Module 6-2
Discrete Convolution Method

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Rounding the Capacity Outage States

- 1. The Proposed Method

- The objective here is to construct the generation system model such that the capacity outage states are multiples of a specified increment. In this manner the number of capacity outage states will be reduced.
- This achieved by rounding the states of the unit which are not multiples of the incremental step to their nearest lower and higher incremental step.
- A two state unit with its down state not a multiple of the incremental step, with this method of round off, will result into a three state unit with modified transition rates. Similarly a three state unit may result into a three state, four state or a five state unit.

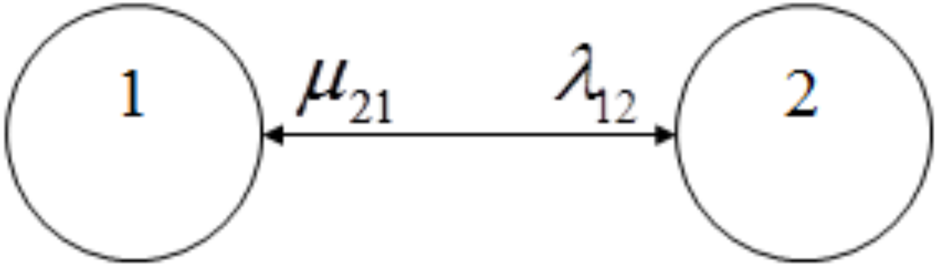
Rounding the Capacity Outage States

- The transition rates are modified in the inverse ratio of the differences between the incremental steps and the unit capacity outage.
- After the transition rates are obtained the probabilities and the frequencies of all the states of unit can be determined. Then this unit is added to the existing generation model using the unit addition algorithm described earlier. The cumulative vectors P and F associated with the generation system model are obtained directly.
- The method of obtaining the modified transition rates is explained later with examples of two state and a three state units separately. Since determining the modified transition rates for a unit which has more than three states is cumbersome, general equations for calculating the cumulative probabilities and frequencies directly after adding the unit are also given.
- From the viewpoint of computational efficiency, it is preferable to use explicit models for two or three state units.

Rounding the Capacity Outage States

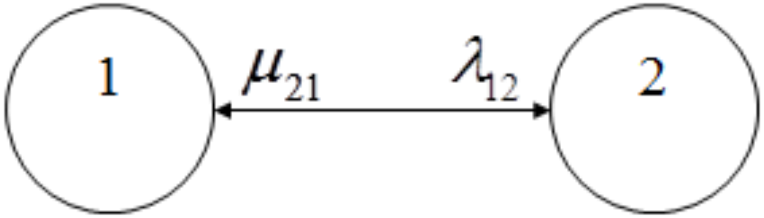
- 2. Two State Unit

- Suppose that generation system model is to be built with capacity outage states multiples of 10 MW. So a two state unit with capacity outages of 0, 56 MW will result into a three state unit with capacity outages of 0, 50, 60 MW. The transition rate diagram without round off is shown in the following figure where, λ_{ij} represents the failure rate from state i to state j and μ_{ij} represents the repair rate from state i to state j.



Two-State Transition Diagram before Rounding Off

Rounding the Capacity Outage States



Two-State Transition Diagram before Rounding Off

- The probabilities and the frequencies of the states 1 and 2 of the unit are obtained as follows.

$$p_1 = \frac{\mu_{21}}{\lambda_{12} + \mu_{21}}$$

$$p_2 = \frac{\lambda_{12}}{\lambda_{12} + \mu_{21}}$$

$$f_{12} = \frac{\lambda_{12}\mu_{21}}{\lambda_{12} + \mu_{21}}$$

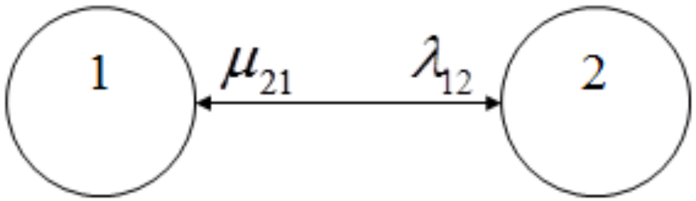
$$f_{21} = \frac{\lambda_{12}\mu_{21}}{\lambda_{12} + \mu_{21}}$$

Where,

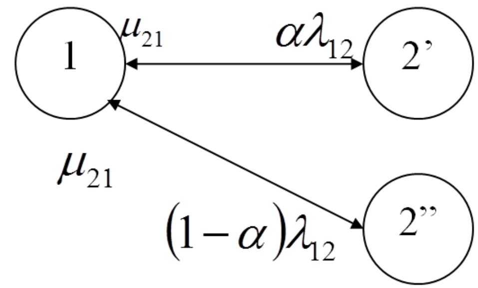
p_i = probability of the unit being in state i

f_{ij} = frequency of transition from unit state i to j

Rounding the Capacity Outage States



Two-State Transition Diagram before Rounding Off

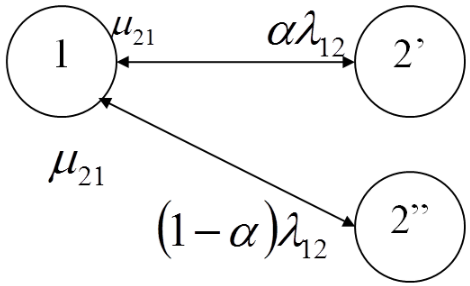


- In the left figure state 2 represents the 56 MW outage capacity. The transition rate diagram after round off is shown in the rightfigure. It can be seen from figure, that the state 2 is split into two states 2' and 2". It should be observed that there is no transition between states 2' and 2".
- α is the parameter which modifies the transition rates. It is the ratio of the difference between the capacity outage of state 2" and unit capacity outage and the increment.
- In this particular case,

$$\alpha = (60 - 56)/10 = 0.4$$

$$1 - \alpha = 0.6$$

Rounding the Capacity Outage States

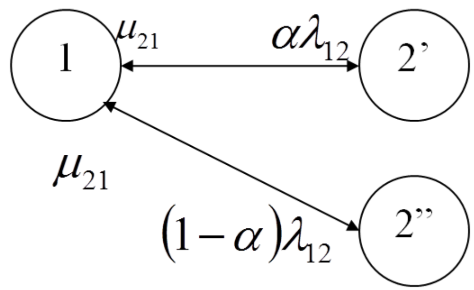


$$\alpha = (60-56)/10 = 0.4$$

$$1-\alpha = 0.6$$

- The transition rates are modified in the inverse ratio of the differences between the capacity outage state of the unit and the capacity outages in the rounded off states. This results in the mean value of capacity outage states 2' and 2" equal to the capacity outage off state 2. In the case of the example, since 56 is closer to 60 than to 50, the number of transitions to state 60 will be more than to state 50. But the total number of transitions to states 2' and 2" should be equivalent to the number of transitions to state 2. So the number of transitions to these modified states should be a fraction of the number of transitions to state 2. This is obtained by multiplying the transition rate by α and $1-\alpha$ which are in the inverse proportion of the differences between the exact capacity outage state and the new incremental capacity outage states.

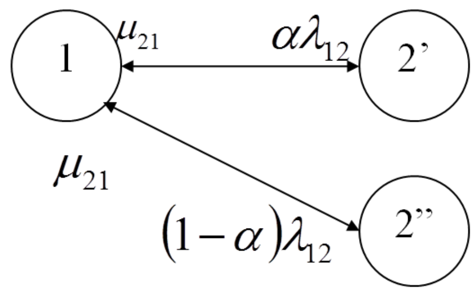
Rounding the Capacity Outage States



- So the transition rates in the above figure are obtained as

$$\lambda_{12'} = \alpha\lambda_{12}$$
$$\lambda_{12''} = (1-\alpha)\lambda_{12}$$
$$\mu_{2''1}, \mu_{2'1} = \mu_{12}$$

Rounding the Capacity Outage States



- It is clear that the repair rates are not modified, since the rate of repair remains the same in both the states. It can be verified that the sum of the transitions to states 2' and 2" in the original figure is equal to the transitions to state 2 in the modified figure i.e.,

$$\lambda_{12} = \lambda_{12'} + \lambda_{12''} = \alpha\lambda_{12} + (1-\alpha)\lambda_{12}$$

- It can be easily shown that the mean sum of the capacity outages of the two rounded off states is equal to the original capacity outage state, i.e.,

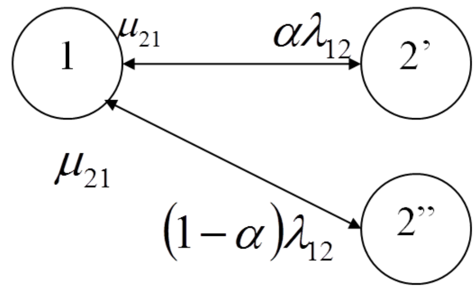
$$c_i = \alpha c_{i1} + (1-\alpha)c_{i2}$$

Where

c_i = original capacity outage state

c_{i1}, c_{i2} = capacity outage states after rounding off

Rounding the Capacity Outage States



- Since the transition rates are modified, the probabilities and frequencies also get modified accordingly as follows.

$$p_{2'} = \frac{\alpha\lambda_{12}}{\lambda_{12} + \mu_{21}} = \alpha p_2$$

$$p_{2''} = \frac{(1-\alpha)\lambda_{12}}{\lambda_{12} + \mu_{21}} = (1-\alpha) p_2$$

$$f_{12'}, f_{2'1} = p_{2'}\mu_{21} = \alpha f_{21}$$

$$f_{12''}, f_{2''1} = p_{2''}\mu_{21} = (1-\alpha) f_{12}$$

- After the modifications are made, the unit is added as usual using the unit addition algorithm.

Rounding the Capacity Outage States

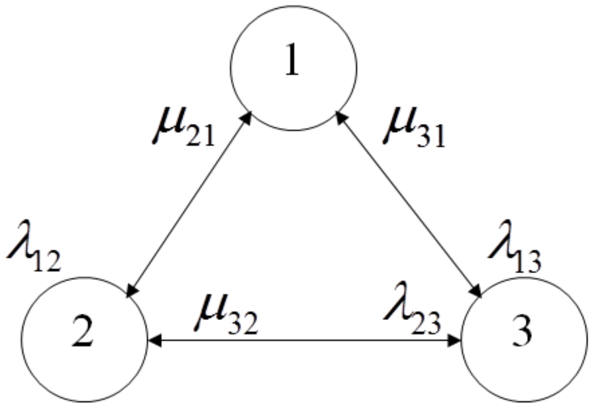
- 3. Three State Unit

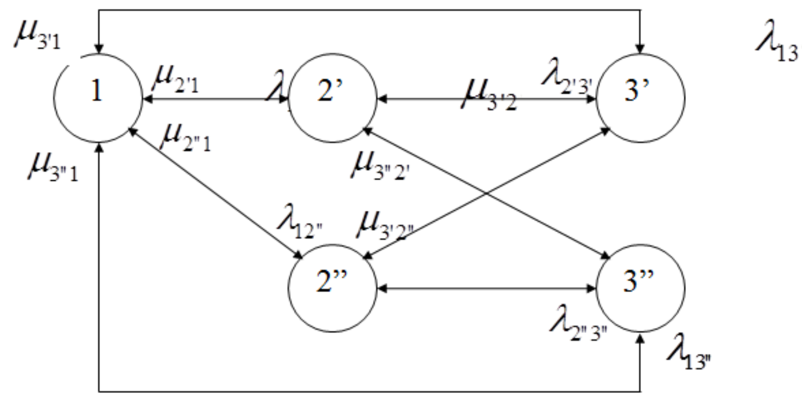
- In this case, a three state unit after rounding off, may result in either a four state unit or a five state unit depending on the capacity outages of the derated and down states. When a state of a unit is an integral multiple of the incremental step there is no need to round off that state. The various possibilities are explained below using an incremental step of 10 MW.
  - When either derated capacity outage or full capacity outage is not an integral multiple of the incremental step, then after rounding off a four state unit will result. For example in case of a unit with capacity outage states of 0, 32, and 50 MW, after rounding off, there will be 0, 30, 40, and 50 MW capacity outage states as shown in the Figures below.

Rounding the Capacity Outage States

- 3. Three State Unit
  - When all the states are not integral multiples of the incremental step then after rounding off, a five state unit results. Example of such a unit is one with capacity outage state of 0, 17, and 43 MW.
  - There is one more possibility that when both the derated and full capacity outage states are in the same range of the incremental step then the resulting unit has only three states. For example a unit with 0, 13, and 19 MW state unit with increment of 10 MW results in 0, 10 and 20 MW state unit. The transition rates in this case have to be modified in a slightly different way when compared to other cases.

Rounding the Capacity Outage States

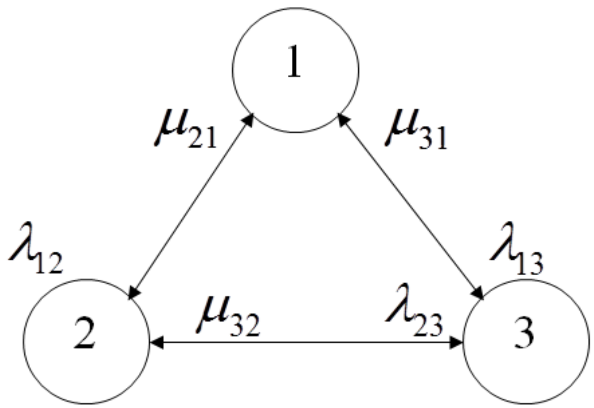


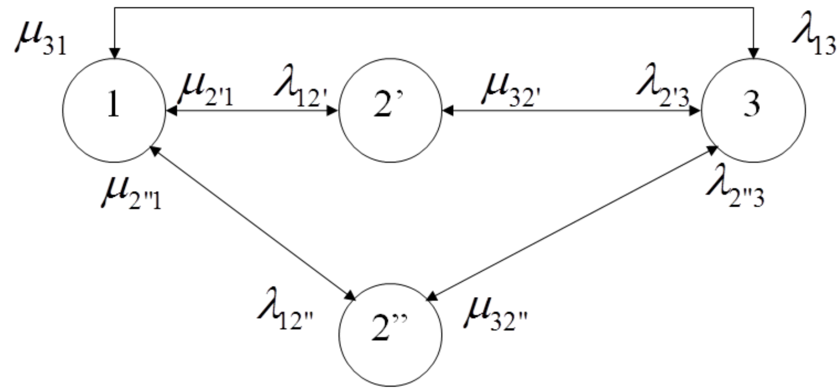


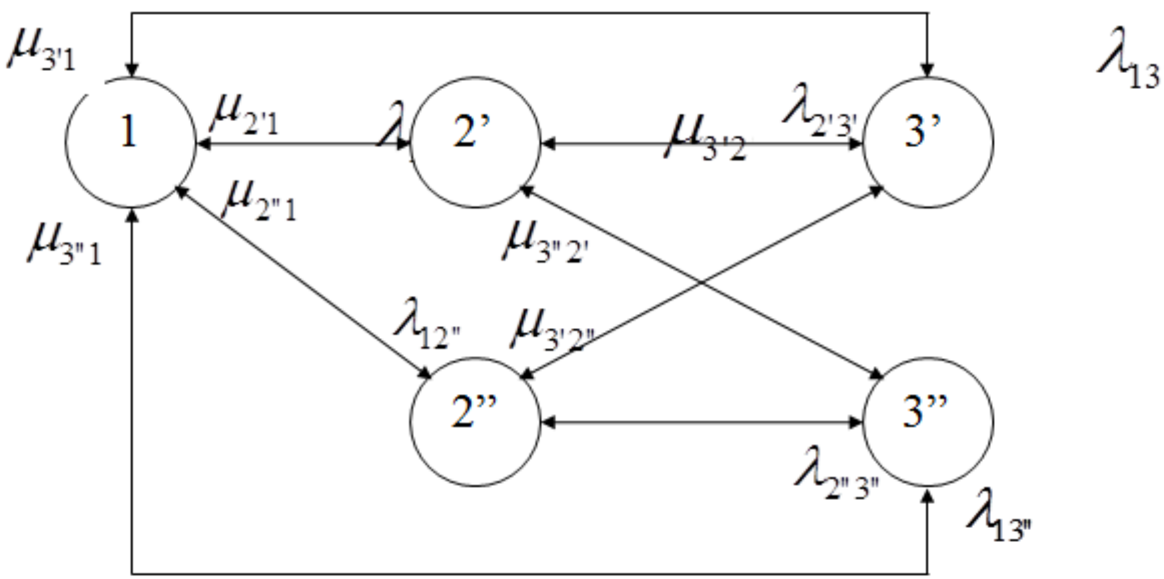
State Transition Diagram after Rounding Off

- The unit round off for a three state unit can be described assuming a three state unit in which the derated and full capacity outage states are not integral multiples of the incremental step of say 10 MW. The state transition diagram without rounding off is as shown in the left figure. The transition diagram after rounding off is as shown in the right figure . The states 2, and 3 in the following figure are split into states 2’, 2”, and 3’, 3” respectively.

Rounding the Capacity Outage States

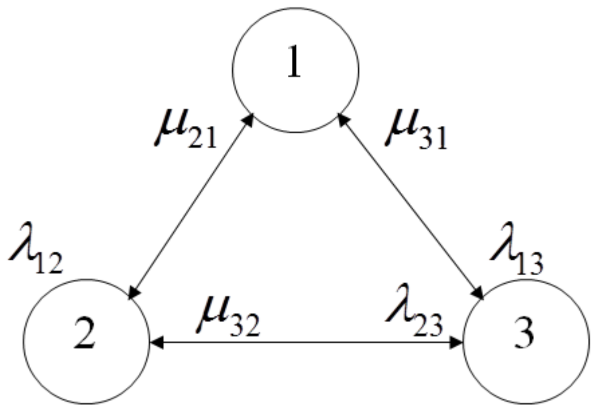


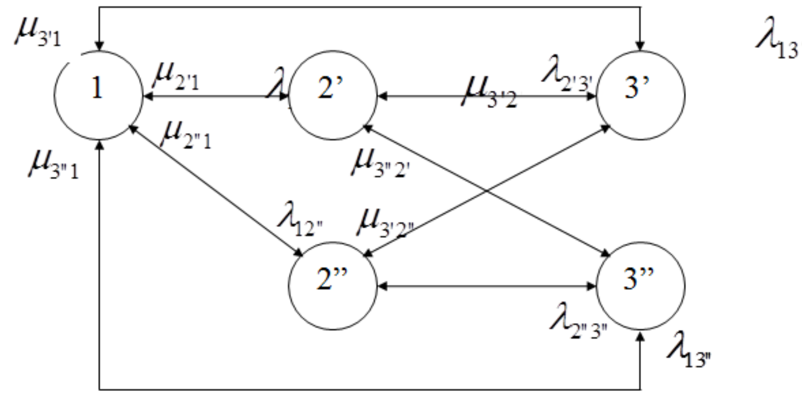




State Transition Diagram after Rounding Off

Rounding the Capacity Outage States

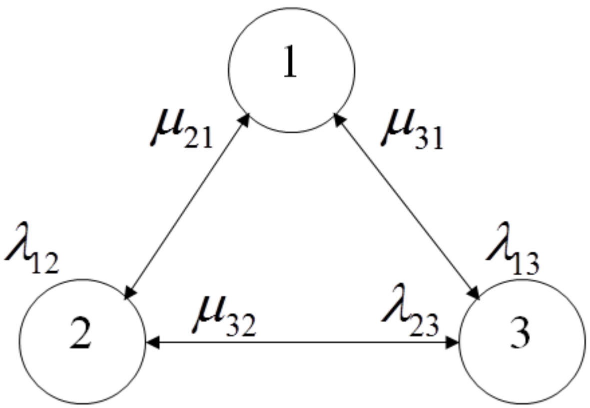


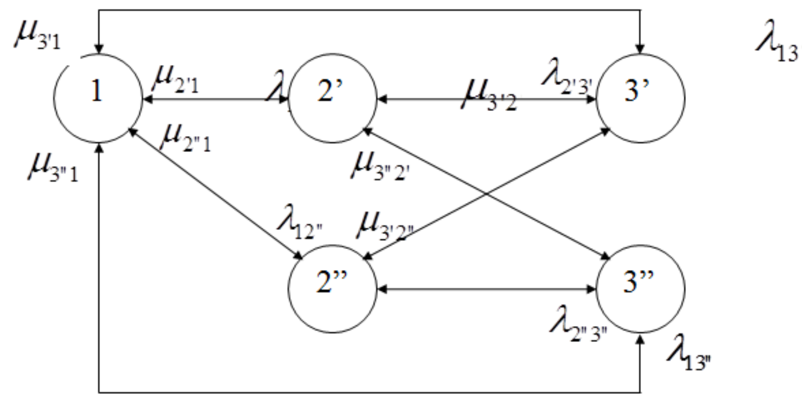


State Transition Diagram after Rounding Off

- As in the case of two state unit, α parameter is required. But in this case two such parameters are required. Let them be called α_1 and α_2 where α_1 is associated with the derated state, and α_2 is associated with the full capacity outage state. Both of them are calculated as described earlier.
- After α_1 , α_2 have been obtained, the various transition rates are calculated as shown. Transition rates $\lambda_{12'}$, $\lambda_{12''}$, $\mu_{2'1}$, $\mu_{2''1}$ are obtained using the following equations, with α replaced by α_1. The other transition rates are obtained using the following equations,

Rounding the Capacity Outage States



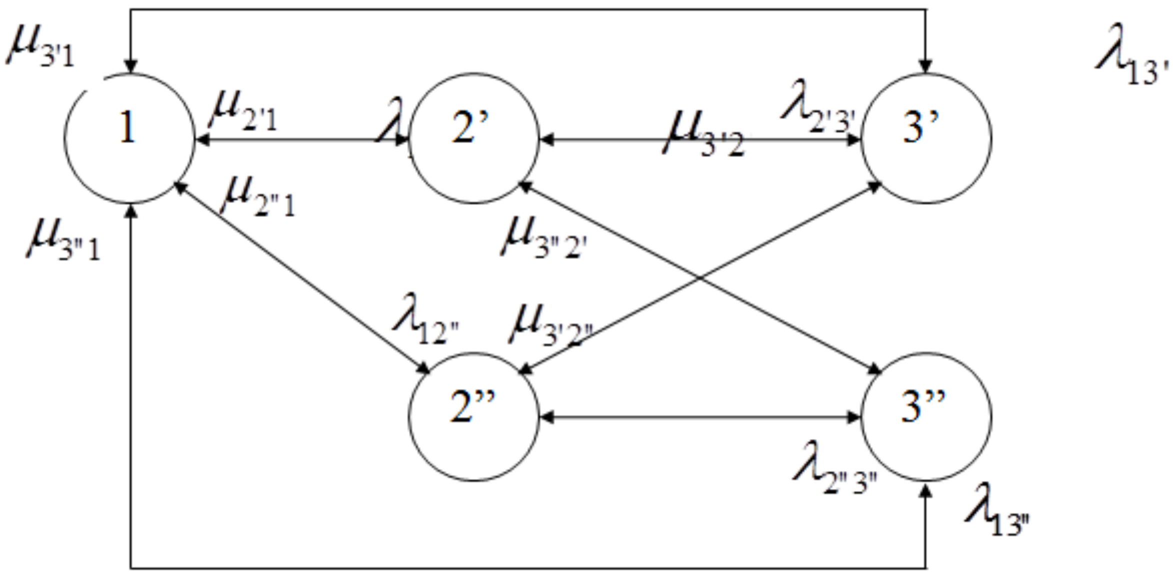


State Transition Diagram after Rounding Off

$$\lambda_{13'} = \alpha_2 \lambda_{13}$$
$$\lambda_{13''} = (1-\alpha_2)\lambda_{13}$$
$$\lambda_{2'3'} = \alpha_2 \lambda_{23}$$
$$\lambda_{2'3''} = (1-\alpha_2)\lambda_{23}$$
$$\lambda_{2''3'} = \alpha_2 \lambda_{23}$$
$$\lambda_{2''3''} = (1-\alpha_2)\lambda_{23}$$

$$\mu_{3'1} = \mu_{31}$$
$$\mu_{3''1} = \mu_{31}$$
$$\mu_{3'2'} = \alpha_1 \mu_{32}$$
$$\mu_{3'2''} = (1-\alpha_1)\mu_{32}$$
$$\mu_{3''2'} = \alpha_1 \mu_{32}$$
$$\mu_{3''2''} = (1-\alpha_1)\mu_{32}$$

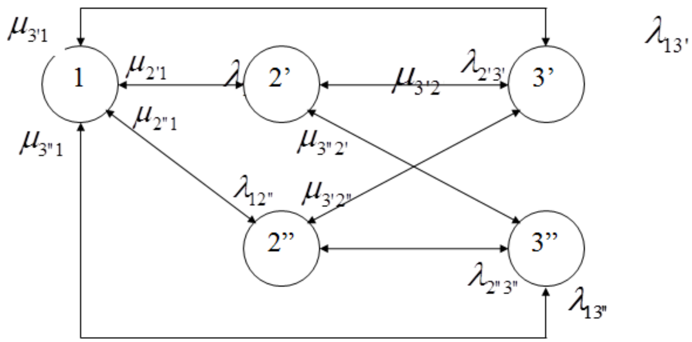
Rounding the Capacity Outage States



State Transition Diagram after Rounding Off

- The frequencies and probabilities of the equivalent unit can be expressed in terms of these values for the original units using equations.

Rounding the Capacity Outage States



State Transition Diagram after Rounding Off

- The frequencies and probabilities of the equivalent unit can be expressed in terms of these values for the original units using equations. Then this unit with modified probabilities and frequencies is added to the existing generation system model.

$$f_{2'1} = \alpha_1 f_{21} \qquad f_{3'2'} = \alpha_1 \alpha_2 f_{32} \qquad p_{2'} = \alpha_1 p_2$$

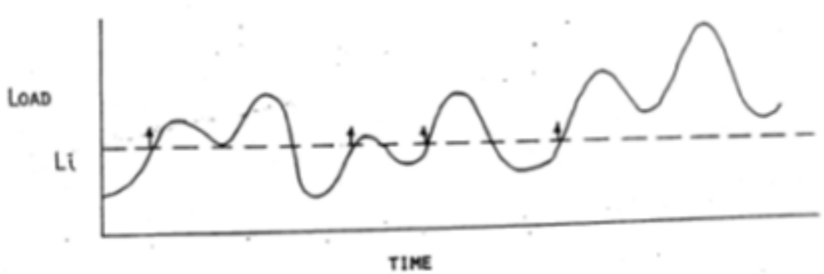
$$f_{2''1} = (1-\alpha_1) f_{21} \qquad f_{3'2''} = (1-\alpha_1)\alpha_2 f_{32} \qquad p_{2''} = (1-\alpha_1) p_2$$

$$f_{3'1} = \alpha_2 f_{31} \qquad f_{3''2'} = \alpha_1 (1-\alpha_2) f_{32} \qquad p_{3'} = \alpha_2 p_3$$

$$f_{3''1} = (1-\alpha_2) f_{31} \qquad f_{3''2''} = (1-\alpha_1)(1-\alpha_2) f_{32} \qquad p_{3''} = (1-\alpha_2) p_3$$

Load Modeling

- Developed from hourly load data
- Developed for discrete load levels Li
- P (Li), F (Li) =Probability and frequency of load greater than or equal to Li MW.
- Li+1 - Li = Z, constant increment



Load Model

$$P_\ell(L_i) = \frac{\text{Number of Hours} \geq L_i}{\text{Total Hours in the Interval}}$$

$$F_\ell(L_i) = \frac{(\text{Number of Transitions from Load Less than } L_i \text{ to Load} \geq L_i)}{\text{Total Hours in the Interval}}$$

Load Modeling

- Load Modeling Algorithm

The load modeling algorithm scans the hourly data and develops the load model of the following form

P (Li), F (Li) = probability and frequency of load equal to or greater than Li

Where

Li+1 - Li = Z, constant

That is, the load model is built for discrete load levels at an increment of Z MW such that L1 = 0, L2 = Z, L3 = 2Z, . . . Li = (i-1) Z. Assume that H_i is the load during the hour i and that load model is to be built from hourly data from hour N1 to N2. The load modeling algorithm consists of the following steps.

Load Modeling

1. NIC = Peak load/Z + 1

   NIC is, therefore, the number of discrete load levels.
2. Set P(Lj), F(Lj) = 0 for j = 1 to NIC.
3. Set i= N1 where i is the hour under consideration.
4. Calculate contribution of ith hour to P(Lj).

   Compute P(Lj) = P(Lj) + 1 for j = 1 to J where J = H_i /Z + 1.
5. Calculate contribution to F(Lj) due to load change from hour i to i +1.

   J1 = H_{i+1} /Z + 1

   J = J + 1

   (i) If J1 < J, then $H_{i+1} \leq H_i$ and therefore there is no contribution to F(Lj). Go to 6.

   (ii) If J1 ≥ J, then $H_{i+1} > H_i$ and there is contribution to frequency.

   Compute F(Lj) = F(Lj) + 1, for j = J to J1.

Load Modeling

6. Advance the hour, $i = i + 1$.

   If $i \leq N2$ go to 4.

7. $P(Lj) = P(Lj)/DEN$

   $f(Lj) = f(Lj)/DEN$

   where $DEN = N2 - N1 + 1$

   The P(Lj) is now probability and F(Lj) is frequency per hour.

This is a very efficient algorithm for computer implementation and develops the entire load model by a single scan of the hourly load data.

Generation Reserve Model

Example

Load → Capacity ↓	100	50	0	
200	100	150	200	
150	50	100	150	← Margins
100	0	50	100	= Cap- Load
50	-50	0	50	
0	-100	-50	0	

↖

Margin $\leq$ -50MW

Generation Reserve Model

$$P(M) = \sum_{i=1}^{m} [P_i - P_{i+1}]\, P_l(C - C_i - M)$$

$$F(M) = F^g(M) + F^l(M)$$

Where

$$F^g(M) = \sum_{i=1}^{m} [F_i - F_{i+1}]\, P_l(C - C_i - M)$$

$$F^l(M) = \sum_{i=1}^{m} [P_i - P_{i+1}]\, F_l(C - C_i - M)$$

P(M), F(M) = probability and frequency margin less than or equal to M

$F^g(M)$, $F^l(M)$= components of F(M) due to generation and load changes respectively

C = installed capacity minus capacity on planned outage.

C_i = capacity outage.

m= number of states in the generation system model.

Generation Reserve Model - Example

Load → Capacity ↓	100	50	0	
200	100	150	200	
150	50	100	150	← Margins
100	0	50	100	= Cap- Load
50	-50	0	50	
0	-100	-50	0	

↖

Margin ≤ -50MW

Generation Reserve Model - Example

To calculate probability of margin $\leq -50\mathrm{MW}$

$$\mathrm{P}(-50) = \sum_{i=1}^{5} [P_i - P_{i+1}] P_l(200 - C_i + 50)$$

When i=1

$$(P_i - P_{i+1})P_l(200 - C_i + 50) = (P_1 - P_2)P_l(200 - 0 + 50)$$

Since $P_l(250) = 0$

$$(P_1 - P_2)P_l(200 - 0 + 50) = 0$$

Similarly for i=2,3, $P_l(200) = P_l(150) = 0$

Then

$$\mathrm{P}(-50) = (P_4 - P_5)P_l(100) + (P_5 - P_6)P_l(50)$$

$(P_6 = 0)$

It can be observed this result match with conditional probability approach

Generation Reserve Model - Example

To calculate frequency of margin $\leq -50\text{MW}$

$$\mathrm{F}(-50) = \mathrm{F}^g(-50) + \mathrm{F}^l(-50)$$

Start with $\mathrm{F}^g(-50)$

$$\mathrm{F}^g(-50) = \sum_{i=1}^{5} [F_i - F_{i+1}]\, P_l(200 - C_i + 50)$$

Similar with probability calculation

$$\mathrm{F}^g(-50) = (F_4 - F_5)P_l(100) + (F_5 - F_6)P_l(50)$$

Since $F_6 = 0$

$$\mathrm{F}^g(-50) = F_4 P_l(100) + F_5[P_l(50) - P_l(100)]$$

It can be observed this result match with conditional probability approach

Generation Reserve Model - Example

For $\mathrm{F}^{l}(-50)$

$$\mathrm{F}^{l}(-50) = \sum_{i=1}^{5} [P_i - P_{i+1}]\, F_l(200 - C_i + 50)$$

When i=1

$$(P_i - P_{i+1})F_l(200 - C_i + 50) = (P_1 - P_2)F_l(200 - 0 + 50)$$

Since $F_l(250) = 0$

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Then

$$\mathrm{F}^{l}(-50) = (P_4 - P_5)F_l(100) + (P_5 - P_6)F_l(50)$$

$(P_6 = 0)$

It can be observed this result match with conditional probability approach

Generation Reserve Model - Example

Load → / Capacity ↓	100	50	0
200	100	150	200
150	50	100	150
100	0	50	100
50	-50	0	50
0	-100	-50	0

← Margins = Cap- Load

F_4 $F_l(100)$ F_5 $F_l(50)$

Margin ≤ -50MW

Generation Reserve Model

The other indices can now be computed as follows:

1. HLOLE

$$HLOLE = P(-\delta) \cdot D$$

where

$-\delta$ = negative margin with smallest absolute value

D = duration of the period of study

2. LOLE

The LOLE can be computed by creating an hourly load curve with load for each hour equal to the daily peak load. Then the load model derived from this hourly load curve is convolved with the generation system model and

$$LOLE = P(-\delta)\, D/24$$

Generation Reserve Model

3. EUE

The expected unserved energy is calculated as,

$$\text{EUE} = \Delta\text{MD}\{\sum_{M=0}^{-L} P(M) - \frac{1}{2}[\text{P}(0) + \text{P}(-\text{L})]\}$$

where

ΔM = increment at which P(M) is computed

D = length of time interval in hours

-L = negative margin with largest absolute value (margins with probabilities less than 10^{-7} can be generally ignored)

Load Forecast Uncertainty

Deviations from the base-line forecast are assumed normally distributed with a mean value of zero.

The standard deviation of the normal distribution is specified as a percent of the annual peak load.

Seven step approximation of normal distribution for uncertainty in load forecasting.

1. Standard Deviation $= \dfrac{\text{\% Uncertainty x Forecast Load, MW}}{100}$

Probability of Actual Load = Forecast

Load Forecast Uncertainty

No. of S.D. From the Mean	Load + Number of Standard Deviations in Col. 1
-3	0.006
-2	0.061
-1	0.242
0	0.382
+1	0.242
+2	0.061
+3	0.006

Create Load Scenarios

n = number of standard deviations corresponding to step i of the discrete probability distribution

Then

H_{ji} = load during hour j corresponding to step i

= H_{jbase} . FMUL

where

H_{jbase} = base line forecast load for hour j

and

FMUL = 1.0 + n (Percent uncertainty)/100

Find Equivalent Load Model

$$\mathbf{P}_\ell\ (\mathbf{L_i}) = \sum_{k=1}^{n} P_{\ell k}(L_i)A_k$$

And

$$\mathbf{F}_\ell\ (\mathbf{L_i}) = \sum_{k=1}^{n} F_{\ell k}(L_i)A_k$$

where

A_k = Probability of kth step of the discrete distribution used to approximate the normal distribution of load forecast uncertainty.

$P_k\ (L_i)$, $F_k\ (L_i)$ = probability and frequency of load equal to or greater than Li MW, for the load scenario corresponding to the kth step.

n = number of steps in the discrete distribution for uncertainty.

Additional Features

- Emergency Assistance
- Firm Imports and Exports
- Planned Outage Scheduling
- Interruptible Loads
- Energy Limited Units

Emergency Assistance

Can be modeled as equivalent multi-state generating unit whose capacity levels, probabilities and frequencies correspond to the possible emergency assistance levels and their probabilities and frequencies.

The equivalent generator is added to the system capacity model at the last so as not to influence the duty cycle of an energy-limited generators and to reflect the unscheduled, last resort, nature of emergency assistance.

Firm Imports and Exports

Firm imports and exports are subtracted from or added to the basic hourly loads.

Planned Outage Scheduling

Reserve levelization while enforcing the forbidden period constraints.

Effective capacity of each unit is used.

$$\mathbf{EC_i = effective\ capacity\ of\ the\ unit\ i}$$
$$\mathbf{= C_i - M \cdot \log R}$$

where

$$\mathbf{R = 1 - FOR_i\,(1 - e^{C_i/M})}$$

M = slope of the capacity outage probability graph with capacity on the axis (linear) and probability on the x-axis (log scale).

and

FOR_i = forced outage rate of unit i

Planned Outage Scheduling

(1) The product of effective capacity and planned outage duration is found for each unit and the resulting products are summed for each plant.

(2) Plants are ranked in priority order based on the sums of unit effective capacity times duration, largest first. Likewise, units are ranked in priority order within a plant on the basis of rated capacity times duration, largest first.

(3) Unit planned outages are then scheduled sequentially using the plant priority order and unit priority within the plant. The scheduling process minimizes the sum of weekly peak load and effective capacity on planned outage in any week while observing the constraints on forbidden periods and only one unit per plant out in any week.

Interruptible Loads

Arrange peaks in descending order.

Choose N highest peaks. N is the number of interruptions allowed.

Interruptible loads are subtracted during these N periods.

Energy Limited Units

Ti = expected period of need for unit i

$= P_{i-1}(0) \bullet I$

where

$P_{i-1}(0)$ = probability of capacity deficiency with (i-1) units committed. This can be calculated by convolving the system capacity model containing the first (i-1) units with the interval load model.

I = Length of study interval in hours.

The study interval is taken to be one week or 168 hours in the GRIP program.

Energy Limited Units

Now, assuming that the unit operates at its maximum output during the period of need, the expected energy output, Ei, is given by

$$E_i = T_i\,[C_i\,P_i + D_i \bullet DFOR_i]$$

where:

C_i, D_i = full and derated capacity of unit i

P_i = probability unit i fully available

$= 1 - FOR_i – DFOR_i$

FOR_i, $DFOR_i$= probability unit i is full outage or is derated

Energy Limited Units

The next step is to compare the energy requirements, E_i, of the unit with the energy (output) available to the unit, Z_i. There are two possibilities:

(1) If $E_i \leq Z_i$, the unit is not energy limited for reliability purposes and the unit capacity model can be combined with the system capacity model reflecting the first (i) units without modification.

(2) If $E_i > Z_i$, the unit is energy limited and the unit state probabilities FOR_i and $DFOR_i$ must be adjusted to reflect the reduced proportion of time the unit can operate at full available capacity output.

Energy Limited Units

The procedure for adjusting the FORi and DFORi parameters is outlined as follows. If Zi is the total (output) energy available to unit i, then:

Z_{id} = energy allocated to the derated state

$$= \mathbf{Z_i} \frac{D_i \cdot DFOR_i}{D_i \cdot DFOR_i + C_i P_i}$$

and

Z_{iu} = energy allocated to the fully available state

$$= \mathbf{Z_i} \frac{C_i P_i}{D_i \cdot DFOR_i + C_i P_i}$$

Energy Limited Units

The modified probability of unit i being in the fully available state, P i, can then be found as follows:

$$\mathbf{P'_i T_i C_i = Z_{iu}}$$

and so

$$\mathbf{P'_i} = \frac{Z_{iu}}{T_i C_i}$$

$$= \frac{Z_i P_i}{(D_i \cdot DFOR_i + C_i P_i) I \cdot P_{i-1}(0)}$$

Energy Limited Units

Similarly, the modified probability of unit i being in the derated state, DFORi, is given by

$$\mathbf{DFOR}'_i = \frac{Z_{id}}{T_i D_i}$$

$$= \frac{Z_i \, . \, DFOR_i}{(D_i \cdot DFOR_i + C_i P_i) I \cdot P_{i-1}(0)}$$

It should be noted, that both Pi and DFORi are modified by the same factor to obtain P'_I and $DFOR'_i$. Because unit state probabilities must add to one, the modified probability of the full outage state, FOR'_i, is given by

$$\mathbf{FOR}'_i = 1 - \mathbf{P}'_i - \mathbf{DFOR}'_i$$