analytical or numerical technique.

6.9.7 *Stress-Strength Time-Dependent Models*

In the previous sections, we considered stress-strength models where stress and strength were independent of time. In real life, however, this may not be necessarily true. The component strength may change with time and a component may experience repeated application of stresses. In other words, the stress or load may follow a random pattern with respect to time t. A hypothetical pattern is shown in Figure 6.7.

This area of mechanical reliability still remains to be explored further. The interested readers are advised to consult references 10, 45, 84, and 87.

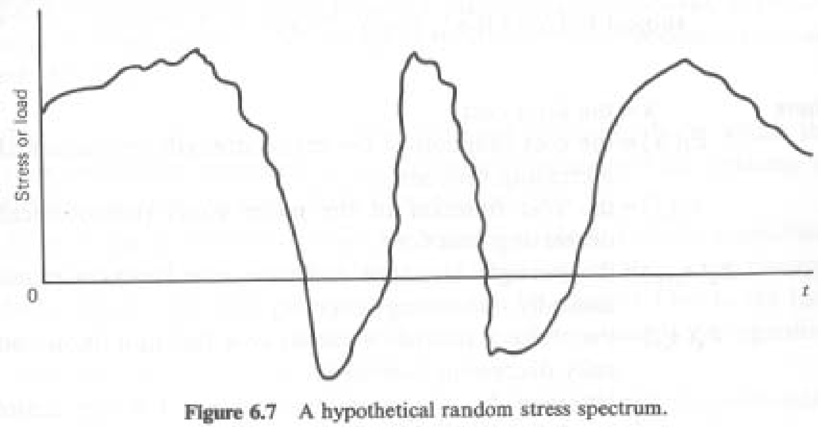

Figure 6.7 A hypothetical random stress spectrum.

6.10 OPTIMIZATION OF MECHANICAL COMPONENT RELIABILITY

A redundant system can be optimized subject to constraints such as cost, weight, and volume. To optimize system reliability, traditional operations research techniques such as Lagrange multiplier, linear, integer, and dynamic programming are applicable. These techniques can be used to optimize reliability of mechanical components, also.

6.10.1 Reliability Optimization of a Mechanical Component with Normally Distributed Stress and Strength

The following reliability equation is taken from reference 45:

$$R=\int_{-n}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy \tag{6.59}$$

where $n=(\bar{S}-\bar{s})(\sigma_{Sth}^2+\sigma_{st}^2)^{-1/2}$

$\bar{s}$ = mean stress

$\bar{S}$ = mean strength

$\sigma_{Sth}, \sigma_{st}$ = standard deviations of strength and stress

It is assumed that to formulate this model, the stress and strength are statistically independent. To maximize component reliability, it is obvious that the value of lower limit of expression (6.59) should be as low as possible. Therefore, the equation to minimize total cost subject to desired component reliability may be formulated as follows:

$$\begin{aligned}&\text{minimize } k=k_1(\bar{S})+k_2(\sigma_{Sth})+k_3(\bar{s})+k_4(\sigma_{st})\\&\text{subject to } (\bar{S}-\bar{s})(\sigma_{Sth}^2+\sigma_{st}^2)^{-1/2}\geq y\end{aligned} \tag{6.60}$$

where k = the total cost

$k_1(\bar{S})$ = the cost function of the mean strength (monotonically increasing function)

$k_3(\bar{s})$ = the cost function of the mean stress (monotonically decreasing function)

$k_2(\sigma_{Sth})$ = the strength standard deviation cost function (monotonically decreasing function)

$k_4(\sigma_{st})$ = the stress standard deviation cost function (monotonically decreasing function)

y = obtained by the coupling equation for the desired reliability level

The Lagrangian equation for the above problem becomes:

$$F(\bar{S},\bar{s},\sigma_{Sth},\sigma_{st},\lambda)=k+\lambda\left[\bar{S}-\bar{s}-y(\sigma_{Sth}^2+\sigma_{st}^2)^{1/2}\right] \tag{6.61}$$

To find optimum solution, differentiate (6.61) with respect to each variable $\lambda, \bar{S}, \bar{s}, \sigma_{Sth}, \sigma_{st}$, and equate each differentiation to zero. The following equations were obtained:

$$\bar{S}-\bar{s}-y(\sigma_{Sth}^2+\sigma_{st}^2)^{1/2}=0 \tag{6.62}$$

$$\dot{k}_4(\sigma_{st})=\lambda y\sigma_{st}\Big/\sqrt{\sigma_{Sth}^2+\sigma_{st}^2} \tag{6.63}$$

$$\ddot{k}_2(\sigma_{Sth})=\lambda y\sigma_{Sth}\Big/\sqrt{\sigma_{Sth}^2+\sigma_{st}^2} \tag{6.64}$$

$$k_3'(\bar{s})=\lambda \tag{6.65}$$

$$k_1''(\bar{S})=-\lambda \tag{6.66}$$

where single overdots and primes represent partial derivative with respect to $\sigma_{st}, \bar{s}$, respectively, and double overdots and primes represent partial derivative with respect to $\sigma_{Sth}, \bar{S}$, respectively. The value of $\bar{S}, \bar{s}, \sigma_{Sth}, \sigma_{st}$, and λ can be found by solving (6.62)–(6.66) to obtain all local optima. To choose a global optimal solution, determine the objective function (6.60) for all the local optimal solutions. For a more detailed analysis and examples on the mechanical component reliability optimization, one should consult references 45 and 95.

6.11 CONCLUDING REMARKS

Although the interference stress-strength modeling is a promising technique for calculating the reliability of a mechanical component, there are several problem areas to be overcome. Some of these problems are outlined as follows:

1. The representative stress and environmental condition under which the component will operate may be difficult to estimate with certainty at the design stage because of the lack of field data.
2. Most of the material properties are time dependent. For some practical purposes this factor may be disregarded because of their slow change, but generally, the time dependency can not be ignored. Due to the lack of variability data of material properties, further assumptions regarding time dependency may be required.
3. Although there is no lack of mathematical techniques or the probabilistic models for the reliability evaluation, further refinement to these

techniques and models will be useful to improve the mechanical reliability prediction.

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