Module 6-1
Discrete Convolution Method

Chanan Singh
Texas A&M University

Basic Approach

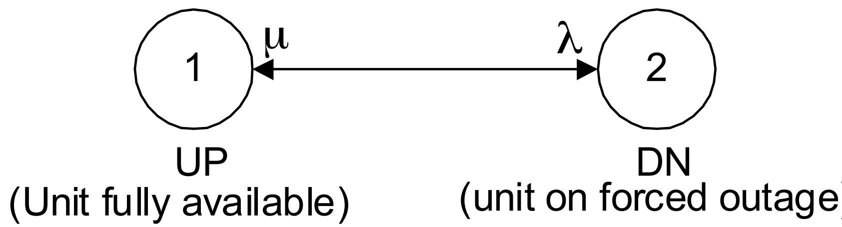
- Generation System Model
- Load Model
- Generation Reserve Model

Other Features are Included by

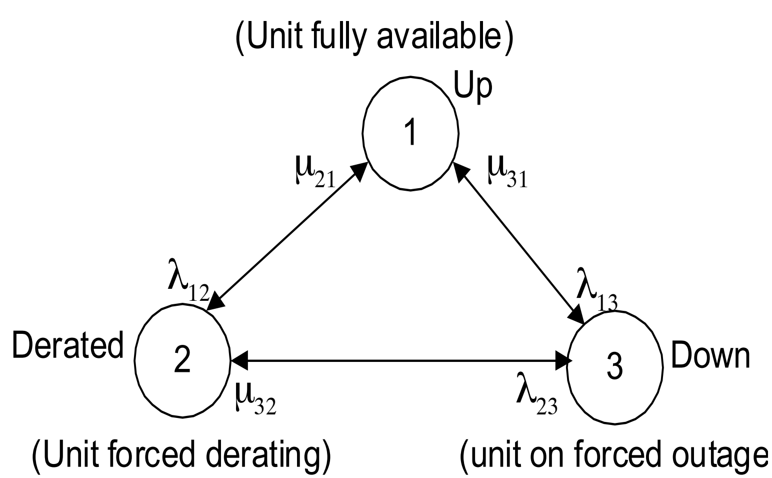
- Modifying Generation System Model
  - Examples:
  - Energy Limited Units
  - Emergency Assistance
- Adjusting Load Model,
  - Examples:
  - Load Forecast Uncertainty
  - Interruptible Loads
  - Firm Interchanges

Generating Unit Model

Two State Unit Model



Three State Unit Model



Generating Unit Model

- If only probabilities are to be computed, then either λ and μ parameters can be used or DFORs (derated forced outage rates) and FORs (forced outage rates) of the units can be utilized.

Then for,

- Two-State Unit

  p2 = FOR

- Three-State Unit

  p2 = DFOR

and

  p3 = FOR

Generating Unit Model

- Another approach which has been used in the past, and is still used by many utilities, is to model forced deratings as equivalent full forced outages. In this approach, all units can be represented by two-state models with

$$p2 = DAFOR$$

where

$$DAFOR = \text{equivalent FOR}$$

Generating Unit Model

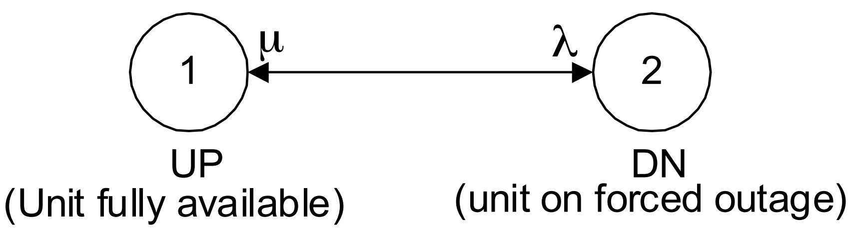
- P_i = probability of the unit state i
- f_{ij} = frequency of transition from unit state i to j

- Two-State Unit

$$p_1 = \frac{\mu}{\lambda + \mu}$$

$$p_2 = \frac{\lambda}{\lambda + \mu}$$

$$f_{12} = f_{21} = \frac{\lambda\mu}{\lambda + \mu}$$



Generating Unit Model

- Three-State Unit

$$P_1 = \frac{Q_1}{Q_1+Q_2+Q_3}$$

$$P_2 = \frac{Q_2}{Q_1+Q_2+Q_3}$$

$$P_3 = \frac{Q_3}{Q_1+Q_2+Q_3}$$

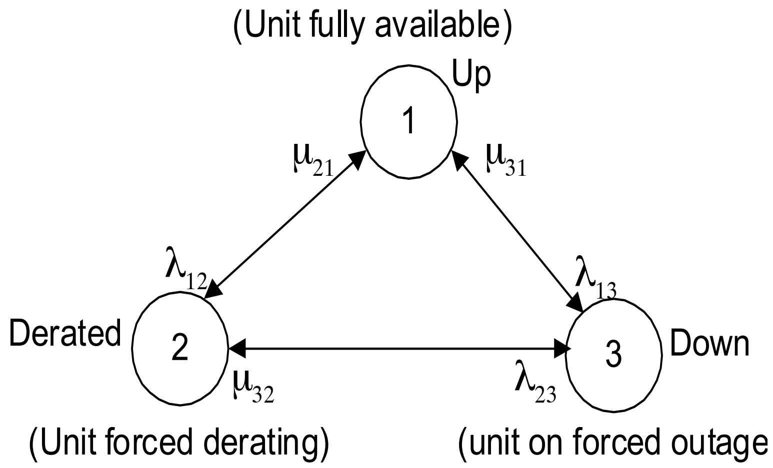
where

$$Q_1 = \mu_{21}\mu_{32} + \mu_{31}\mu_{21} + \mu_{31}\lambda_{23}$$

$$Q_2 = \mu_{32}\lambda_{12} + \mu_{32}\lambda_{13} + \lambda_{12}\mu_{31}$$

and

$$Q_3 = \lambda_{12}\lambda_{23} + \lambda_{13}\mu_{21} + \lambda_{13}\lambda_{23}$$



Modeling Immature Units

- $V(t) = Vi - m * t$

where

V(t)	= parameter value at time t
Vi	= initial value of the parameter
m	= (Vi - Vf)/D
Vf	= mature value of the parameter

and

D	= duration of immaturity

Generation System Model

- Described by three vectors C, P and F:

Ci = ith element of C

= one of the discrete capacity outage levels

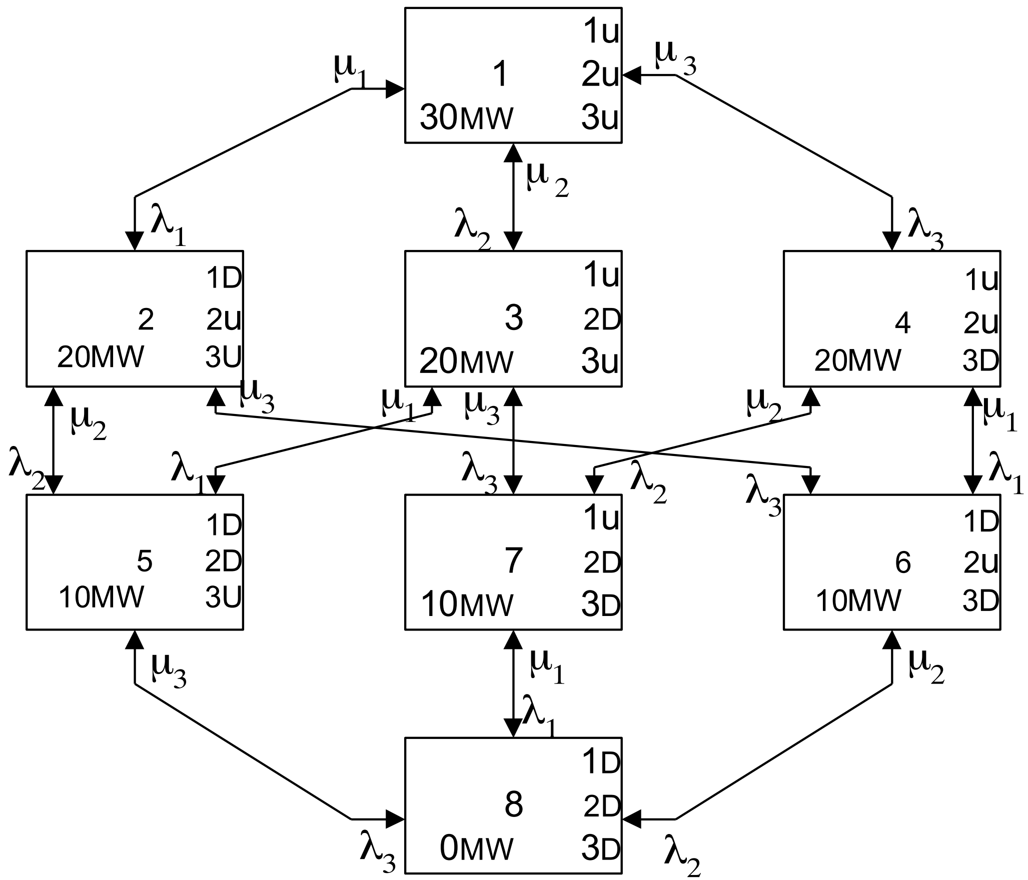
Pi = ith element of P

= probability of capacity outage greater than or equal to Ci

Fi = ith element of F

= frequency of capacity outage greater than or equal to Ci

Exact State and Cumulative Probability and Frequency



Exact State and Cumulative Probability and Frequency

Capacity	Capacity Out	Exact State Prob.	Cum. Prob. P(Cap out $\geq$ X)
30	0	$P'_1 = P\,(\text{Capout} = 0)$	$P_1 = P_2 + P'_1$
20	10	$P'_2 = P\,(\text{Capout} = 10)$	$P_2 = P_3 + P'_2$
10	20	$P'_3 = P\,(\text{Capout} = 20)$	$P_3 = P_4 + P'_3$
0	30	$P'_4 = P\,(\text{Capout} = 30)$	$P_4 = P'_4$

Exact State and Cumulative Probability and Frequency

- Exact State Frequency

$$F\,(Capout = 20) = p_5\,(\mu_1 + \mu_2 + \lambda_3) + p_7\,(\mu_2 + \mu_3 + \lambda_1) + p_6\,(\mu_1 + \mu_3 + \lambda_2)$$

$$F\,(Capout \geq 20) \neq F\,(Capout = 20) + F\,(Capout = 30)$$

Unit Addition Algorithm - Example

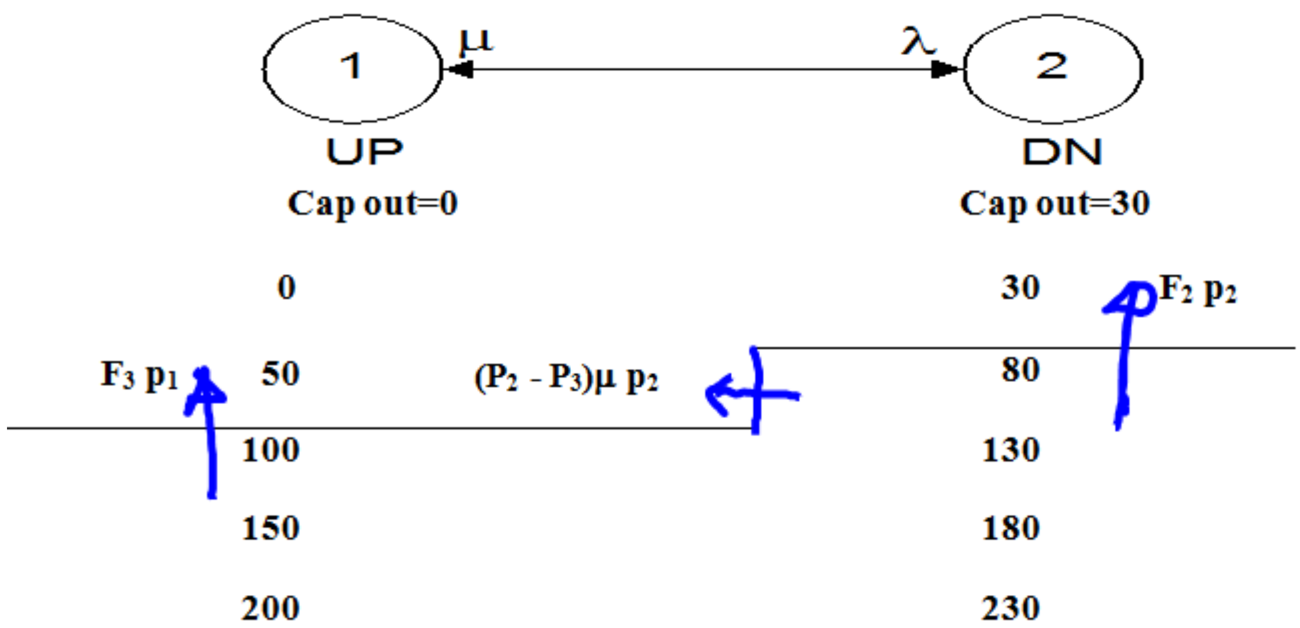
- Assume that a capacity outage table exists:

i	C_i	P_i	F_i
1	0	P_1	F_1
2	50	P_2	F_2
3	100	P_3	F_3
4	150	P_4	F_4
5	200	P_5	F_5

- Add a 2- State Unit of 30 MW

Unit Addition Algorithm - Example

- Add a 2- State Unit of 30 MW



Unit Addition Algorithm - Example

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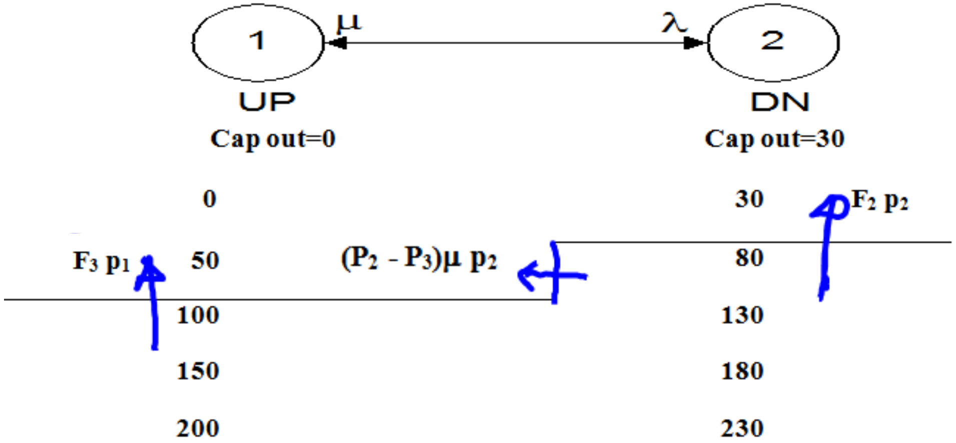

$P(C \geq 80)$ = Probability of capacity outage greater than or equal to 80MW

$= P_2 p_2 + P_3 p_1$

$F(C \geq 80)$ = Frequency of capacity outage $\geq$ 80MW

$= F_2 p_2 + F_3 p_1 + (P_2 - P_3)\mu p_2$

Unit Addition Algorithm

- The algorithm described in this section is used for embedding a unit model in the generation system model.

- Assume that the capacity outage states are arranged in ascending order of magnitude. Let x_i be the capacity outage in state i. The addition of a 3-state unit, results in three subsets of states (refer to Fig 1.).

$$S_1 = \{x_i\}$$
$$S_2 = \{x_i + C_D\}$$
$$S_3 = \{x_i + C_T\}$$

where

C_T = capacity of unit being added.
C_D = amount of capacity lost when unit being added is derated.

Unit Addition Algorithm

	S_1		S_2		S_3	
	X_1		$X_1 + C_D$		$X_1 + C_T$	
	X_2		$X_2 + C_D$		$X_2 + C_T$	
	X_3		$X_3 + C_D$		$X_3 + C_T$	
	•		•		•	
	•		•	$F_j p_2$	•	$F_k p_3$
	X_k		$X_k + C_D$		$X_k + C_T$	
	•		•	$(P_k-P_j)f_{32}$	•	
$F_i p_1$	X_j		$X_j + C_D$		$X_j + C_T$	
	•	$(P_j-P_i)f_{21}$	•		•	$(P_k-P_i)f_{31}$
	X_i		$X_i + C_D$		$X_i + C_T$	
	•		•		•	
	•		•		•	

Fig. 1 State Frequency Diagram for Unit Addition

Unit Addition Algorithm

- These three subsets, arranged as three columns in Fig. 1, have an equal number of states and in each the capacity outages are arranged in an ascending order. Assuming that a capacity equal to or greater than X is defined by states equal to and greater than i, j and k in S1, S2 and S3 respectively,

$$P(X) = P_i\, p_1 + P_j\, p_2 + P_k\, p_3$$

and

$$F(X) = G(X) + N(X)$$

where

$$G(X) = F_i\, p_1 + F_j\, p_2 + F_k\, p_3$$

$$N(X) = (P_j - P_i)\, f_{21} + (P_k - P_i)\, f_{31} + (P_k - P_j)\, f_{32}$$

and

P_i, F_i = probability and frequency of capacity outage equal to or greater than x_i.

- It should be noted that G(X) represents the frequency due to change in the states of the units other than the unit being added.

Exact State and Cumulative Probability and Frequency

Capacity	Exact State Prob.	Cum. Prob. P(Cap out $\geq$ X)
30	$P'_1 = P\,(Cap = 30)$	$P_1 = P_2 + P'_1$
20	$P'_2 = P\,(Cap = 20)$	$P_2 = P_3 + P'_2$
10	$P'_3 = P\,(Cap = 10)$	$P_3 = P_4 + P'_3$
0	$P'_4 = P\,(Cap = 0)$	$P_4 = P'_4$

Exact State Frequency

$$F\,(Cap = 10) \;=\; p_5\,(\mu_1 + \mu_2 + \lambda_3) + p_7\,(\mu_2 + \mu_3 + \lambda_1) + p_6\,(\mu_1 + \mu_3 + \lambda_2)$$

$$F\,(Cap \leq 10) \;\neq\; F\,(Cap = 10) + F\,(Cap = 0)$$