

ECEN643
Problem Solving Session

Content

There will be 5 problems.

Problem 1-4 use the same system but utilize different methods to compare the process and results.

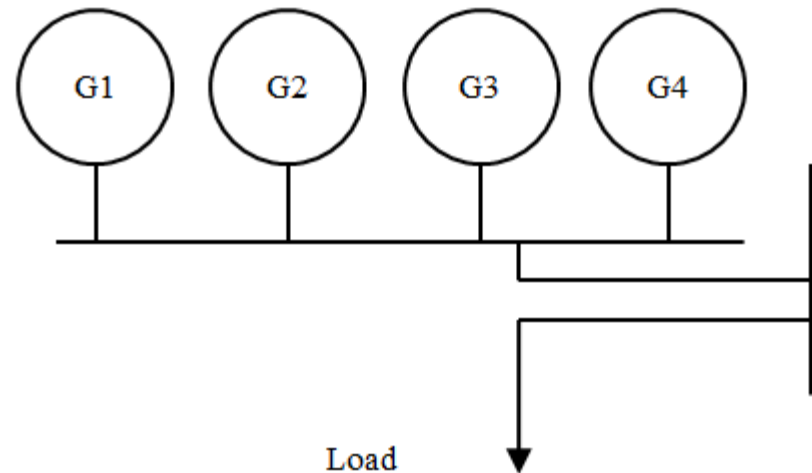
Problem 5 is for better understanding the concepts of frequency.

Problem 1

There are four generators. Each generator either has full capacity of 50 MW or 0 MW when failed. The failure rate of each generator is 0.1/day and mean repair time is 24 hours.

The daily load cycle:

Hours	Load (MW)
1- 4	45
4- 8	105
8- 12	155
12- 16	105
16- 24	45



Question:

Build generation system model with unit addition algorithm.

Problem 1

For a single generator

$$\lambda = 0.1/day, \mu = 1/day$$

$$p_u = \frac{\mu}{\lambda + \mu} = \frac{10}{11}$$

$$p_d = \frac{\lambda}{\lambda + \mu} = \frac{1}{11}$$

$$f_G = \frac{\lambda\mu}{\lambda + \mu} = \frac{1}{11}/day$$

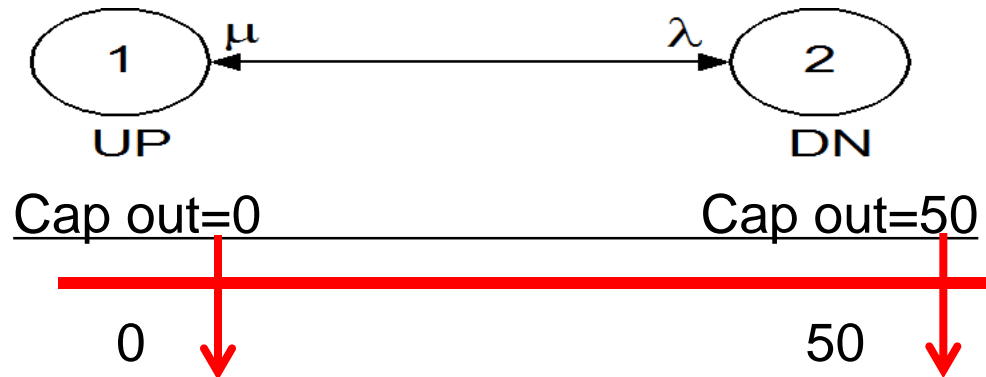
Problem 1

Try to build generation system model using unit addition method.

We start with 0 generator scenario

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	0	1	0

The next step is to add one generator, assume increment=50MW

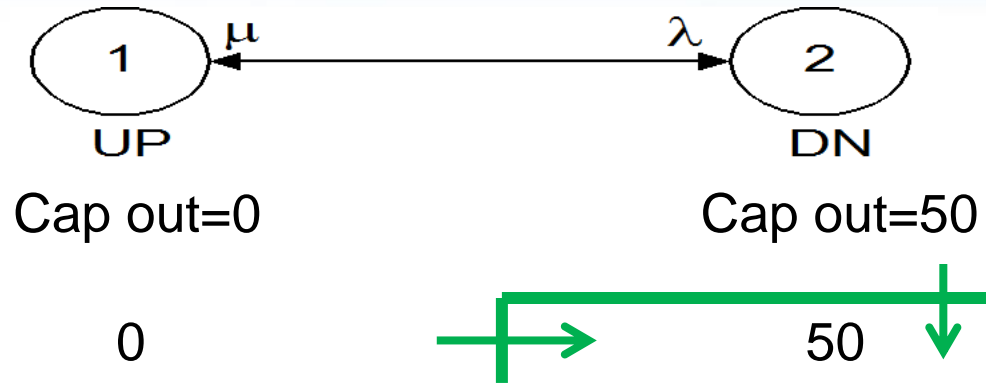


$$P(\text{Cap out} \geq 0) = P_1 p_u + P_1 p_d = P_1 = 1$$

$$F(\text{Cap out} \geq 0) = F_1 p_u + F_1 p_d = F_1 = 0$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	0	1	0



$$P(\text{Cap out} \geq 50) = P_1 p_d = p_d = \frac{1}{11}$$

$$F(\text{Cap out} \geq 50) = F_1 p_d + f_G P_1 = f_G = \frac{1}{11} / \text{day}$$

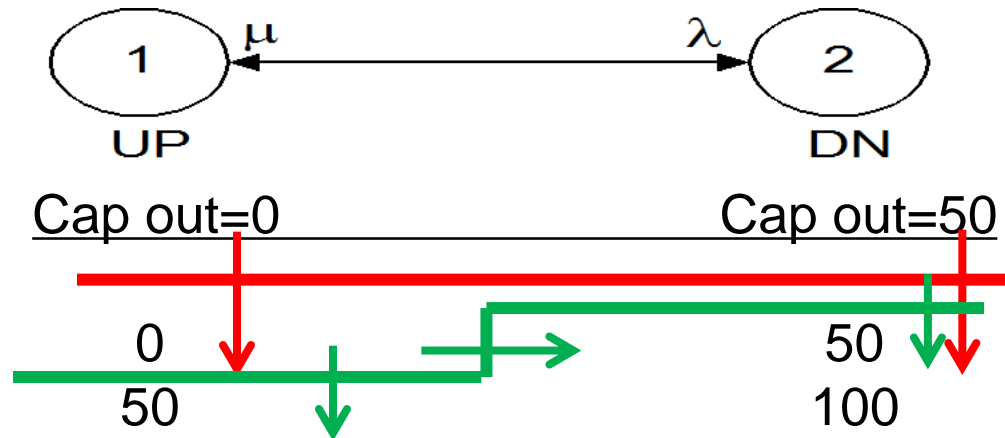
Thus with one unit added to the system, we have

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	50	1	0
50	0	$\frac{1}{11}$	$\frac{1}{11}$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	50	1	0
50	0	$\frac{1}{11}$	$\frac{1}{11}$

Then the next step is to add 2nd generator into the system



Similarly, we have

$$P(\text{Cap out} \geq 0) = P_1 p_u + P_1 p_d = P_1 = 1$$

$$F(\text{Cap out} \geq 0) = F_1 p_u + F_1 p_d = F_1 = 0$$

$$P(\text{Cap out} \geq 50) = P_2 p_u + P_1 p_d = \frac{10}{121} + \frac{1}{11} = \frac{21}{121}$$

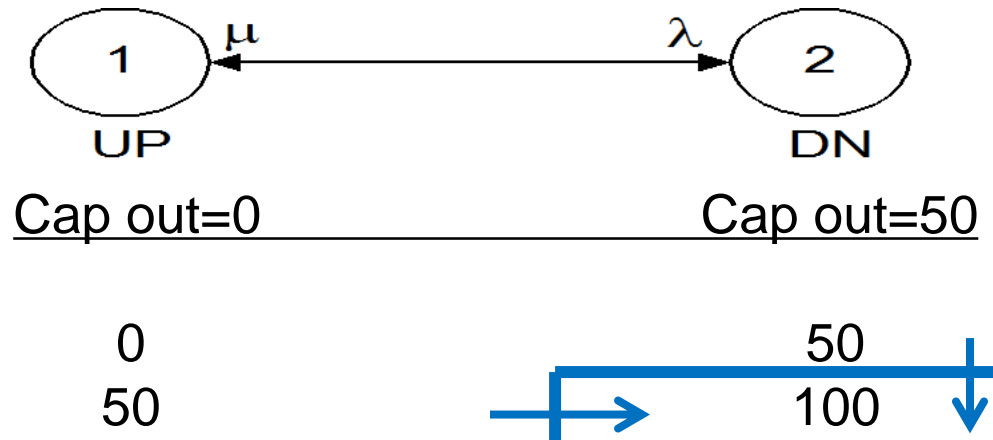
$$F(\text{Cap out} \geq 50) = F_2 p_u + F_1 p_d + f_G (P_1 - P_2) = \frac{10}{121} + 0 + \frac{10}{121}$$

$$= \frac{20}{121} \text{ /day}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	50	1	0
50	0	$\frac{1}{11}$	$\frac{1}{11}$

Then the next step is to add 2nd generator into the system



New cap out is 100MW

$$P(\text{Cap out} \geq 100) = P_2 p_d = \frac{1}{121}$$

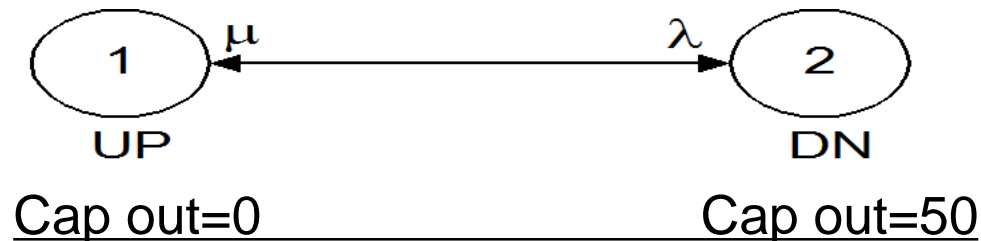
$$F(\text{Cap out} \geq 100) = F_2 p_d + f_G P_2 = \frac{1}{121} + \frac{1}{121} = \frac{2}{121} / \text{day}$$

Problem 1

Then we could summarize the 2 generator system into a new table

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	100	1	0
50	50	$\frac{21}{121}$	$\frac{20}{121}$
100	0	$\frac{1}{121}$	$\frac{2}{121}$

Then the next step is to add 3rd generator into the system

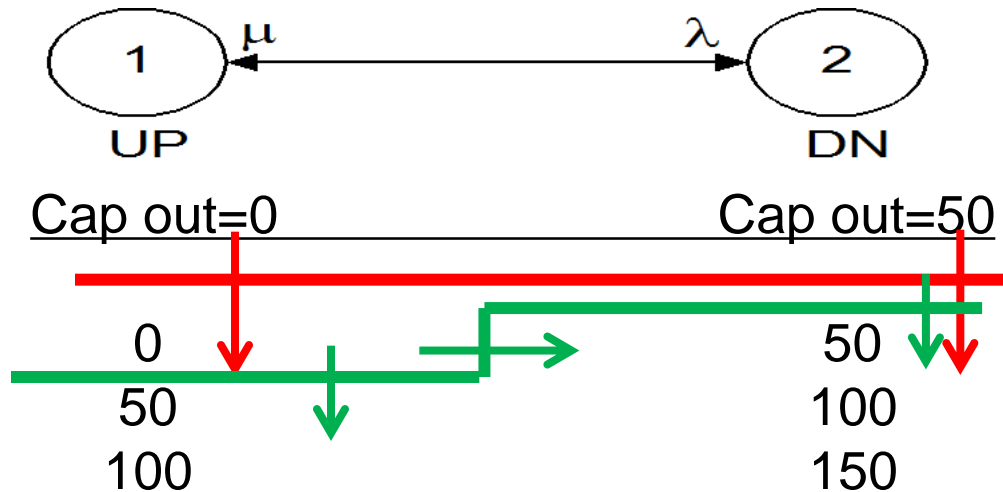


0	50
50	100
100	150

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	100	1	0
50	50	$\frac{21}{121}$	$\frac{20}{121}$
100	0	$\frac{1}{121}$	$\frac{2}{121}$

Then the next step is to add 3rd generator into the system



Similarly, we have

$$P(\text{Cap out} \geq 0) = P_1 p_u + P_1 p_d = P_1 = 1$$

$$F(\text{Cap out} \geq 0) = F_1 p_u + F_1 p_d = F_1 = 0$$

$$P(\text{Cap out} \geq 50) = P_2 p_u + P_1 p_d = \frac{210}{1331} + \frac{1}{11} = \frac{331}{1331}$$

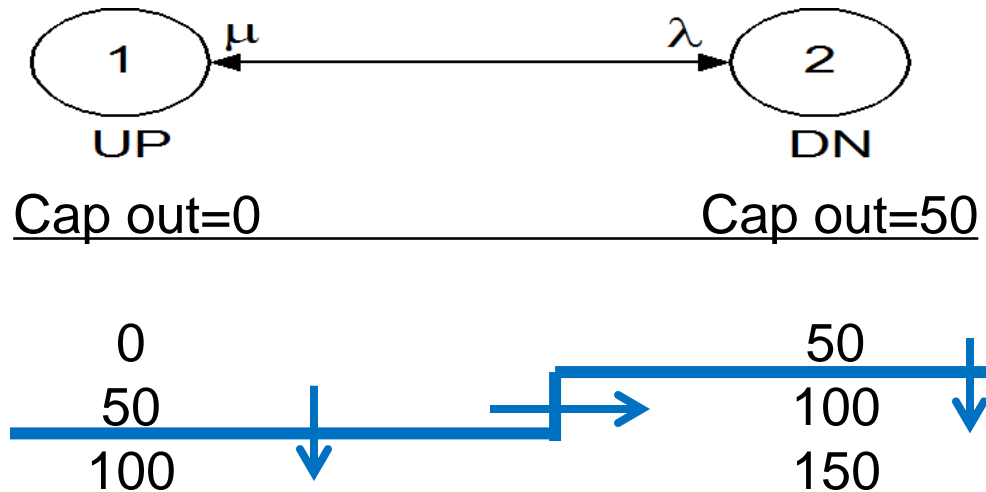
$$F(\text{Cap out} \geq 50) = F_2 p_u + F_1 p_d + f_G (P_1 - P_2) = \frac{200}{1331} + 0 + \frac{100}{1331}$$

$$= \frac{300}{1331} \text{ /day}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	100	1	0
50	50	$\frac{21}{121}$	$\frac{20}{121}$
100	0	$\frac{1}{121}$	$\frac{2}{121}$

Then the next step is to add 3rd generator into the system



Similarly, we have

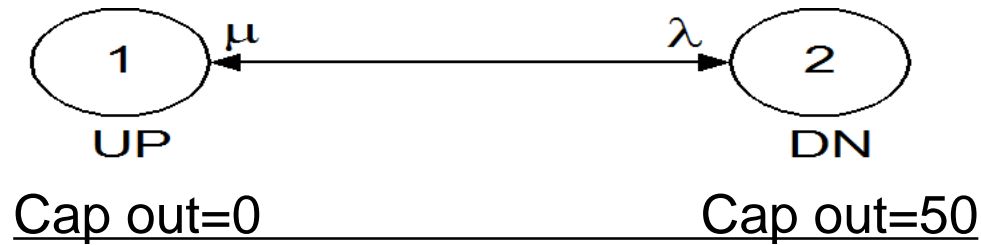
$$P(\text{Cap out} \geq 100) = P_3 p_u + P_2 p_d = \frac{10}{1331} + \frac{21}{1331} = \frac{31}{1331}$$

$$\begin{aligned}
 F(\text{Cap out} \geq 100) &= F_3 p_u + F_2 p_d + f_G (P_2 - P_3) = \frac{20}{1331} + \frac{20}{1331} + \frac{20}{1331} \\
 &= \frac{60}{1331} / \text{day}
 \end{aligned}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	100	1	0
50	50	$\frac{21}{121}$	$\frac{20}{121}$
100	0	$\frac{1}{121}$	$\frac{2}{121}$

Then the next step is to add 3rd generator into the system



New cap out is 150MW

$$P(\text{Cap out} \geq 150) = P_3 p_d = \frac{1}{1331}$$

$$F(\text{Cap out} \geq 150) = F_3 p_d + f_G P_3 = \frac{2}{1331} + \frac{1}{1331} = \frac{3}{1331} / \text{day}$$

Problem 1

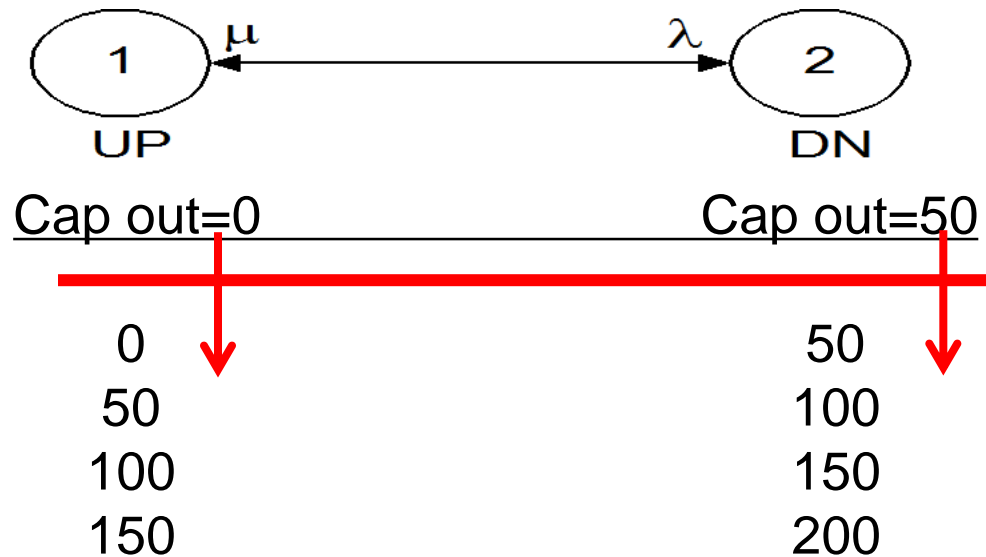
Then we could summarize the 3 generator system into a new table

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$

Then the last step is to add 4th generator to the system

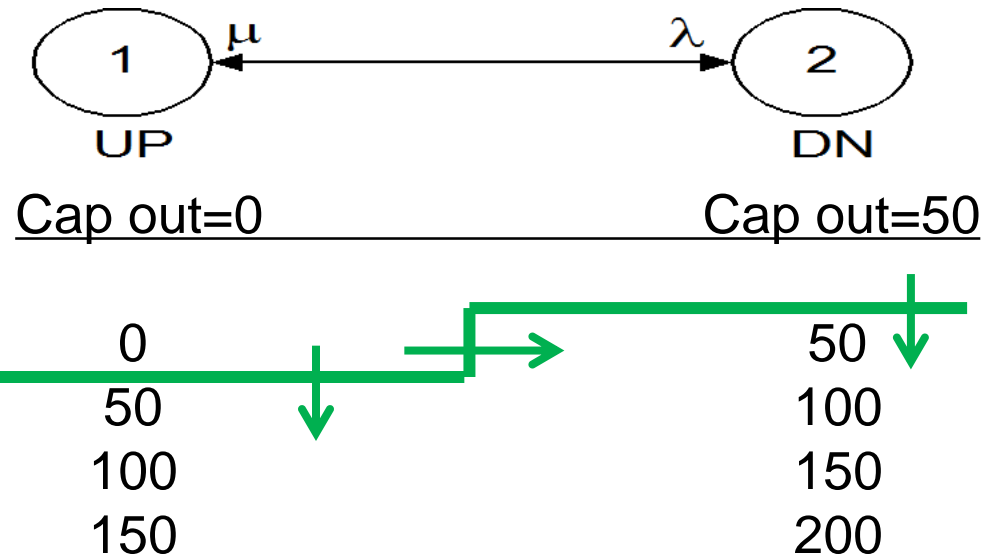


$$P(\text{Cap out} \geq 0) = P_1 p_u + P_1 p_d = P_1 = 1$$

$$F(\text{Cap out} \geq 0) = F_1 p_u + F_1 p_d = F_1 = 0$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$



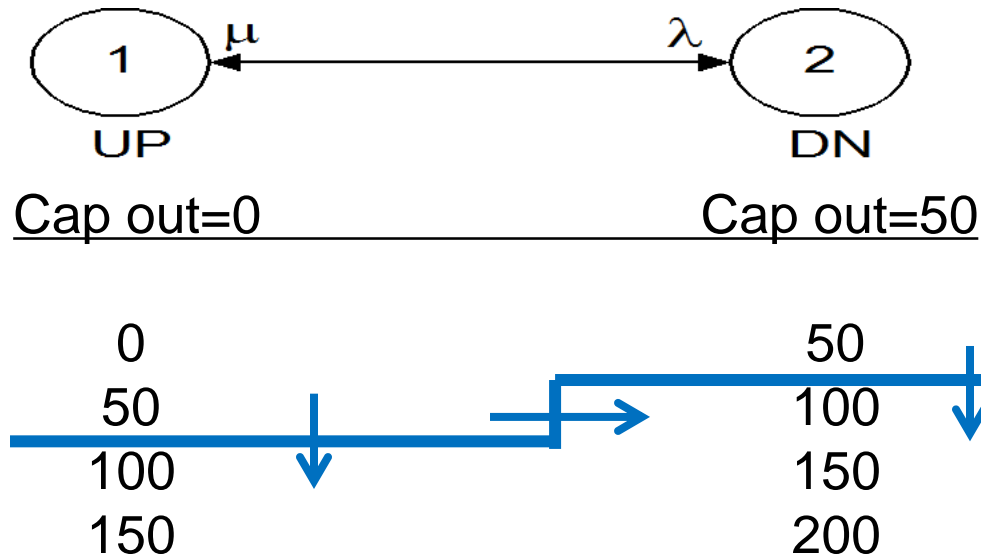
$$P(\text{Cap out} \geq 50) = P_2 p_u + P_1 p_d = \frac{3310}{14641} + \frac{1}{11} = \frac{4641}{14641}$$

$$F(\text{Cap out} \geq 50) = F_2 p_u + F_1 p_d + f_G (P_1 - P_2) = \frac{3000}{14641} + 0 + \frac{1000}{14641}$$

$$= \frac{4000}{14641} \text{ /day}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$



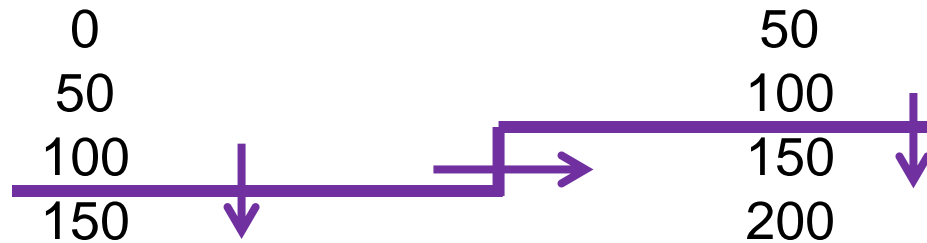
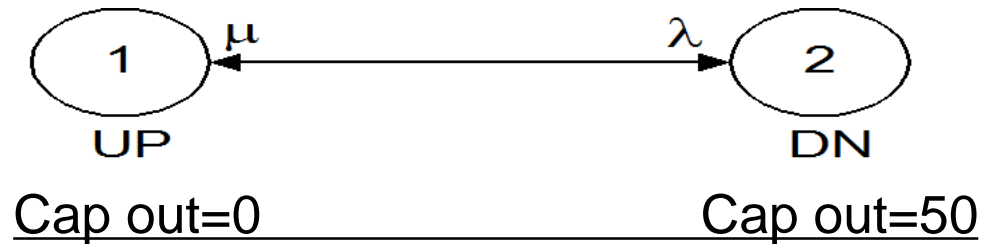
$$P(\text{Cap out} \geq 100) = P_3 p_u + P_2 p_d = \frac{310}{14641} + \frac{331}{14641} = \frac{641}{14641}$$

$$F(\text{Cap out} \geq 100) = F_3 p_u + F_2 p_d + f_G (P_2 - P_3) = \frac{600}{14641} + \frac{300}{14641} + \frac{300}{14641}$$

$$= \frac{1200}{14641} / \text{day}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$



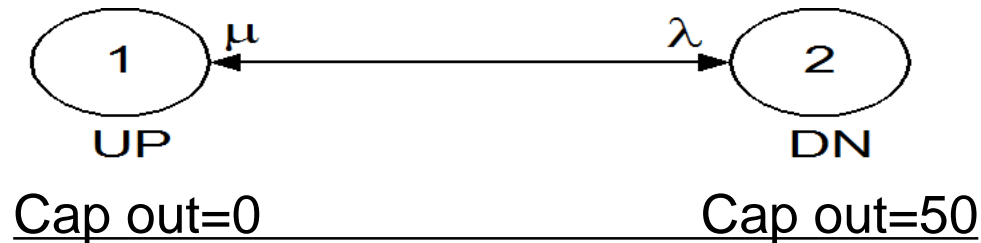
$$P(\text{Cap out} \geq 150) = P_4 p_u + P_3 p_d = \frac{10}{14641} + \frac{31}{14641} = \frac{41}{14641}$$

$$F(\text{Cap out} \geq 150) = F_4 p_u + F_3 p_d + f_G (P_3 - P_4) = \frac{30}{14641} + \frac{60}{14641} + \frac{30}{14641}$$

$$= \frac{120}{14641} / \text{day}$$

Problem 1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	150	1	0
50	100	$\frac{331}{1331}$	$\frac{300}{1331}$
100	50	$\frac{31}{1331}$	$\frac{60}{1331}$
150	0	$\frac{1}{1331}$	$\frac{3}{1331}$



New cap out is 200MW

$$P(\text{Cap out} \geq 200) = P_4 p_d = \frac{1}{14641}$$

$$F(\text{Cap out} \geq 200) = F_4 p_d + f_G P_4 = \frac{3}{14641} + \frac{1}{14641} = \frac{4}{14641} / \text{day}$$

Problem 1

Then we could summarize the 4 generator system into a new table

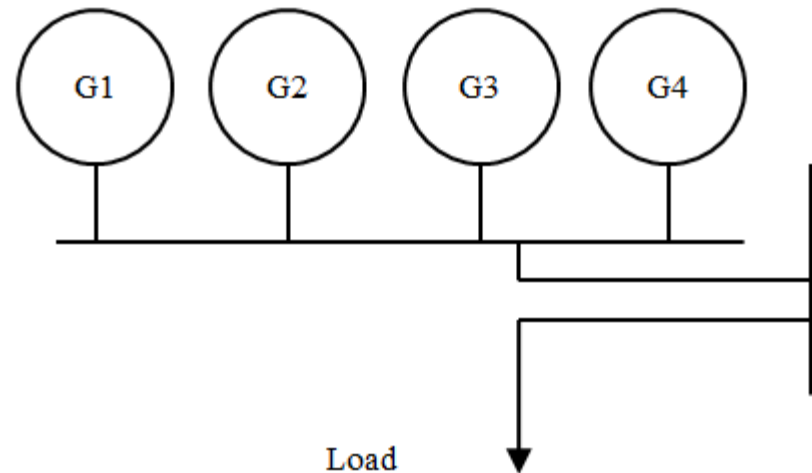
Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Problem 2

There are four generators. Each generator either has full capacity of 50 MW or 0 MW when failed. The failure rate of each generator is 0.1/day and mean repair time is 24 hours.

The daily load cycle:

Hours	Load (MW)
1- 4	45
4- 8	105
8- 12	155
12- 16	105
16- 24	45



Question:

Build generation system model in Markov approach

Problem 2

For a single generator

$$\lambda = 0.1/day, \mu = 1/day$$

$$p_u = \frac{\mu}{\lambda + \mu} = \frac{10}{11}$$

$$p_d = \frac{\lambda}{\lambda + \mu} = \frac{1}{11}$$

$$f_G = \frac{\lambda\mu}{\lambda + \mu} = \frac{1}{11}/day$$

Problem 2

Since we have four 2-state generators, if we write out all the states, there will be $2^4 = 16$ states in the state space.

Then merge all the states with same generation capacity, we will have 5 states

State 1: 4 generators up, 200MW

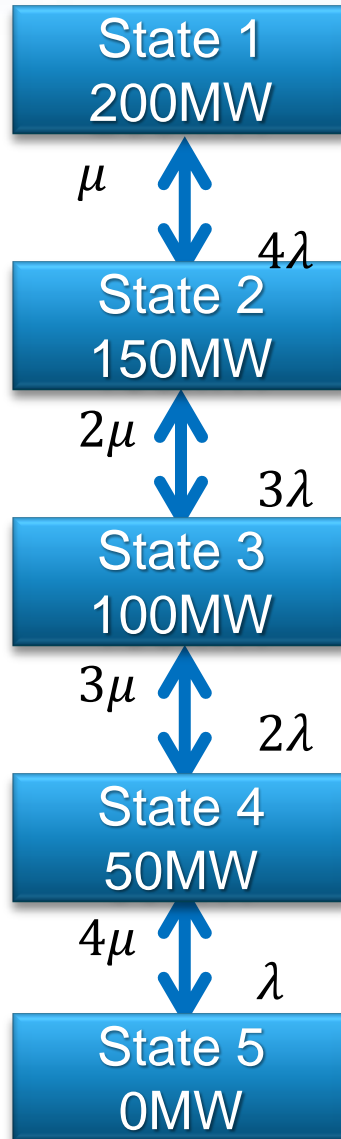
State 2: 3 generators up, 150MW

State 3: 2 generators up, 100MW

State 4: 1 generators up, 50MW

State 5: 0 generators up, 0MW

Problem 2



Problem 2

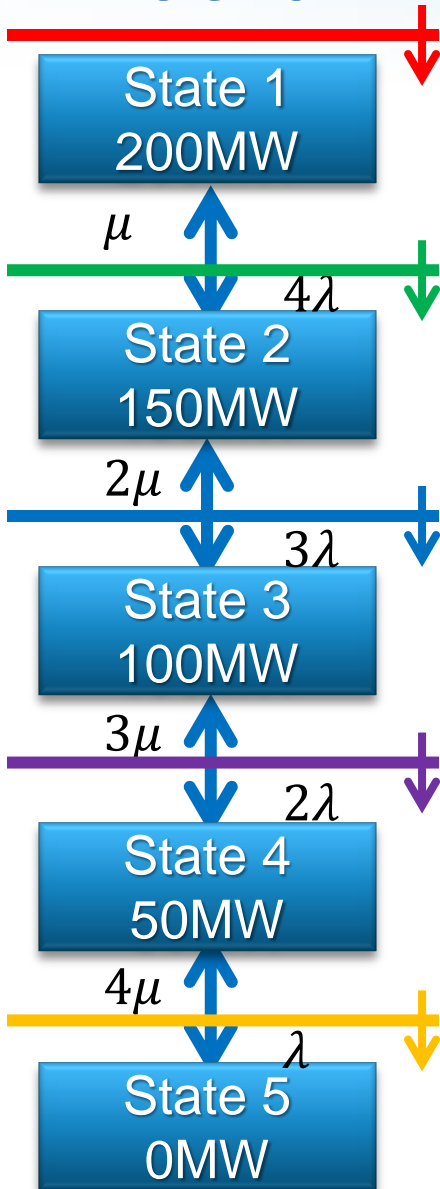
With transition diagram in previous page, it is possible to find every probability and frequency.

Here, we obtain the same values in problem 1 as a comparison between Markov approach and unit addition algorithm.

Firstly calculate probability of each state

$$P_{4u} = p_u^4 = \left(\frac{10}{11}\right)^4 = \frac{10000}{14641}$$
$$P_{3u} = \binom{4}{3} p_u^3 p_d = 4 \left(\frac{10}{11}\right)^3 \frac{1}{11} = \frac{4000}{14641}$$
$$P_{2u} = \binom{4}{2} p_u^2 p_d^2 = 6 \left(\frac{10}{11}\right)^2 \left(\frac{1}{11}\right)^2 = \frac{600}{14641}$$
$$P_{1u} = \binom{4}{1} p_u p_d^3 = 4 \frac{10}{11} \left(\frac{1}{11}\right)^3 = \frac{40}{14641}$$
$$P_{0u} = p_d^4 = \left(\frac{1}{11}\right)^4 = \frac{1}{14641}$$

Problem 2



$$P(\text{Cap out} \geq 0) = 1$$

$$F(\text{Cap out} \geq 0) = 0$$

$$P(\text{Cap out} \geq 50) = 1 - P_{4u} = 1 - \frac{10000}{14641} = \frac{4641}{14641}$$

$$F(\text{Cap out} \geq 50) = P_{4u} * 4\lambda = \frac{10000}{14641} * \frac{4}{10} = \frac{4000}{14641}$$

$$P(\text{Cap out} \geq 100) = 1 - P_{4u} - P_{3u}$$

$$= 1 - \frac{10000}{14641} - \frac{4000}{14641} = \frac{641}{14641}$$

$$F(\text{Cap out} \geq 100) = P_{3u} * 3\lambda = \frac{4000}{14641} * \frac{3}{10} = \frac{1200}{14641}$$

$$P(\text{Cap out} \geq 150) = P_{1u} + P_{0u} = \frac{600}{14641}$$

$$F(\text{Cap out} \geq 150) = P_{2u} * 2\lambda = \frac{600}{14641} * \frac{2}{10} = \frac{120}{14641}$$

$$P(\text{Cap out} \geq 200) = P_{0u} = \frac{1}{14641}$$

$$F(\text{Cap out} \geq 200) = P_{1u} * \lambda = \frac{40}{14641} * \frac{1}{10} = \frac{4}{14641}$$

Problem 1&2 Comparison

It can be observed that Markov approach and unit addition method arrived at the same results.

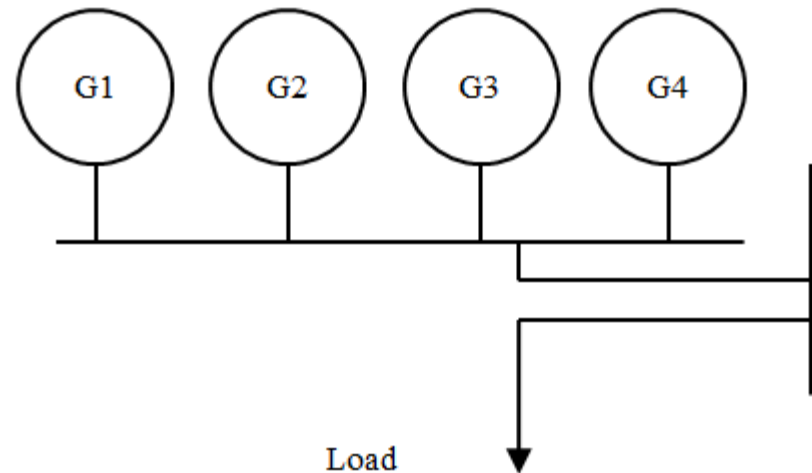
- Markov approach is straight forward and easy to understand, but when it comes to actual system with large number of components, this method require tremendous computation time and efforts.
- Unit addition method is widely used in computer software, which could be easily adapted to large system.

Problem 3

There are four generators. Each generator either has full capacity of 50 MW or 0 MW when failed. The failure rate of each generator is 0.1/day and mean repair time is 24 hours.

The daily load cycle:

Hours	Load (MW)
1- 4	45
4- 8	105
8- 12	155
12- 16	105
16- 24	45



Question:

Using conditional probability approach, obtain HLOLE and frequency of loss of load on a yearly basis.

Problem 3

$$HLOLE = LOLP * 8760 \text{ (hrs)}$$

In order to find the LOLP, build load model with 3 states

Hours	Load (MW)
1- 4	45
4- 8	105
8- 12	155
12- 16	105
16- 24	45

State 1: Load=45 MW, $p_1 = \frac{1}{2}$

State 2: Load=105 MW, $p_2 = \frac{1}{3}$

State 3: Load=155 MW, $p_3 = \frac{1}{6}$

Also

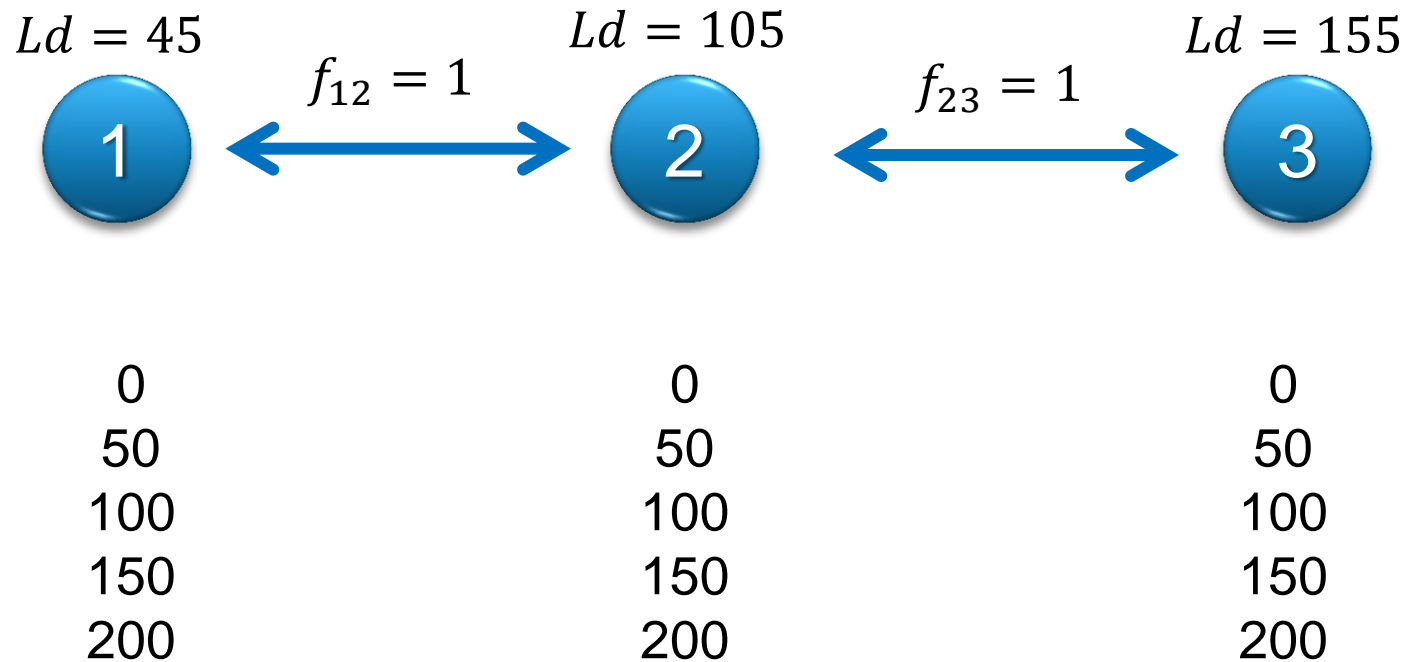
$$f_{12} = 1/\text{day}$$

$$f_{23} = 1/\text{day}$$

$$f_{13} = 0/\text{day}$$

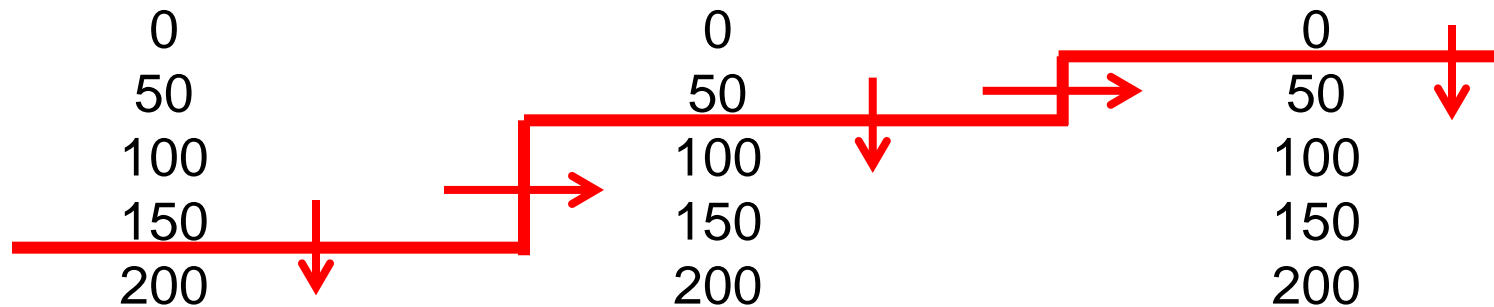
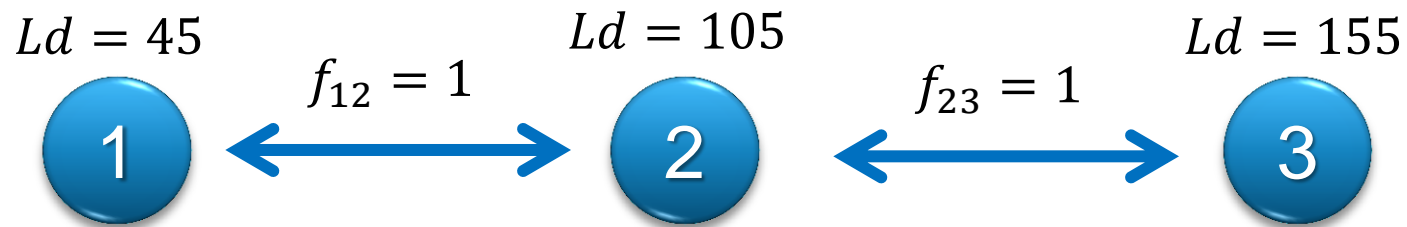
Problem 3

Using conditional probability approach to find LOLP



Problem 3

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

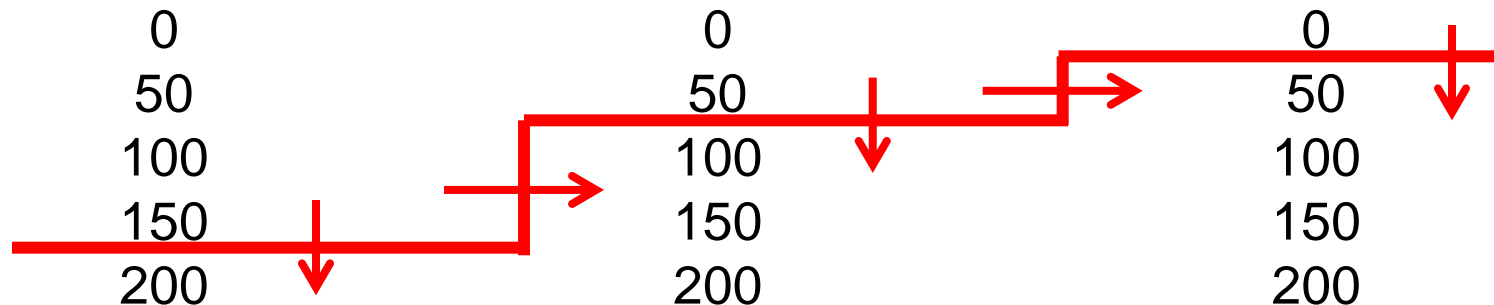
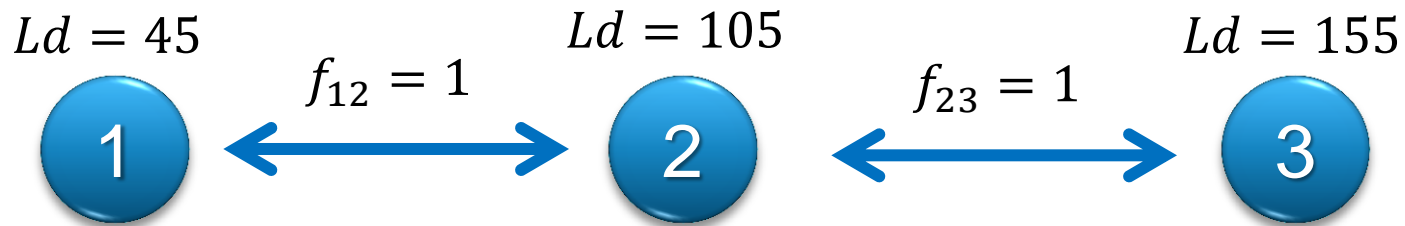


$$LOLP = P_5 p_1 + P_3 p_2 + P_2 p_3 = \frac{1}{29282} + \frac{641}{43923} + \frac{4641}{87846} = \frac{193}{2861}$$

$$HLOLE = LOLP * 8760 \approx 590.94 \text{ (hrs)}$$

Problem 3

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$



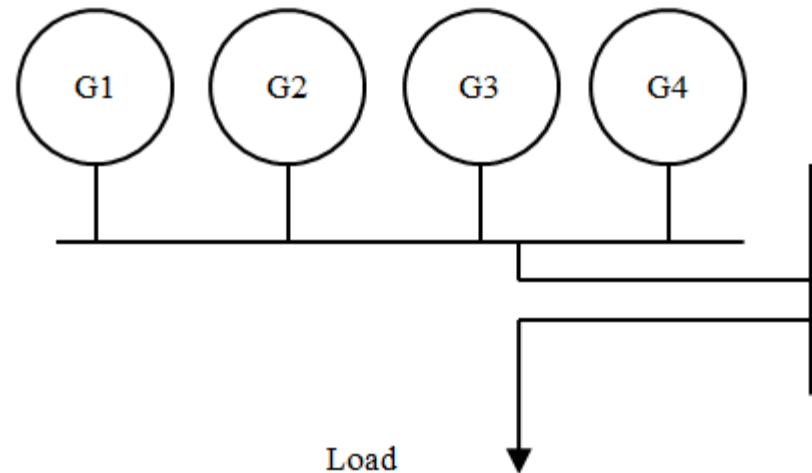
$$\begin{aligned}
 FF &= F_5 p_1 + F_3 p_2 + F_2 p_3 + f_{12}(P_3 - P_5) + f_{23}(P_2 - P_3) = \frac{1337}{3429} / \text{day} \\
 &\approx 0.3899 / \text{day} = 142.317 / \text{year}
 \end{aligned}$$

Problem 4

There are four generators. Each generator either has full capacity of 50 MW or 0 MW when failed. The failure rate of each generator is 0.1/day and mean repair time is 24 hours.

The daily load cycle:

Hours	Load (MW)
1- 4	45
4- 8	105
8- 12	155
12- 16	105
16- 24	45



Question:

Using generation reserve model, obtain HLOLE and frequency of loss of load on a yearly basis.

Problem 4

$$HLOLE = LOLP * 8760 \text{ (hrs)}$$

In order to find the LOLP with generation reserve model, build load model in a cumulative way, similar with generation model

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Problem 4

With generation model and load model, we could construct generation reserve model

Capacity \ Load	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

Problem 4

To find LOLP, calculate probability of $margin \leq -5MW$

Capacity \ Load	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$P(M) = \sum_{i=1}^m [P_i - P_{i+1}] P_l(C - C_i - M)$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$P(-5) = \sum_{i=1}^5 [P_i - P_{i+1}] P_l(C - C_i + 5)$$

When $i=1$

$$(P_i - P_{i+1})P_l(200 - C_i + 5) = (P_1 - P_2)P_l(200 - 0 + 5)$$

Since $P_l(205) = 0$

$$(P_1 - P_2)P_l(205) = 0$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$\begin{aligned}
 P(-5) &= (P_2 - P_3)P_l(155) + (P_3 - P_4)P_l(105) + (P_4 - P_5)P_l(55) + (P_5)P_l(5) \\
 &= \left(\frac{4641}{14641} - \frac{641}{14641}\right)\frac{1}{6} + \left(\frac{641}{14641} - \frac{41}{14641}\right)\frac{1}{2} + \left(\frac{41}{14641} - \frac{1}{14641}\right)\frac{1}{2} + \frac{1}{14641} * 1 \\
 &= \frac{1013}{22247} + \frac{300}{14641} + \frac{20}{14641} + \frac{1}{14641} = \frac{193}{2861} = LOLP \\
 HLOLE &= LOLP * 8760 \approx 590.94 \text{ (hrs)}
 \end{aligned}$$

Problem 4

To find FF, calculate frequency of $margin \leq -5MW$

Capacity \ Load	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$F(M) = F^g(M) + F^l(M)$$

$$F^g(M) = \sum_{i=1}^m [F_i - F_{i+1}] P_i (C - C_i - M)$$

$$F^l(M) = \sum_{i=1}^m [P_i - P_{i+1}] F_i (C - C_i - M)$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$F^g(-5) = \sum_{i=1}^5 [F_i - F_{i+1}] P_l(C - C_i + 5)$$

When $i=1$

$$(F_i - F_{i+1})P_l(200 - C_i + 5) = (F_1 - F_2)P_l(200 - 0 + 5)$$

Since $P_l(205) = 0$

$$(F_1 - F_2)P_l(205) = 0$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity \	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$\begin{aligned}
 F^g(-5) &= (F_2 - F_3)P_l(155) + (F_3 - F_4)P_l(105) + (F_4 - F_5)P_l(55) + (F_5)P_l(5) \\
 &= \left(\frac{4000}{14641} - \frac{1200}{14641} \right) \frac{1}{6} + \left(\frac{1200}{14641} - \frac{120}{14641} \right) \frac{1}{2} + \left(\frac{120}{14641} - \frac{4}{14641} \right) \frac{1}{2} + \frac{4}{14641} * 1 \\
 &= \frac{174}{5459} + \frac{540}{14641} + \frac{58}{14641} + \frac{4}{14641} = \frac{397}{5439} / \text{day}
 \end{aligned}$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$F^l(-5) = \sum_{i=1}^5 [P_i - P_{i+1}] F_l(200 - C_i + 5)$$

When $i=1$

$$(P_i - P_{i+1}) F_l(200 - C_i + 5) = (P_1 - P_2) F_l(200 - 0 + 5)$$

Since $F_l(205) = 0$

$$(P_1 - P_2) F_l(205) = 0$$

Load	$P_l(\text{Load} \geq X)$	$F_l(\text{Load} \geq X)$
45	1	0
105	$\frac{1}{2}$	1
155	$\frac{1}{6}$	1

Cap out	Capacity	$P(\text{Cap out} \geq X)$	$F(\text{Cap out} \geq X)$
0	200	1	0
50	150	$\frac{4641}{14641}$	$\frac{4000}{14641}$
100	100	$\frac{641}{14641}$	$\frac{1200}{14641}$
150	50	$\frac{41}{14641}$	$\frac{120}{14641}$
200	0	$\frac{1}{14641}$	$\frac{4}{14641}$

Load Capacity	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$\begin{aligned}
 F^l(-5) &= (P_2 - P_3)F_l(155) + (P_3 - P_4)F_l(105) + (P_4 - P_5)F_l(55) + (P_5)F_l(5) \\
 &= \left(\frac{4641}{14641} - \frac{641}{14641} \right) 1 + \left(\frac{641}{14641} - \frac{41}{14641} \right) 1 + \left(\frac{41}{14641} - \frac{1}{14641} \right) 1 + \frac{1}{14641} * 0 \\
 &= \frac{4000}{14641} + \frac{600}{14641} + \frac{40}{14641} = \frac{4640}{14641} / \text{day}
 \end{aligned}$$

Problem 4

To find FF, calculate frequency of $margin \leq -5MW$

Capacity \ Load	155	105	45
200	45	95	155
150	-5	45	105
100	-55	-5	55
50	-105	-55	5
0	-155	-105	-45

$$F(-5) = F^g(-5) + F^l(-5) = \frac{1337}{3429} / \text{day} \approx 0.3899 / \text{day} = 142.317 / \text{year}$$

Problem 3&4 Comparison

It can be observed that conditional probability approach and generation reserve model arrived at the same results.

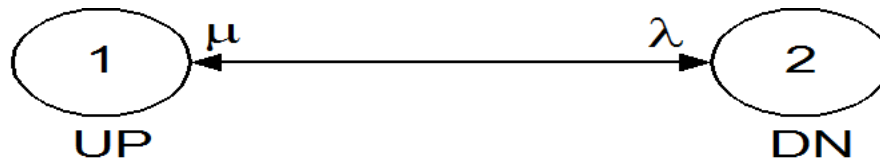
- Conditional probability approach is more straightforward for hand calculation, while generation reserve model is easy to adapt with computer computation
- This problem solving session is also a demonstration of how to use generation reserve model

Problem 5

There one generators with full capacity of 50 MW and 0 MW when failed, its failure rate is 0.1/day and mean repair time is 24 hours. Assume generator was in up state at $t=0$

Find:

1. Frequency of failure's function with time
2. Frequency of repair's function with time



Problem 5

For the generator

$$\lambda = 0.1/\text{day}, \mu = 1/\text{day}$$

$$p_u(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

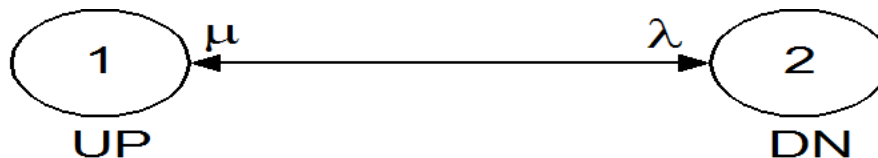
$$p_d(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Then for frequency of failure

$$f_f(t) = p_u(t)\lambda = \frac{\lambda\mu}{\lambda + \mu} + \frac{\lambda^2}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

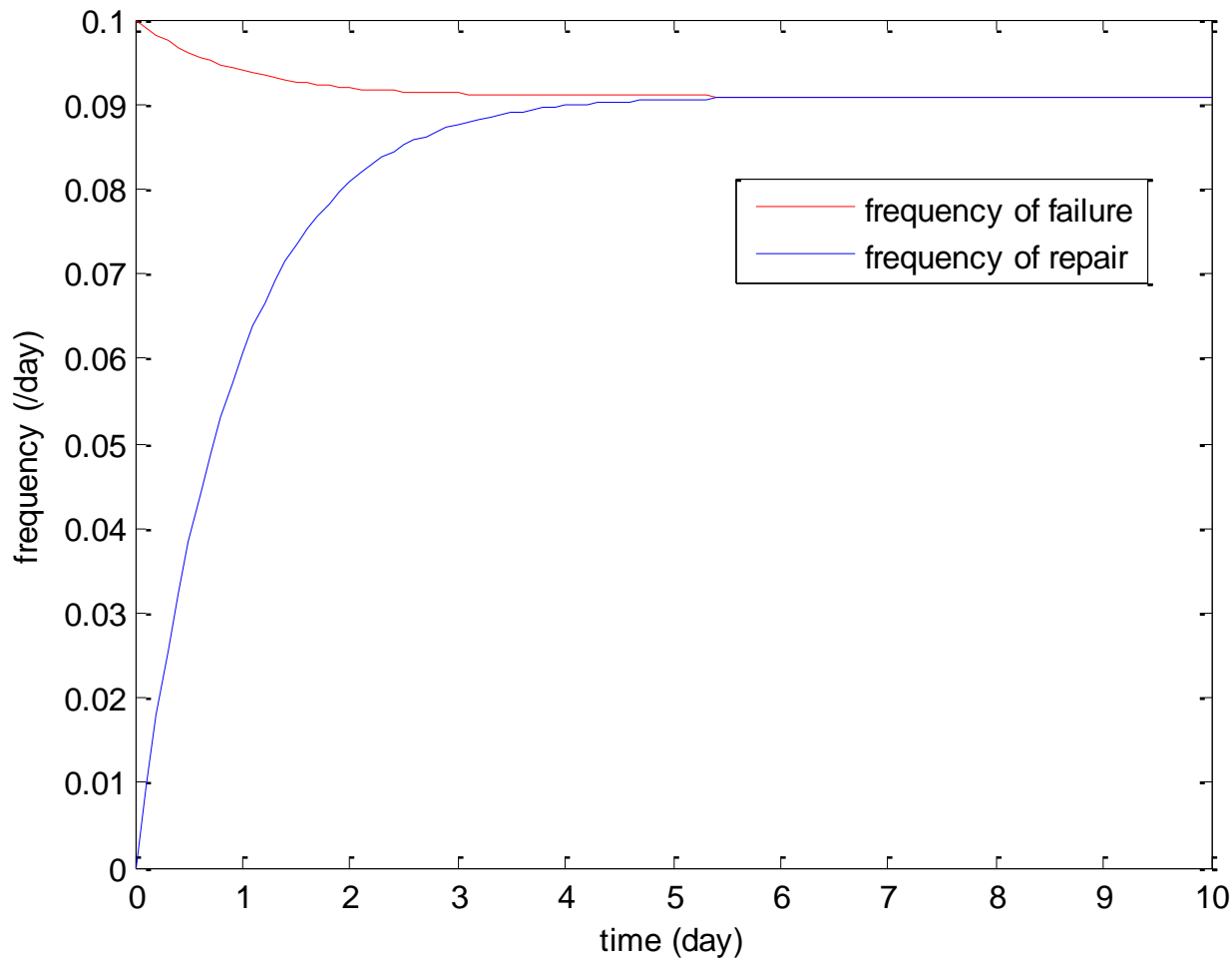
For frequency of repair

$$f_r(t) = p_d(t)\mu = \frac{\lambda\mu}{\lambda + \mu} - \frac{\lambda\mu}{\lambda + \mu} e^{-(\lambda + \mu)t}$$



Problem 5

The idea is to show that frequency balance is only valid when it reaches steady state



Thank you!

